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A Review on Efflux Time

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Abstract: In Chemical industry, storage vessels appear in a large variety of geometries. The reasons for the choice of the typical shape or geometry may be attributed to convenience, insulation requirements, floor space, material costs, corrosion and safety considerations. The time required to drain these vessels off their liquid contents is known as efflux time and this is of crucial importance in many emergency situations besides productivity considerations. Literature reports theoretical and experimental works for arriving at efflux time. The present review focuses on the literature available on efflux time. The scope for future work is also presented in this paper.

Key words: Chemical industry · Storage vessels · Efflux time · Productivity · Geometry

INTRODUCTION

Processing and storage vessels in the chemical and related industries appear in a large variety of shapes. The time required to drain these vessels off their liquid contents is known as efflux time [1] and this is of crucial importance in many emergency situations besides productivity considerations. This is of considerable interest in a variety of industries like chemical, food and pharmaceutical [2]. Mathematical analysis of efflux time for Newtonian liquid (below its bubble point) through restricted orifice for annular (both horizontal and vertical) containers is carried out by Hart and Sommerfeld and [1]. The authors mentioned that two fundamental equations must be invoked while solving draining problems from the storage tank. The first equation is related to the mass balance and a second equation is to the discharge coefficient. The two equations reported were $A\frac{dh}{dt} = -VA_0$ where A and A_0 refer to cross sectional area of tank and restricted orifice respectively, V refers to the linear velocity of liquid in the tank and $\frac{dh}{dt}$, the time variation of liquid level in the tank and h is the height of liquid in the tank and $V = C_0\sqrt{2gh}$ where C_o refers to discharge coefficient. They arrived at a general mathematical expression for efflux time given by $t = \frac{1}{C_0 A_0\sqrt{2g}}\int_0^H \frac{A}{\sqrt{h}}dh$, where H is the height of liquid before draining. If C_o=1, the equation for efflux time becomes Toricelli theorem.

Sommerfeld [3] derived efflux time equation for five new configurations. They are parallelepiped, vertical elliptical cylinder, regular tetrahedron, pyramid and paraboloid. The author stated that such shapes may be of use for academic purposes.

When drainage occurs through an orifice drain hole located at the bottom of the vessel, formulas for computing the drainage time required have been summarized by foster [4] for a number of vessel shapes -vertical, horizontal, cylindrical and spherical.

They mentioned that the discharge coefficient (C_d) is constant for Newtonian fluid in turbulent flow, but it depends on the shape of orifice. They considered discharge coefficient as 0.61 for sharp edged orifice, 0.8 for short flush mounted tube and 0.98 for rounded orifice.

However, Delozier *et al.* [5] reported a C_d value of 0.75 for an undergraduate experiment on efflux time through a drain hole at the bottom of a horizontal cylinder with flat ends.

Work is also reported for comparing the efflux times for different geometries of vessels through a circular hole. The author compared the efflux time for cylindrical,

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spherical, cone and inverse cone, hemispherical shapes of for tanks [6] and derived the following expression for efflux time (T)

$T = K\frac{V}{S\alpha\sqrt{gh}}$ V is the volume of liquid in the tank, S is the cross sectional area of tank and h is the height of the liquid in the tank and K is the coefficient given by

$$K = \frac{\int_0^H \frac{A(u)du}{\sqrt{u}}}{\int_0^H A(u)du}\sqrt{H}$$

Higher values of K suggest higher draining time. They arrived at K values of 3.2 for inverse cone, 2 for cylinder, 1.6 for sphere, 1.4 for hemisphere.

Jouse [7] carried out the mathematical analysis of efflux time for draining a cylindrical tank through restricted orifices of different diameters. The mathematical analysis is based on the assumption of pseudo steady state conditions. They mentioned that the pseudo steady state assumption is valid for cross sectional ratio of tank to orifice as low as 100. It is also mentioned that, unlike a free falling particle which travels at constant acceleration during its fall, the free surface of a liquid decelerates continuously while draining. It is also highlighted that during draining of a liquid from a cylindrical tank through restricted orifice, Froude number remains constant and is independent of initial height of liquid in the tank.

Subbarao [8] developed expressions for efflux time for different geometries of vessels through restricted orifice of same diameter. The author reported the following order for efflux time for the geometries considered.

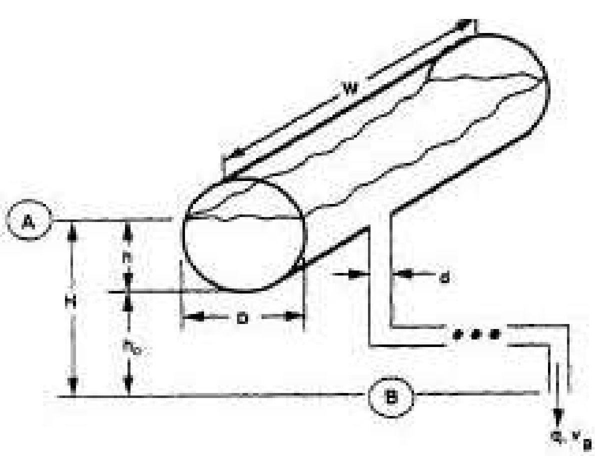

Fig. 1: Horizontal cylindrical tank with drain piping

Efflux time for cylinder >Efflux time for sphere >Efflux time for cone.

Somemrfeld and Stallybrass [2] derived expression for efflux time for the case of a horizontal cylindrical vessel with associated drain piping. The configuration they considered is shown below.

They reported the following equation for time (τ) required to drain the tank from some initial level ($H_1 = h_1 + h_0$) to some final level $H_2(= h_2 + h_0)$

$$\tau = \frac{G(H_1) - G(H_2)}{\alpha}$$

Where the function G is defined as

$$G = \int_{h_0}^{Hi} \left[\frac{2R + h_0 - H)(H - h_0)}{H}\right]^{1/2} dH$$

i = 1or 2

and $\alpha = \frac{s}{2W}\left(\frac{2g}{1 + f\frac{L}{d}}\right)^{1/2}$

Where *s* is cross sectional area of piping, *f* is friction factor and *W* is length of the tank, *L* is the length of piping, *d* is diameter of exit pipe.

Van Donngen and Roche. Jr [9] carried out efflux time analysis from cylindrical tanks with exit pipes and fittings in the Reynolds number range of 40,000-60,000. They mentioned that under turbulent flow conditions in the exit pipe, the efflux time can be related to the height of the liquid (H_1) relative to the bottom of the exit pipe H, by.

$t_{eff} = K_1^* (H_1^{3/7} - H^{3/7})$ where K_1 is a constant given by

$K_1 = \frac{7}{3} * (\frac{r}{r_0})^2 [\frac{0.0791 L_e \mu^{1/4}}{2^{1/4} g \rho^{1/4} r_0^{5/4}}]^{4/7}$ where r is radius of cylindrical tank, r_0 is the radius of exit pipe, L_e is the total equivalent length of the exit pipe and fittings. They also mentioned that the goal of the experiment is that an abstract term L_e does have physical significance, a term that can be directly measured and observed through proper data analysis. They further stated that the analysis of set of data from different runs where the length of exit pipe has been changed can clearly demonstrate the concept of 'entrance effect'. The equivalent length of the pipe can be calculated that would have the same pressure drop or flow resistance caused by entrance effect.

When change in friction factor is ignored and an average value is used that that represents the average of flow regimes and pipe roughness, they used the following equation for efflux time.

$t_{eff} = K_2^* (H_1^{1/2} - H^{1/2})$ where $K_2 = 2(\frac{r}{r_0})^2 \left[\frac{f_{avg} L_e}{g r_0}\right]^{1/2}$, r is radius of cylindrical tank, r_0 is radius of exit pipe, f_{avg} is average friction factor and L_e is equivalent length. They used the following equation for friction factor for calculating the efflux time.

$$f = \frac{0.0316}{\mathrm{Re}^{0.25}}$$

They also mentioned that under such high Reynolds numbers, there is a possibility of existence of vena-contracta immediately downstream of contraction or piping that might contain trapped air or vapour.

Morrison [10] also modelled the efflux time equation for using computational techniques through an exit pipe for turbulent flow in the exit pipe. The Reynolds number considered is around 6,400. The author considered a contraction coefficient of 3.8 while arriving at efflux time. The maximum efflux time reported is 35 seconds. The tank to pipe cross sectional area in the work is 228.

The tank drainage problem in a cylindrical tank is studied in detail by Joye and Barret [11]. They derived the following equation for efflux time for draining the contents of a storage vessel through exit pipe for turbulent flow in the exit pipe.

$$t_{eff} = \frac{D^2}{d_2}\sqrt{\frac{2\left(4fL/d + \sum K\right)}{g}}\left(\sqrt{H_i + L_v} - \sqrt{H_f + L_v}\right)$$

t_{eff} is the efflux time to drain the tank from fluid height H_i to H_f, L_v is the vertical drop of the exit pipe, D is the diameter of tank, d is diameter of exit pipe, k is the resistance coefficient to account for fittings in the line, L is the length of straight pipe. While deriving the above expression, they assumed the friction factor to remain constant. They considered a contraction coefficient value of 1.5. They reported a deviation of 8% between experimental values and their model for turbulent flow in the exit pipe. They also used the following efflux time equation reported by Bird *et al.* for laminar flow in the exit pipe for verifying the experimental values.

$t = \frac{32\mu L D^2}{\rho g d^4}\ln(1 + \frac{H}{L})$, t is efflux time, D is the diameter of tank, L is the length of exit pipe, μ and ρ are the viscosity and density of liquid respectively. H is the initial height of liquid in the tank.

Mathematical equation efflux time from a cylindrical tank (Where the flow is essentially laminar) for turbulent flow through an exit pipe is reported by subbarao and co-researchers [12]. Their analysis is based on macroscopic balances. They mentioned that macroscopic balances are used for making preliminary estimate of an Engineering problem. They also made an assumption of constant friction factor for deriving the expression. They simplified the efflux time equation to the following form

$t = \sqrt{\frac{2}{g_m}}\left(\sqrt{H+L} - \sqrt{L}\right)$ and named it as modified form of Torricelli equation. g_m is the modified form of acceleration due to gravity and is given by $g_m = \frac{g}{\left(1 + 4f\frac{L}{d}\right)\left(\frac{A_t}{A_p}\right)^2}$, f is the friction factor in the pipe line and L is the length of exit pipe and A_t and A_p are cross sectional areas of tank and exit pipe respectively. They defined Froude number as $\frac{g_m}{g}$ α $(Fr)^2$ where (Fr) denotes the Froude number. They mentioned that the equation so developed will be of use for finding the minimum time required for draining the contents of the storage vessel. While deriving the above equation, the authors have not considered the contraction coefficient, friction in the tank, flow within the tank and roughness of the walls. Even though the mathematical equation developed suggests that complete draining could be achieved, they could not achieve complete draining due to surface tension forces.

They performed experiments for a tank of 0.27m dia and exit pipe of dia.4X10^{-3}m. They used the following friction factor equation to verify the validity of the model with experimental work $f = 0.0014 + \frac{0.125}{\mathrm{Re}^{0.32}}$ (known as Drew correlation). They mentioned that the advantage of using the equation is that it is valid in the Range of Reynolds number starting from 3000 to 3X10^6. Even though the mathematical equation developed suggests that complete draining can be achieved, they could not achieve complete draining due to surface tension forces.

The authors fine tuned the above friction factor equation and developed the Fouling equation to validate the experimental data.

$$f = 0.0014 + \frac{0.125}{\mathrm{Re}^{0.25}}$$

The equation so developed took into account the contraction coefficient, the flow with in the cylindrical tank and the friction in the pipe line. They verified the validity of the fine-tuned friction factor equation for 0.32m [13] and 0.34m [14] dia tanks while keeping the exit pipe dia at $4X10^{-3}$m. They also performed experiments for 0.25m, 0.5m, 0,75m and 1m lengths of exit pipes. The deviations between theoretical and experimental efflux times with fine tuned friction factor equation is observed to be less for 0.75m and 1m length of exit pipes possibly due to establishment of fully developed flow. The deviation is more for 0.5m dia and 0.25m dia exit pipes. They also noted that as the diameter of the tank increased, the deviation between theoretical and experimental values of efflux time also reduced. This is because as the diameter of the tank is increased, pseudo steady state conditions can prevail.

They observed that when a liquid is drained from a cylindrical tank through an exit pipe, Froude number remained constant and is only influenced by length and diameter of the exit pipe. They further studied the effect of water soluble polyacrylamide (PAM) polymer on drag reduction. The concentrations of PAM considered are 40, 30, 20 & 10 ppm and arrived at 10ppm optimum concentration. In the concentration range considered, they assumed the polymer solutions to behave like Newtonian fluids. They also mentioned that polymer solutions decrease the efflux time and hence increase the Froude number.

The authors also developed the following equation for efflux time [15].

$$t = \sqrt{\frac{2}{g_m}}\left(\sqrt{H+L} - \sqrt{L}\right) \text{ where } g_m = \frac{g}{\left(1 + 4f\frac{L}{d} + K_c\right)\left(\frac{A_t}{A_p}\right)^2} \text{ where}$$

K_c is the contraction coefficient.

The authors considered the contraction coefficient values of 1.5 reported by Joye and Barret and 3.8 reported by Morrison and mentioned that their experimental values were close to theoretical values for a contraction coefficient value of 3.8. They used the following friction factor equation reported by Bird *et al*. [16] for calculating the friction factor which in turn is used for calculating the efflux time.

$$f = \frac{0.046}{\mathrm{Re}^{0.25}}$$

They noticed that the error in efflux time equation using the friction factor equation reported by Bird *et al*. is much more than the friction factor equation reported by Drew.

Gopal singh and co-workers [17] performed experiments for draining a Newtonian liquid through exit piping system using polyacrylamide and polythene oxide polymer solutions. They reported that polythene oxide is a better drag reducing agent for laminar flow in the exit pipe where as polyacrylamide is a better drag reducing agent when the flow is turbulent. They observed that optimum concentration using polyacrylamide is 10ppm for laminar flow and 5 ppm for turbulent flow. The optimum concentration using polythene oxide is 20ppm for laminar flow and 40ppm for turbulent flow.

Subbarao and co-researchers performed experiments for understanding the hydrodynamics of a Newtonian liquid while draining through two exit pipe system for turbulent flow in the exit pipe [18]. They derived the following equation for efflux time.

$$t = \sqrt{\frac{2}{g_m'}}\left(\sqrt{H+L} - \sqrt{L}\right)$$

$$g_m' = \frac{g}{\left(1 + 8f\frac{L}{d}\right)\left(\frac{A_t}{A_p}\right)^2}$$

g_m' is modified form of acceleration due to gravity for two exit pipe system. While deriving the above equation, they considered equal dia. of exit pipes and hence made an assumption that the velocity of fluid in each of the pipes is same. However, the authors did not verify this assumption. They used friction factor equation reported by Drew while evaluating the efflux time.

They carried out studies for two exit pipes each of $4X10^{-3}$m dia and single exit pipe length of 0.75m. They observed a maximum deviation of 12.7% between experimental values and theoretical values of efflux time. The less deviation is due to reduced cross sectional area for flow leading to possibility of eliminating the vortices at the entrance of the exit pipe. They also mentioned that the ratio of efflux times for single exit pipe system with out polymer additions to that of two exit pipe system is at 1.7 for the tank diameters considered. They also compared efflux time for single exit pipe system in presence of polymers to that of two-exit pipe system (without polymers) and arrived at

1.3 for the tank diameters considered. They studied the effect of polyacrylamide polymer on drag reduction. They performed experiments with 10, 5, 2.5 & 1 ppm polymer solutions and arrived at an optimum of 10ppm. They also concluded that maximum drag reduction is 24% for two exit pipe system as against 26% for single exit pipe system. They also carried out experiments with polymer solutions for exit pipe diameter of $8X10^{-3}$m and a tank of 0.32m. They noticed no reduction in drag. Hence, they concluded that drag reduction is effective only for ratios of cross section of tank to exit pipe >1600. This ratio also establishes the saturation limit of Froude number upon addition of water soluble polyacrylamide solution.

Subbarao [19] reported that while draining a liquid from a large cylindrical storage vessel through an exit pipe, the flow in the tank is essentially laminar and turbulent in the pipe depending on the diameter of the exit pipe and physical properties of the liquid to be drained. The author also stated that during draining, the liquid experiences friction and this friction is a measure of drag. This drag increases drastically when flow transforms from laminar in the tank to turbulent in the exit pipe. Hence, drag reduction options are to be explored. They performed their experiments with water for carrying out efflux time studies, since water is a Newtonian fluid and happens to be used in most of the applications. Besides this, it is a good solvent that offers excellent resistance to shear degradation of polymer additives.

Subbarao [20] mentioned that for the case of a liquid drained from a storage vessel (where the flow is essentially laminar) through an exit pipe (when the flow is turbulent), drag reduction takes place in the laminar side i.e in the tank. They reported a maximum drag reduction of 26% with single exit pipe using polyacrylamide polymer solution and a maximum drag reduction of 24% in the two exit pipe system.

Subbarao *et al.* [21] also used the friction factor equation reported by Bird *et al*. [16] for verifying the mathematical equation for efflux time for two-exit pipe system. They mentioned that the error in arriving at theoretical efflux time using is more than that calculated based on the friction factor equation reported by drew.

Subbarao and co-researchers [22] developed equation for efflux time of the following form of Toricelli equation and named it as modified form of Toricelli equation.

$$t = \sqrt{\frac{2}{g_m'}}\left(\sqrt{H+L} - \sqrt{L}\right)$$

$$g_m' = \frac{g}{\left(1 + 8f\frac{L}{d} + K_c\right)\left(\frac{A_t}{A_p}\right)^2}$$

Where K_c is the contraction coefficient. The authors considered 0.27m, 0.32m and 0.34m dia. tanks and two exit pipes each of $4X10^{-3}$m dia. They reported that Contraction coefficient value is influenced by ratio of cross sectional area of tank to pipe and hence used different values of contraction coefficient for different ratios of tank to pipe cross section for calculating the efflux time and comparing with the experimental values.

For two-exit pipe system, Subbarao *et al*. [23] derived the following equation for efflux time (t_{eff}) for the case of turbulent flow in the exit pipe

$$\frac{t_{eff}}{\left(\frac{\mu d^2}{\rho g^4}\right)^{1/7}} = 0.6044 * \left[\left(1 + \frac{H}{L}\right)^{3/7} - 1\right]\frac{A_t}{A_p}\frac{L}{d}$$

$\theta_1 = 0.8133 * \left[\left(1 + \frac{H}{L}\right)^{3/7} - 1\right]\frac{A_t}{A_p}\frac{L}{d}$ Where dimension less time θ is given by

$$\theta = \frac{t_{eff}}{\left(\frac{\mu d^2}{\rho g^4}\right)^{1/7}}$$

Where H is the initial height of liquid in the tank, L is the length of the exit pipe, A_t is the cross sectional area of tank, A_p is the cross sectional area of exit pipe and d & L are diameter and length of the exit pipe respectively.

The above equation even though is derived for variable friction factor can also be used for constant friction factor as well.

They performed experiments for fixed exit pipe lengths and reported a maximum deviation of 16% between theoretical and experimental values. They also mentioned that the variation of Reynolds number (and hence the friction factor) with initial height of liquid is marginal and hence the assumption of constant friction factor is justified. They also mentioned that during draining, the Froude number remains constant and influenced by diameter and length of the exit pipe.

Subbarao *et al*. [24] reported efflux time models for draining a Newtonian liquid from a cylindrical storage vessel (Where the flow is laminar) through an exit piping system (When the flow is turbulent) without assuming constant friction factor. The efflux time equation is written in terms of dimensionless groups as shown below.

Where

$\theta_1 = 0.8133 * \left[\left(1+\frac{H_1}{L}\right)^{3/7} - 1\right]\frac{D_1^2}{d^2}\frac{L}{d}$ Where dimension less time θ_1 is given by

$$\theta_1 = \frac{t_1}{\left(\frac{\mu d^2}{\rho g^4}\right)^{1/7}}$$

D_1 is the dia. of cylindrical tank, d is dia. of exit pipe, H_1 is initial height of liquid in the tank, L is the length of the exit pipe, t_1 is efflux time and ρ is the density of liquid, μ is the viscosity of liquid.

They also derived the following efflux time equation for a conical tank [25]:

$$\theta_2 = 0.35 * \frac{D_2^2}{H_2^2}\left(\frac{L}{d}\right)^3 X_2$$

$$X_2 = \left\{\left[\frac{7}{17}\left(1+\frac{H_2}{L}\right)^{17/7} - 1\right] + \left[\frac{7}{3}\left(1+\frac{H_2}{L}\right)^{3/7} - 1\right] - \left[\frac{7}{5}\left(1+\frac{H2}{L}\right)^{10/7} - 1\right]\right\}$$

Where

Where H_2 the height of liquid in the tank, L is the length of the exit pipe, D_2 is maximum diameter of cone and d is diameter of exit pipe.

The authors also carried out comparison of efflux times between cylinder and cone (for draining through exit pipe of same diameter). The ratio of efflux times is reported to be a function of height of liquid and length of the exit pipe. It is also mentioned that efflux time for cylinder is greater than that of cone.

The authors derived efflux time equation for sphere as well and obtained the following relation [26]:

$$\theta_2 = 1.393\left(\frac{L}{d}\right)^3 X_2$$

Where $\theta_2 = \frac{t_2}{\left(\frac{\mu d^2}{\rho g^4}\right)^{1/7}}$

$$X_2 = \left\{\begin{array}{l} \frac{7R_2}{5L}\left[\left(1+\frac{H_2}{L}\right)^{10/7} - 1\right] - \frac{14R_2}{3L}\left[\left(1+\frac{H_2}{L}\right)^{3/7} - 1\right] \\ -\left[\frac{7}{17}\left(1+\frac{H_2}{L}\right)^{17/7} - 1\right] - \left[\frac{7}{3}\left(1+\frac{H_2}{L}\right)^{3/7} - 1\right] + \left[\frac{7}{5}\left(1+\frac{H_2}{L}\right)^{10/7} - 1\right] \end{array}\right\}$$

Where t_2 is the efflux time for spherical tank, R_2 is radius of sphere; H_2 is height of liquid in the sphere, d is the diameter of exit pipe, L is the length of the exit pipe.

Work is also reported for comparing the efflux time for spherical tank with that of a conical tank for draining the same volume of liquid through an exit pipe [27]. The authors concluded that cone drains faster than a sphere. How faster is the draining time is influenced by height and length of the exit pipe.

Work is also reported for draining a Newtonian liquid through an exit pipe for laminar flow conditions in the exit pipe [28]. It is mentioned that efflux time so obtained will be useful for arriving at the maximum draining time required for draining the contents of the storage vessels. They authors compared the efflux time equations for cylindrical, conical and spherical tank for laminar flow in the exit pipe of same diameter and found the following order for efflux times.

Efflux time for cylinder > efflux time for sphere > efflux time for sphere.

Scope for future work: Future work can be focussed on reduction in efflux time using dilute solutions of different water soluble polymers for conical and spherical tanks. Work can also be focussed on developing models for concentrated polymer solutions for different geometries of vessels through restricted orifice as well as through exit pipe. The efflux time equation so developed can be verified with experimental efflux time values.

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