

## Applying GTMA and Fuzzy Shannon's Entropy for Vendor Selection: A Case Study

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**Abstract:** Vendor selection is one of the most important decision making problems including both qualitative and quantitative factors to identify vendors with the highest potential for meeting a firm's needs consistently and at an acceptable cost and plays a key role in supply chain management (SCM). The purpose of this paper is applying a new integrated method to vendor selection. Proposed approach is based on fuzzy Shannon's Entropy and GTMA (graph theory and matrix approach) methods. Fuzzy Shannon's Entropy method is used in determining the weights of the criteria by decision makers and then rankings of vendors are determined by GTMA method. We apply the integrated approach in a real case to demonstrate the application of the proposed method.

**Key words:** Graph theory and matrix approach (GTMA) • Entropy • Fuzzy logic • Vendor selection

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### INTRODUCTION

In today's highly competitive and interrelated manufacturing environment, the performance of the vendor becomes a key element in a company's success, or failure. Vendor selection decisions are an important component of production and logistics management for many companies. These decisions are typically complicated, for several reasons. First, potential options may need to be evaluated on more than one criterion. A second complication is the fact that individual vendors may have different performance characteristics for different criteria. A third complication arises from internal policy constraints and externally imposed system constraints placed on the buying process. The nature of vendor selection decision usually is complex, unstructured and inherently a multiple criteria problem [1]. Handfield *et al.* [2] illustrated the use of AHP as a decision support model to help managers understand the tradeoffs between environmental dimensions. Gunasekaran *et al.* [3] established a framework consisting of three-level indices: strategic performance, tactical performance and operational performance. Feng *et al.* [4] presented a stochastic integer programming approach for simultaneous selection of tolerances and suppliers based on the quality loss function and process capability indices. Oliveria *et al.* [5] developed a multicriteria model

for assigning new orders to service vendors. Kwang *et al.* [6] combined a scoring method and fuzzy expert systems for vendor assessment and presented a case study. Cebi *et al.* [7] structured vendor selection problem in terms of an integrated lexicographic goal programming (LGP) and AHP model, including both quantitative and qualitative conflicting factors. Cengiz *et al.* [8] applied the fuzzy AHP method for solving the vendor selection problem. Ibrahim *et al.* [9] used activity-based costing and fuzzy present-worth techniques for vendor selection. Kumar *et al.* [10] presented a fuzzy goal programming approach for the vendor selection problem in a supply chain. Ge *et al.* [11] developed an integrated AHP and preemptive goal programming (PGP)-based multicriteria decision-making methodology to account both qualitative and quantitative factors in supplier selection. Pi *et al.* [12] presented a supplier evaluation and selection approach using Taguchi's loss function and AHP. Degraeve *et al.* [13] used total cost of ownership information for evaluating a firm's strategic procurement options. The approach was used to develop a decision support system at a European multinational steel company. Shyur *et al.* [14] proposed a hybrid MCDM model using ANP and TOPSIS methods for strategic vendor selection. Sucky [15] proposed a dynamic decision making approach based on the principle of hierarchical planning for strategic vendor selection. Cao *et al.* [16] discussed the aspects of optimizing vendor

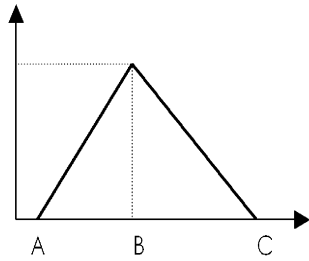


Fig. 1: A triangular fuzzy number  $\mu_{\tilde{A}}(X)$

selection in a two-stage outsourcing process. Wadhwa *et al.* [17] presented multi-objective optimization methods including goal programming and compromise programming. Amid *et al.* [18] proposed a multi-objective linear model for supplier selection in a supply chain. Rao [19] proposed a combined AHP and genetic algorithm (GA) method for the vendor selection problem. The rest of the paper is organized as follows: The following section presents a concise treatment of the basic concepts of fuzzy set theory. Section 3 presents the methodology. The application of the proposed method is addressed in Section 4. Finally, conclusions are provided in Section 5.

**Fuzzy Sets and Fuzzy Numbers:** Fuzzy set theory, which was introduced by Zadeh [20] to deal with problems in which a source of vagueness is involved, has been utilized for incorporating imprecise data into the decision framework. A fuzzy set  $\tilde{A}$  can be defined mathematically by a membership function  $\mu_{\tilde{A}}(X)$ , which assigns each element  $x$  in the universe of discourse  $X$  a real number in the interval  $[0,1]$ . A triangular fuzzy number  $\tilde{A}$  can be defined by a triplet  $(a, b, c)$  as illustrated in Fig 1.

The membership function  $\mu_{\tilde{A}}(X)$  is defined as

$$\mu_{\tilde{A}}(X) = \begin{cases} \frac{x-a}{b-a} & a \leq x \leq b \\ \frac{x-a}{b-a} & b \leq x \leq c \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

Basic arithmetic operations on triangular fuzzy numbers  $A_1 = (a_1, b_1, c_1)$ , where  $a_1 = b_1 = c_1$  and  $A_2 = (a_2, b_2, c_2)$ , where  $a_2 = b_2 = c_2$ , can be shown as follows:

Addition:  $A_1 \oplus A_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$  (2)

Subtraction:  $A_1 \ominus A_2 = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$  (3)

**Multiplication:** If  $k$  is a scalar

$$K \otimes A_1 = \begin{cases} (ka_1, kb_1, kc_1), & k > 0 \\ (ka_1, kb_1, kc_1), & k < 0 \end{cases}$$

$A_1 \otimes A_2 \approx (a_1 a_2, b_1 b_2, c_1 c_2)$ , if  $a_1 \geq 0, a_2 \geq 0$  (4)

Division:  $A_1 \oslash A_2 \sim (\frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2})$ , if  $a_1 \geq 0, a_2 \geq 0$  (5)

Although multiplication and division operations on triangular fuzzy numbers do not necessarily yield a triangular fuzzy number, triangular fuzzy number approximations can be used for many practical applications [21]. Triangular fuzzy numbers are appropriate for quantifying the vague information about most decision problems including personnel selection (e.g. rating for creativity, personality, leadership, etc.). The primary reason for using triangular fuzzy numbers can be stated as their intuitive and computational-efficient representation [22]. A linguistic variable is defined as a variable whose values are not numbers, but words or sentences in natural or artificial language. The concept of a linguistic variable appears as a useful means for providing approximate characterization of phenomena that are too complex or ill-defined to be described in conventional quantitative terms [23].

**Research Methodology:** In this paper, the weights of each criterion are calculated using fuzzy Shannon’s Entropy. After that, GTMA is utilized to rank the alternatives. Finally, we select the best vendor based on these results.

**Fuzzy Shannon’s Entropy Based on A- Level Sets:** Hosseinzadeh *et al.* [24] extend the Shannon entropy for the imprecise data, especially interval and fuzzy data cases. In this paper we obtain the weights of criteria based on their method. The steps of fuzzy Shannon’s Entropy explained as follow [24]:

**Step 1:** transforming fuzzy data into interval data by using the  $\alpha$ -level sets:

The  $\alpha$ -level set of a fuzzy variable  $\tilde{x}_{ij}$  is defined by a set of elements that belong to the fuzzy variable  $\tilde{x}_{ij}$  with membership of at least  $\alpha$  i.e.  $(\tilde{x}_{ij})_\alpha = \{x_{ij} \in R | \mu_{\tilde{x}_{ij}}(x_{ij}) \geq \alpha\}$

The  $\alpha$ -level set can also be expressed in the following interval form:

$$\left[ (\tilde{x}_{ij})'_a, (\tilde{x}_{ij})^U_a \right] = \left[ \min_{x_{ij}} \{x_{ij} \in R | \mu_{\tilde{x}_{ij}}(x_{ij}) \geq a\}, \max_{x_{ij}} \{x_{ij} \in R | \mu_{\tilde{x}_{ij}}(x_{ij}) \geq a\} \right] \quad (6)$$

where  $0 < \alpha \leq 1$ . By setting different levels of confidence, namely  $1-\alpha$ , fuzzy data are accordingly transformed into different  $\alpha$ -level sets  $\left\{ (\tilde{x}_{ij})'_a \mid 0 < a \leq 1 \right\}$ , which are all intervals.

**Step 2:** The normalized values  $P'_{ij}$  and  $P''_{ij}$  are calculated as:

$$P'_{ij} = \frac{x'_{ij}}{\sum_{i=1}^m x'_{ij}}, P''_{ij} = \frac{x''_{ij}}{\sum_{i=1}^m x''_{ij}}, \quad j=1, \dots, m, I=1, \dots, n \quad (7)$$

**Step 3:** Lower bound  $h'_i$  and upper bound  $h''_i$  of interval entropy can be obtained by:

$$h'_i = \min \left\{ -h_0 \sum_{i=1}^m P'_{ij} \cdot \ln P'_{ij}, -h_0 \sum_{i=1}^m P''_{ij} \cdot \ln P''_{ij} \right\}, i=1, \dots, n$$

and

$$h''_i = \min \left\{ -h_0 \sum_{i=1}^m P'_{ij} \cdot \ln P'_{ij}, -h_0 \sum_{i=1}^m P''_{ij} \cdot \ln P''_{ij} \right\}, i=1, \dots, n \quad (8)$$

Where  $h_0$  is equal to  $P'_{ij}$  and  $P''_{ij} \cdot \ln P'_{ij}$  or  $P''_{ij} \cdot \ln P''_{ij}$  is defined as 0 if  $P'_{ij} = 0$  or  $P''_{ij} = 0$ .

**Step 4:** Set the lower and the upper bound of the interval of diversification  $d'_i$  and  $d''_i$  as the degree of diversification as follows:

$$d'_i = 1 - h'_i, d''_i = 1 - h''_i, i=1, \dots, n \quad (9)$$

Step 5: Set  $w_i^L = \frac{d_i^L}{\sum_{s=1}^n d_s^L}, w_i^U = \frac{d_i^U}{\sum_{s=1}^n d_s^U}$ ,

$I=1, \dots, n$  as the lower and upper bound of interval weight of attribute  $i$ .

**Graph Theory and Matrix Approach:** A graph  $G = (V, E)$  consists of a set of objects  $V = \{v_1, v_2, \dots\}$  called vertices or nodes and another set  $E = \{e_1, e_2, \dots\}$ , of which the elements are called edges, such that each edge  $e_k$  is identified with a pair of vertices. The vertices  $v_i$  and  $v_j$  associated with edge  $e_k$  are called the end vertices of  $e_k$ .

The most common representation of a graph is by means of a diagram, in which the vertices are represented by small points or circles and each edge as a line segment joining its end vertices. The application of graph theory was known centuries ago, when the long standing problem of the Konigsberg bridge was solved by Leonhard Euler in 1736 by means of a graph. Since then, graph theory has proved its mettle in various fields of science and technology such as physics, chemistry, mathematics, communication science, computer technology, electrical engineering, sociology, economics, operations research, linguistics, internet, etc. Graph theory has served an important purpose in the modeling of systems, network analysis, functional representation, conceptual modeling, diagnosis, etc. Graph theory is not only effective in dealing with the structure (physical or abstract) of the system, explicitly or implicitly, but also useful in handling problems of structural relationship. The theory is intimately related to many branches of mathematics including group theory, matrix theory, numerical analysis, probability, topology and combinatory. The advanced theory of graphs and their applications are well documented [25-34].

**Methodology of GTMA**

**The Main Steps Are Given below [1]:** Step 1: Identify the pertinent attributes and the alternatives involved in the decision-making problem under consideration. Obtain the values of the attributes ( $A_i$ ) and their relative importance ( $a_{ij}$ ). An objective or subjective value, or its range, may be assigned to each identified attribute as a limiting value or threshold value for its acceptance for the considered decision-making problem. An alternative with each of its selection attributes, meeting the acceptance value, may be short-listed. After short-listing the alternatives, the main task in choosing the alternative is to see how it serves the considered attributes.

**Step 2:**

- Develop the attributes digraph considering the identified pertinent attributes and their relative importance. The number of nodes shall be equal to the number of attributes considered in Step 1 above. The edges and their directions will be decided upon based on the interrelations among the attributes ( $a_{ij}$ ).
- Develop the attributes matrix for the attributes digraph. This will be the  $M \times M$  matrix with diagonal elements as  $A_i$  and off-diagonal elements as  $a_{ij}$ .
- Obtain the permanent function for the attributes matrix.

- Substitute the values of  $A_i$  and  $a_{ij}$ , obtained in step 1.
- Arrange the alternatives in the descending order of the index. The alternative having the highest value of index is the best choice for the decision-making problem under consideration.
- Obtain the identification set for each alternative.
- Evaluate the coefficients of dissimilarity and similarity. List also the values of the coefficients for all possible combinations.
- Document the results for future analysis/reference.

**Step 3:** Take a final decision, keeping practical considerations in mind. All possible constraints likely to be experienced by the user are looked into during this stage. These include constraints such as: availability or assured supply, management constraints, political constraints, economic constraints, environmental constraints, etc. However, compromise may be made in favor of an alternative with a higher index.

**A Numerical Application of Proposed Approach:** In this section, we presented a case study to demonstrate the application of proposed method for a firm that manufactures LPG and CNG components. The company purchases a significant number of parts used on the assembly line from domestic and foreign vendors. The company had divided all purchased parts into 5 groups, including  $A_1, A_2, A_3, A_4$  and  $A_5$ . Selecting the best vendor has a great importance for this company.

But it is hard to choose the most suitable one among the other vendors. In the application, firstly through the literature investigation and studying other papers that are related to vendor selection, ten criteria are selected. These criteria include Capacity ( $C_1$ ), Availability of Raw materials ( $C_2$ ), Geographic Location ( $C_3$ ), Shipment Accuracy ( $C_4$ ), Cost ( $C_5$ ), Customer Service ( $C_6$ ), Total Order Lead time ( $C_7$ ), Trade Restrictions ( $C_8$ ), Supplier's Selling Price ( $C_9$ ), Commitment to Quality ( $C_{10}$ ). In this paper we have 10 decision maker.

**Fuzzy Shannon's Entropy:** In fuzzy Shannon's Entropy, firstly, the criteria and alternatives' importance weights must be compared. Afterwards, the comparisons about the criteria and alternatives and the weight calculation need to be made. Thus, the evaluation of the criteria according to the main goal and the evaluation of the alternatives for these criteria must be realized. Then, after all these evaluation procedure, the weights of the alternatives can be calculated. In the second step, these weights are used to GTMA calculation for the final evaluation. The aggregate decision matrix for Shannon's Entropy can be seen from Table 1.

After forming decision matrix, we transformed fuzzy data of Table 1 into interval data. For transforming fuzzy data into interval data, we consider  $\alpha = 0.3$ . The interval decision matrix for the criteria can be seen from Table 2.

Table 1: Aggregate decision matrix for fuzzy Shannon's Entropy

DM	$C_1$	$C_2$	$C_3$	...	$C_{10}$
$A_1$	(5.83, 8.33,10.0)	(6.66, 9.16,10.0)	(2.83, 5.00,7.50)	...	(3.16, 5.00,6.66)
$A_2$	(3.33, 5.83,8.33)	(1.00, 1.50,3.33)	(2.00, 4.16,6.66)	...	(2.50, 5.00,7.50)
$A_3$	(2.00, 4.16,6.66)	(1.50, 3.33,5.83)	(6.66, 9.16,10.0)	...	(3.66, 5.33,7.50)
$A_4$	(2.00, 4.16,6.66)	(4.16, 6.66,9.16)	(6.66, 9.16,10.0)	...	(1.00, 2.00,4.16)
$A_5$	(5.83, 8.33,10.0)	(2.00, 4.16,6.66)	(1.50, 3.33,5.83)	...	(3.66, 5.33,7.50)

Table 2: Interval decision matrix

DM	$C_1$	$C_2$	$C_3$	...	$C_{10}$
$A_1$	[6.58,9.50]	[7.41,9.75]	[3.48,6.75]	...	[3.71,6.16]
$A_2$	[4.08,7.58]	[1.15,2.78]	[2.65,5.91]	...	[3.25,6.75]
$sA_3$	[2.65,5.91]	[2.05,5.08]	[7.41,9.75]	...	[4.16,6.85]
$A_4$	[2.65,5.91]	[4.91,8.41]	[7.41,9.75]	...	[1.30,3.51]
$A_5$	[6.58,9.50]	[2.65,5.91]	[2.05,5.08]	...	[4.16,6.85]

Table 3: The normalized interval decision matrix

DM	$C_1$	$C_2$	$C_3$	...	$C_{10}$
$A_1$	[0.171,0.421]	[0.232,0.536]	[0.093,0.293]	...	[0.123,0.371]
$A_2$	[0.106,0.336]	[0.035,0.153]	[0.071,0.257]	...	[0.107,0.406]
$A_3$	[0.068,0.262]	[0.064,0.279]	[0.199,0.423]	...	[0.138,0.412]
$A_4$	[0.068,0.262]	[0.153,0.462]	[0.199,0.423]	...	[0.043,0.211]
$A_5$	[0.171,0.421]	[0.082,0.325]	[0.055,0.220]	...	[0.138,0.412]

Table 4: The values of  $h_i', h_i'', d_i'$  and  $d_i''$

	$[h_i', h_i'']$	$[d_i', d_i'']$
C <sub>1</sub>	[0.52,0.78]	[0.21,0.47]
C <sub>2</sub>	[0.49,0.73]	[0.26,0.50]
C <sub>3</sub>	[0.52,0.60]	[0.23,0.47]
C <sub>4</sub>	[0.46,0.74]	[0.25,0.53]
C <sub>5</sub>	[0.49,0.75]	[0.24,0.50]
C <sub>6</sub>	[0.57,0.77]	[0.22,0.42]
C <sub>7</sub>	[0.55,0.78]	[0.21,0.44]
C <sub>8</sub>	[0.57,0.78]	[0.21,0.42]
C <sub>9</sub>	[0.52,0.77]	[0.22,0.47]
C <sub>10</sub>	[0.51,0.77]	[0.22,0.48]

Table 5: The interval and crisp weight of criteria

	$[w_i^L, w_i^U]$	W <sub>i</sub>
C <sub>1</sub>	[0.094,0.099]	0.097
C <sub>2</sub>	[0.107,0.112]	0.110
C <sub>3</sub>	[0.0995,0.0996]	0.100
C <sub>4</sub>	[0.110,0.113]	0.112
C <sub>5</sub>	[0.1050,0.1059]	0.105
C <sub>6</sub>	[0.08,0.09]	0.093
C <sub>7</sub>	[0.093,0.094]	0.094
C <sub>8</sub>	[0.088,0.094]	0.092
C <sub>9</sub>	[0.097,0.099]	0.099
C <sub>10</sub>	[0.095,0.102]	0.099

Table 6: Decision matrix of GTMA

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>	C <sub>7</sub>	C <sub>8</sub>	C <sub>9</sub>	C <sub>10</sub>
A <sub>1</sub>	8.13	8.75	5.08	8.75	2.29	6.08	3.88	6.67	3.04	4.96
A <sub>2</sub>	5.83	1.83	4.25	5.00	8.75	4.63	5.71	7.29	4.63	5.00
A <sub>3</sub>	4.25	3.50	8.75	2.29	5.83	7.29	5.46	6.71	3.88	5.46
A <sub>4</sub>	4.25	6.67	8.75	2.75	3.50	8.13	6.46	5.25	6.54	2.29
A <sub>5</sub>	8.13	4.25	3.50	3.50	5.83	5.25	7.29	5.71	7.29	5.46
MAX	8.13	8.75	8.75	8.75	8.75	8.13	7.29	7.29	7.29	5.46

Table 7: Normalized decision matrix of GTMA

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>	C <sub>7</sub>	C <sub>8</sub>	C <sub>9</sub>	C <sub>10</sub>
A <sub>1</sub>	1.000	1.000	0.581	1.000	0.262	0.749	0.531	0.914	0.417	0.908
A <sub>2</sub>	0.718	0.210	0.486	0.571	1.000	0.569	0.783	1.000	0.634	0.916
A <sub>3</sub>	0.523	0.400	1.000	0.262	0.667	0.897	0.749	0.920	0.531	1.000
A <sub>4</sub>	0.523	0.762	1.000	0.314	0.400	1.000	0.886	0.720	0.897	0.420
A <sub>5</sub>	1.000	0.486	0.400	0.400	0.667	0.646	1.000	0.783	1.000	1.000

Table 8: Pair-wise comparison of criteria with respect to each other

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>	C <sub>7</sub>	C <sub>8</sub>	C <sub>9</sub>	C <sub>10</sub>
C <sub>1</sub>	-	0.469	0.494	0.464	0.479	0.510	0.509	0.515	0.496	0.495
C <sub>2</sub>	0.531	-	0.525	0.495	0.510	0.541	0.540	0.546	0.527	0.527
C <sub>3</sub>	0.506	0.475	-	0.471	0.486	0.516	0.515	0.521	0.503	0.502
C <sub>4</sub>	0.536	0.505	0.529	-	0.515	0.545	0.545	0.550	0.532	0.531
C <sub>5</sub>	0.521	0.490	0.514	0.485	-	0.530	0.530	0.535	0.517	0.516
C <sub>6</sub>	0.490	0.459	0.484	0.455	0.470	-	0.499	0.505	0.487	0.486
C <sub>7</sub>	0.491	0.460	0.485	0.455	0.470	0.501	-	0.506	0.487	0.486
C <sub>8</sub>	0.485	0.454	0.479	0.450	0.465	0.495	0.494	-	0.482	0.481
C <sub>9</sub>	0.504	0.473	0.497	0.468	0.483	0.513	0.513	0.518	-	0.499
C <sub>10</sub>	0.505	0.473	0.498	0.469	0.484	0.514	0.514	0.519	0.501	-
W <sub>j</sub>	0.097	0.110	0.100	0.112	0.105	0.093	0.094	0.092	0.099	0.099

Table 9: Pair-wise comparison of criteria with respect to A<sub>1</sub>

A <sub>1</sub>	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>	C <sub>7</sub>	C <sub>8</sub>	C <sub>9</sub>	C <sub>10</sub>
C <sub>1</sub>	1.000	0.469	0.494	0.464	0.479	0.510	0.509	0.515	0.496	0.495
C <sub>2</sub>	0.531	1.000	0.525	0.495	0.510	0.541	0.540	0.546	0.527	0.527
C <sub>3</sub>	0.506	0.475	0.581	0.471	0.486	0.516	0.515	0.521	0.503	0.502
C <sub>4</sub>	0.536	0.505	0.529	1.000	0.515	0.545	0.545	0.550	0.532	0.531
C <sub>5</sub>	0.521	0.490	0.514	0.485	0.262	0.530	0.530	0.535	0.517	0.516
C <sub>6</sub>	0.490	0.459	0.484	0.455	0.470	0.749	0.499	0.505	0.487	0.486
C <sub>7</sub>	0.491	0.460	0.485	0.455	0.470	0.501	0.531	0.506	0.487	0.486
C <sub>8</sub>	0.485	0.454	0.479	0.450	0.465	0.495	0.494	0.914	0.482	0.481
C <sub>9</sub>	0.504	0.473	0.497	0.468	0.483	0.513	0.513	0.518	0.417	0.499
C <sub>10</sub>	0.505	0.473	0.498	0.469	0.484	0.514	0.514	0.519	0.501	0.908

Table 10: Pair-wise comparison of criteria with respect to A<sub>2</sub>

A <sub>2</sub>	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>	C <sub>7</sub>	C <sub>8</sub>	C <sub>9</sub>	C <sub>10</sub>
C <sub>1</sub>	0.718	0.469	0.494	0.464	0.479	0.510	0.509	0.515	0.496	0.495
C <sub>2</sub>	0.531	0.210	0.525	0.495	0.510	0.541	0.540	0.546	0.527	0.527
C <sub>3</sub>	0.506	0.475	0.486	0.471	0.486	0.516	0.515	0.521	0.503	0.502
C <sub>4</sub>	0.536	0.505	0.529	0.571	0.515	0.545	0.545	0.550	0.532	0.531
C <sub>5</sub>	0.521	0.490	0.514	0.485	1.000	0.530	0.530	0.535	0.517	0.516
C <sub>6</sub>	0.490	0.459	0.484	0.455	0.470	0.569	0.499	0.505	0.487	0.486
C <sub>7</sub>	0.491	0.460	0.485	0.455	0.470	0.501	0.783	0.506	0.487	0.486
C <sub>8</sub>	0.485	0.454	0.479	0.450	0.465	0.495	0.494	1.000	0.482	0.481
C <sub>9</sub>	0.504	0.473	0.497	0.468	0.483	0.513	0.513	0.518	0.634	0.499
C <sub>10</sub>	0.505	0.473	0.498	0.469	0.484	0.514	0.514	0.519	0.501	0.916

Table 11: Pair-wise comparison of criteria with respect to A<sub>3</sub>

A <sub>3</sub>	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>	C <sub>7</sub>	C <sub>8</sub>	C <sub>9</sub>	C <sub>10</sub>
C <sub>1</sub>	0.523	0.469	0.494	0.464	0.479	0.510	0.509	0.515	0.496	0.495
C <sub>2</sub>	0.531	0.400	0.525	0.495	0.510	0.541	0.540	0.546	0.527	0.527
C <sub>3</sub>	0.506	0.475	1.000	0.471	0.486	0.516	0.515	0.521	0.503	0.502
C <sub>4</sub>	0.536	0.505	0.529	0.262	0.515	0.545	0.545	0.550	0.532	0.531
C <sub>5</sub>	0.521	0.490	0.514	0.485	0.667	0.530	0.530	0.535	0.517	0.516
C <sub>6</sub>	0.490	0.459	0.484	0.455	0.470	0.897	0.499	0.505	0.487	0.486
C <sub>7</sub>	0.491	0.460	0.485	0.455	0.470	0.501	0.749	0.506	0.487	0.486
C <sub>8</sub>	0.485	0.454	0.479	0.450	0.465	0.495	0.494	0.920	0.482	0.481
C <sub>9</sub>	0.504	0.473	0.497	0.468	0.483	0.513	0.513	0.518	0.531	0.499
C <sub>10</sub>	0.505	0.473	0.498	0.469	0.484	0.514	0.514	0.519	0.501	1.000

Table 12: Pair-wise comparison of criteria with respect to A<sub>4</sub>

A <sub>4</sub>	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>	C <sub>7</sub>	C <sub>8</sub>	C <sub>9</sub>	C <sub>10</sub>
C <sub>1</sub>	0.523	0.469	0.494	0.464	0.479	0.510	0.509	0.515	0.496	0.495
C <sub>2</sub>	0.531	0.762	0.525	0.495	0.510	0.541	0.540	0.546	0.527	0.527
C <sub>3</sub>	0.506	0.475	1.000	0.471	0.486	0.516	0.515	0.521	0.503	0.502
C <sub>4</sub>	0.536	0.505	0.529	0.314	0.515	0.545	0.545	0.550	0.532	0.531
C <sub>5</sub>	0.521	0.490	0.514	0.485	0.400	0.530	0.530	0.535	0.517	0.516
C <sub>6</sub>	0.490	0.459	0.484	0.455	0.470	1.000	0.499	0.505	0.487	0.486
C <sub>7</sub>	0.491	0.460	0.485	0.455	0.470	0.501	0.886	0.506	0.487	0.486
C <sub>8</sub>	0.485	0.454	0.479	0.450	0.465	0.495	0.494	0.720	0.482	0.481
C <sub>9</sub>	0.504	0.473	0.497	0.468	0.483	0.513	0.513	0.518	0.897	0.499
C <sub>10</sub>	0.505	0.473	0.498	0.469	0.484	0.514	0.514	0.519	0.501	0.420

Table 13: Pair-wise comparison of criteria with respect to  $A_5$

$A_5$	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7$	$C_8$	$C_9$	$C_{10}$
$C_1$	1.000	0.469	0.494	0.464	0.479	0.510	0.509	0.515	0.496	0.495
$C_2$	0.531	0.486	0.525	0.495	0.510	0.541	0.540	0.546	0.527	0.527
$C_3$	0.506	0.475	0.400	0.471	0.486	0.516	0.515	0.521	0.503	0.502
$C_4$	0.536	0.505	0.529	0.400	0.515	0.545	0.545	0.550	0.532	0.531
$C_5$	0.521	0.490	0.514	0.485	0.667	0.530	0.530	0.535	0.517	0.516
$C_6$	0.490	0.459	0.484	0.455	0.470	0.646	0.499	0.505	0.487	0.486
$C_7$	0.491	0.460	0.485	0.455	0.470	0.501	1.000	0.506	0.487	0.486
$C_8$	0.485	0.454	0.479	0.450	0.465	0.495	0.494	0.783	0.482	0.481
$C_9$	0.504	0.473	0.497	0.468	0.483	0.513	0.513	0.518	1.000	0.499
$C_{10}$	0.505	0.473	0.498	0.469	0.484	0.514	0.514	0.519	0.501	1.000

Table 14: Permanent matrix of each alternative

Alternative	Permanent matrix
$A_1$	5543.90
$A_2$	5051.20
$A_3$	5107.40
$A_4$	5081.70
$A_5$	5574.30

Table 15: Ranking alternative

Alternative	Permanent matrix	Rank
$A_1$	5543.90	2
$A_2$	5051.20	5
$A_3$	5107.40	3
$A_4$	5081.70	4
$A_5$	5574.30	1

Then, according to Eq. (7), we normalized the interval decision matrix. The normalized interval decision matrix is shown in Table 3.

In the next step, we calculate the lower bound  $h_i'$  and upper bound  $h_i''$  of criteria based on the Eq. (8). After that the degrees of diversification are calculated using Equation (9), as shown in Table 4.

Finally, the interval weight and crisp weight are calculated, as shown in Table 5.

The GTMA method: The weights of the criteria are calculated by fuzzy Shannon's Entropy up to now and then these values can be used in GTMA. After calculating the weights, we formed the decision matrix of GTMA that shows in Table 6.

In the next step, we normalized the decision matrix of GTMA that shows in Table 7.

Then, according to GTMA method, we carry out pair-wise comparison with respect to their weight that shows from Table 8 to Table 13.

After that we calculate the permanent matrix using of MATLAB software. The permanent matrix of each alternative is indicated in Table 14.

Finally, we rank all vendors with respect to their permanent matrix that shows in Table 15.

According to Table 15, the fifth vendor is the best vendor among other vendors and other vendors ranked as follow:  $A_5 > A_1 > A_3 > A_4 > A_2$

### CONCLUSION

The objective of vendor selection is to identify vendors with the highest potential for meeting a company's needs consistently and at an acceptable cost. Selection is a broad comparison of vendors based on a common set of criteria and measures. However, the level of details used for examining potential vendors may vary depending on a company's needs. The overall goal of selection is to identify high potential vendors and their quota allocations. An effective and appropriate vendor assessment method is therefore crucial to the competitiveness of companies. In this paper, fuzzy Shannon's Entropy and GTMA are combined that GTMA uses fuzzy Shannon's Entropy result weights as input weights. Then a real case study is presented to show applicability and performance of the method. It can be

said that using linguistic variables makes the evaluation process more realistic. Because evaluation is not an exact process and has fuzziness in its body. Here, the usage of fuzzy Shannon's Entropy weights in GTMA makes the application more realistic and reliable. As a future direction, other decision-making methods such as fuzzy ELECTRE, Fuzzy Prioritization Method (FPM) and Fuzzy GTMA can be used in this area.

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