

Determination of Portfolio Through Fuzzy Data Envelopment Analysis in Companies Accepted in Tehran Securities Exchange

¹Mansour Garkaz and ²Fatemeh Pesarakloo

¹Department of Accounting & Management, Ali Abad Katoul Branch,
Islamic Azad University, Ali Abad Katoul, Iran

²Department of Accounting & Management,
Borujerd Branch, Islamic Azad University, Borujerd, Iran

Abstract: In this research, the aim was to select a portfolio from accepted companies in Tehran Securities Exchange through Data Envelopment Analysis approach and Fuzzy Logic. Efficiency measurement of companies via DEA can assist investors as an approach in selecting companies. For this purpose, first, we determined variables considered important by a typical investor in choosing significant stocks. These variables are consisted of revenues total, cost of goods and financial expenses of each company. The determination of the efficiencies of twenty one companies was done via DEA Model of Girod and Triantis (1998) accompanied with CCR FUZZY designs in a four-year period of 2006-2009. In the end, it was revealed that no company had efficiency 1 through utilizing triangular fuzzy numbers regarding revenues and expanses of companies and CCR Fuzzy Model; however, the efficiency of four companies were found to be more than 0.8. Calculation the efficiency of these four companies revealed that the obtained efficiency is different from the portfolio efficiency and market efficiency.

Key words: Portfolio • Data Envelopment Analysis • Fuzzy numbers

INTRODUCTION

Today, global economy has assumed widespread dimensions, among which capital market development such as securities exchanges can be mentioned. Stock investment has turned into an important means for personal saving. The issue of optimally selecting the portfolio is one of the problems for which no precise and given pattern has been presented. In some cases, the investor is confronted with some options, which are vague. Vagueness of decision analysis leads to an incomplete analysis; this is where the utilization of fuzzy series seems helpful, which lessens the need for precise data in decision making. In non-parametric approaches the companies' efficiency will be assessed through mathematical linear planning techniques. In this approach, it is not necessary to assess the production function; in addition, if the agency in question has several input and outputs, then there will not be any problem in assessment. DEA is one of the aforementioned approaches, which is a theoretical framework for performance analysis and efficiency measurement.

Once Harry Markowitz presented his paper regarding variance-mean return portfolio selection in 1952, he was considered as a guide in a new age of financial economy. His outstanding work brought him the Noble prize in economics in 1990 and laid the foundations for developing CAMP by William Sharp (1962), Lintner (1965) and Mosin (1966) [1].

Kabnurkar [2] suggested DEA as a tool for measuring industrial efficiency in 2001, which is a research method for automatic calculation of assigned weights for assessing inputs/outputs of production unit in investigation within the operation.

In addition, he stated that fuzzy series theory enables us to utilize "the Principle of incompatibility", which provides the possibility of obtaining decision based on qualitative data.

Chi-Ming Lin [3] examined the portfolio optimization issue from genetic engineering viewpoint in 2006, which presented his two propositions for solving the problem. He also designed the genetic algorithm for solving the pertinent optimization issues; the result of his work offers a reliable approach for the optimal portfolio issue.

His goal of applying this variance-mean approach was to determine the rate of optimal investment period for securities according to the expected return rate.

Additionally, Aranha Claus [4] mentioned the genetic algorithm in portfolio management in 2007 and achieved a system, which is capable of dealing with the previous huge flawed materials. He developed the two objective and ranking approaches in his work in order to indicate the measurement model of transactions value and the effect of certain market fluctuations.

Lee, Chang (2012) discuss the particular issues, the ones which help an organization reach its goals.

in selecting a portfolio of projects while there is a restriction for capital resources. They suggested an integrated model for addressing problems through Knapsack Formulation and Data Envelopment Analysis and Fuzzy Series Theory [5].

Masumeh Sheikh Moradi [6] concludes her dissertation by calculating the MALMQUIST Productivity Index (MPI) through modified Russell Model for Iran Insurance Company branches in Tehran with fuzzy data. In this work, she considered MALMQUIST Productivity Index of fuzzy cost and revenue by fuzzy data and then calculated branches' efficiencies via FDCCR model with weight control and without weight control. Applying modified Russell Model for calculating the MALMQUIST Productivity Index and utilizing weight control, which determine all the inefficiencies of the unit under assessment and exert input and output indices on the problem, respectively are two features of this dissertation.

Tiryaki and Ahlatcioglu [7] selects portfolio by employing AHP hierarchy model. He considered external and internal factors and also investor's goals in his research. Consequently, the obtained results indicated that the fuzzy AHP is appropriate for decision making on problems. The applied approach for formulating the hierarchy is adopted from Satty *et al.* research, which examined the stock selection problem in Istanbul securities.

Ha & Leo (2009) applied the genetic algorithm as a tool for solving their models in a research under the title of mean-variance model for portfolio selection with random fuzzy efficiency. In this research, new models of mean-variance model were presented based on Markowitz theory for portfolio selection with random fuzzy investment efficiencies. For solving the presented portfolio selection models, first, variance formula were shown as random fuzzy variables and then the variance formula were utilized in a way that main

stock portfolio selection turned to equivalent linear programming. Next, the genetic algorithms were employed for solving the models [11].

KHOSHFETRAT (2010) in his research examined the problem that whether evaluating the performance of activities or organization by common data envelopment analysis models, requires crisp input/output data. However, they used fuzzy data since the precise calculation of data is not always possible.

This paper focuses on the fuzzy CCR model and proposes a new method for determining the lower bounds of fuzzy inputs and outputs. This improves the weak efficiency frontiers of the corresponding production possibility set. Also a numerical example illustrates the capability of the proposed method [9].

SHIANG-TAI LIC (2011) presented a fuzzy model for the optimization of fuzzy portfolio.

This paper discusses the fuzzy portfolio optimization problem where the asset returns are represented by fuzzy data. Since the parameters are fuzzy numbers, the gain of return is a fuzzy number as well. A pair of two-level mathematical programs is formulated to calculate the upper bound and lower bound of the return of the portfolio optimization problem. Based on the duality theorem and by applying the variable transformation technique, the pair of two-level mathematical programs is transformed into a pair of ordinary one-level linear programs so they can be manipulated. It is found that the calculated results conform to an essential idea in finance and economics that the greater the amount of risk that an investor is willing to take on, the greater the potential return [10].

Mullen & Powers (2000) applied DEA for the securities selection problem.

A technique has been presented which employs Data Envelopment Analysis to select the most desirable securities from a list of the largest market cap stocks. Within the context of this analysis and assumptions, it is shown that of these 185 stocks evaluated, fourteen are found to be relatively efficient, or dominant, while there are four others found to be "near-efficient." One of the advantages of using DEA is that for the DEA-inefficient securities, information is provided disclosing how much reduction of inputs or how much augmentation is needed for these inefficient securities to become DEA-efficient. In other words, DEA can inform the decision-maker which alternatives are consistently the best when several attributes are considered, but it also provides information as to how much improvement is needed for each alternative to become efficient with respect to inputs (input reduction) and outputs (output augmentation).

It must be noted that if a prospective investor were to use this approach for selection of stocks, consideration must be made to weighting issues. The weighting for the initial model ($0.2 < \text{Ratio} < 5$) was intentionally generalized. The sensitivity analysis where the weighting schemes were "tightened" was also generalized. Different investors, of course, will have very specific priorities and weights can be engineered to reflect these priorities [11].

Gordan & Baptista (2010) conducted a research on active portfolio management based on Alpha factor. They stated in their research that:

Active portfolio management often involves the objective of selecting a portfolio with minimum tracking error variance (TEV) for some expected gain in return over a benchmark.

Our paper proposes an appealing method to lessen this suboptimality that involves the objective of selecting a portfolio from the set of portfolios that have minimum TEV for various levels of *ex-ante* alpha, which we refer to as the alpha-TEV frontier. Since practitioners commonly use *ex-post* alpha to assess the performance of managers, the use of this frontier aligns the objectives of managers with how their performance is evaluated. Furthermore, sensible choices of *ex-ante* alpha lead to the selection of portfolios that are less risky (in variance terms) than the portfolios that active managers would otherwise select [12].

Anagnostopoulos & Mamanis (2010) presented an optimal portfolio model with three objectives and discrete variables.

They formulate the portfolio selection as a tri-objective optimization problem so as to find tradeoffs between risk, return and the number of securities in the portfolio.

Furthermore, quantity and class constraints are introduced into the model in order to limit the proportion of the portfolio invested in assets with common characteristics and to avoid very small holdings. Since the proposed portfolio selection model involves mixed integer decision variables and multiple objectives finding the exact efficient frontier may be very hard. Nevertheless, finding a good approximation of the efficient surface which provides the investor with a diverse set of portfolios capturing all possible tradeoffs between the objectives within limited computational time is usually acceptable.

They experiment with the current state of the art evolutionary multiobjective optimization techniques,

namely the Non-dominated Sorting Genetic Algorithm II (NSGA-II), Pareto Envelope-based Selection Algorithm (PESA) and Strength Pareto Evolutionary Algorithm 2 (SPEA2), for solving the mixed-integer multiobjective optimization problem and provide a performance comparison among them using metrics proposed by the community [13].

MATERIAL AND METHOD

This research aims to examine all the companies, which were the members of Tehran Securities Exchange between March 21st, 2006 to March 20th, 2009. However, statistical samples of companies were selected since the examination of all the existing companies could take a considerable amount of time. This collection includes the following conditions:

- The information of financial statements of the companies of between the beginnings of 2006 to the end of 2009 is available.
- The companies must not be financially detrimental.
- The companies must be present and active in Tehran Securities Exchange from the beginnings of 2006 to the end of 2009.
- The fiscal year of the companies must end at the date of March 20th.
- The companies must not be a part of banks, credit or investment institutions.

After the determination of the statistical sample in question, the triangular fuzzy numbers were used, which were related to the four-year period of 2006-2009 and were comprised of revenues total, cost of goods and financial expenses.

The years 2006-2007 and 2008-2009 were taken as time periods t and $t+1$, respectively. For instance, in the following table, the third, fourth and fifth columns from left side are the left sheet, the center and the right sheet of the first input (cost of goods) at the time t , respectively. The sixth, seventh and eighth columns are the left sheet, the center and the sheet of the second input (financial expenses) at the time t , respectively. In addition, the ninth, tenth and eleventh columns are the left sheet, center and the right sheet of our output (revenues total) at the time t , respectively. A similar table is designed for the period $t+1$ as well. These numbers are pertinent to the first two samples.

Row	Company's Name	IM11	IL11	IU11	IM21	IL21	IU11	OM11	OL11	OU11
		85-86	85-86	85-86	85-86	85-86	85-86	85-86	85-86	85-86
1	DMU1	433040	490367	547694	24577	26627	28677	581682	640576.5	699471
2	DMU2	345809	369078.5	392348	15057	20243.5	25430	434797	469442.5	504088

DEA Model Selection: In the method we applied, the DEA Model of Girod and Triantis (1998) and Fuzzy Production Designs were used. Based on the definitions presented in this approach, the DEA-CCR Model was resulted as follows:

$$\begin{aligned} & \text{Max} \sum_i \sigma_i \left[(y_{i,k}^{0*} - y_{i,k}^{1*}) \mu_y + y_{i,k}^{1*} \right] \\ \text{s.t:} & \sum_j \pi_j \left[x_{j,k}^{0*} - (x_{j,k}^{0*} - x_{j,k}^{1*}) \mu_x \right] = 1 \\ & \sum_j \pi_j \times \left[x_{j,k}^0 - (x_{j,k}^0 - x_{j,k}^1) \mu_x \right] + \sum_i \delta_i \times \left[(y_{i,k}^0 - y_{i,k}^1) \mu_y + y_{i,k}^1 \right] \leq 0 \\ & \delta_i \geq \varepsilon, \pi_j \geq \varepsilon \end{aligned}$$

According to Carlsson & Korhonen (1986), the abovementioned model leads to a an answer, whereas.

$$\mu_x = \mu_y = \mu = \min(\mu_x, \mu_y)$$

Consequently, the model in question is presented as follows:

$$\begin{aligned} & \text{Max} \sum_i \sigma_i \left[(y_{i,k}^{0*} - y_{i,k}^{1*}) \mu + y_{i,k}^{1*} \right] \\ \text{s.t:} & \sum_j \pi_j \times \left[x_{j,k}^{0*} - (x_{j,k}^{0*} - x_{j,k}^{1*}) \mu_x \right] = 1 \\ & \sum_j \pi_j \times \left[x_{j,k}^0 - (x_{j,k}^0 - x_{j,k}^1) \mu_x \right] + \sum_i \delta_i \times \\ & - \sum_j \left[(y_{i,k}^0 - y_{i,k}^1) \mu_y + y_{i,k}^1 \right] \leq 0 \\ & \delta_i \geq \varepsilon, \pi_j \geq \varepsilon \end{aligned}$$

Finally, the μ membership function (which is considered as a parameter here) in predetermined distances is various and variable for including the differences of the properties of efficiency diagram. $\mu = 0$ and $\mu = 1$ show the maximum optimistic quantities of technique efficiency and extreme conservative quantities (pessimistic), respectively (9).

If 0 and 1 represent the impossible borders and free risk for the input data, respectively; therefore, the membership function of j^{th} fuzzy input ($X_{j,k}$) for k^{th} DMU is as follows:

$$\mu_x(x_{j,k}) = \frac{x_{j,k}^0 - x_{j,k}}{x_{j,k}^0 - x_{j,k}^1}$$

Similarly, the membership function of i^{th} fuzzy output ($y_{i,k}$) DMU_k is as follows:

$$\mu_y(x_{i,k}) = \frac{y_{i,k} - y_{i,k}^1}{y_{i,k}^0 - y_{i,k}^1}$$

Both $x_{j,k}$ and $y_{i,k}$ can be stated in terms of free risk and impossible borders (1).

The results pertinent to two typical companies are summarized in tables 1, 2 and 3 as follow:

Row	DMU	$\mu = 0$	$\mu = 1$
1	DMU1	0.3262	0.7803
2	DMU2	0.3053	0.7026

1- Efficiency Table at the time t

Row	DMU	$\mu = 0$	$\mu = 1$
1	DMU1	0.6428	0.6900
2	DMU2	0.6177	0.6719

2- Efficiency Table at the time t+1

Row	DMU	Efficiency
1	DMU1	0.6098
2	DMU2	0.5744

3- Efficiency Mean Table

For constituting the portfolio, four companies whose efficiencies were more than 0.8 were selected and the pertinent portfolio efficiency was calculated:

The first efficiency is related to those companies that were chosen based on the model and the second efficiency is related to the random portfolio efficiency.

$$\bar{R}_P = \sum w_i R_i$$

$$\bar{R}_{P1} = \frac{1}{4} (36.65 + 52.5 + 54.45 + 42.7)$$

$$= 46.575$$

$$\bar{R}_{P2} = \frac{1}{4} (-8.79 + 25.72 + 31.02 + 13.79)$$

$$= 15.435$$

$$46.575 > 15.435$$

Moreover, there is a difference between the resulted portfolio efficiency based on the Fuzzy Data Envelopment Analysis and the mean market efficiency.

$$46.575 > 46.30$$

The obtained results indicated that there was a difference between the resulted portfolio efficiency based on the Fuzzy Data Envelopment Analysis, the market efficiency and the random efficiency.

It is recommended to investors to employ FUZZY and DEA approaches for assessing the admitted companies in the securities exchange and making decisions regarding the investment with respect to the interests and the factors affecting decision.

REFERENCES

1. James R. Morris and John P. Daley, 2009. Introduction to Financial Models for Management and Planning, CRC press, pp: 535-536.
2. Kabnurkar Amit, 2001. Mathematical Modeling For Data Envelopment Analysis With Fuzzy Restrictions on Weights,
3. Chi-Ming Lin and Mitsuogen, 2006. An Effective Decision- Based Genetic Algorithm Approach to Multi objective Portfolio Optimization Problem, Applied Mathematical Sci., pp: 19.
4. Aranha. Claus, 2007. Portfolio Management with Cost Model Using Multi Objective Genetic Algorithms, Graduate school of Frontier sciences The university of Tokyo, August.
5. Ping-Teng Chang and Jung-Hua Lee, 2012. A fuzzy dea and knapsack formulation integrated model for project selection, www.sciencedirect.com,
6. Sheikh Moradi and Masumeh, 2008. Calculation of Malmquist productivity index of fuzzy cost and revenue in Iran insurance branches in Tehran, M.S. dissertation. Tehran Science & Research University.
7. Tiryaki. Fatma and Ahlatcioglu. Beyza, 2009. Fuzzy Portfolio Selection Using Fuzzy analytic hierarchy process, www.elsevier.com,
8. Hao, F.F. and Y.K. Liu, 2009. Meanvariance models for portfolio selection with fuzzy random returns, springer,
9. Khoshfetrat Sahar, Daneshvar Sahand, 2011. Improving weak efficiency frontiers in the fuzzy data envelopment analysis models, Appl. Mathematical Modelling.
10. Shiang-Tai Liu, 2011. A fuzzy modeling for fuzzy portfolio optimization, www.sciencedirect.com.
11. Powers, J. and P.R. McMullen, 2000. Using Data Envelopment Analysis to Select Efficient Large Market Cap Securities, J. Business and Management.
12. Alexander. Gordon, J. and M. Baptista. Alexandre, 2010. Active portfolio management with benchmarking: A frontier based on alpha, www.sciencedirect.com.
13. Anagnostopoulos, K.P. and G. Mamanis, 2010. A portfolio optimization model with three objectives and discrete variables, www.sciencedirect.com.