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Spin-Orbital Effect in Hi-Density Low-Temp Plasma with Ssc Potential in Hydrogen-Like Atoms

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Abstract: In this article Dirac's solutions was used to study the spin-orbital interaction effects of an orbital particle on ionic energy levels, using ssc potential for hydrogen like elements to find their energy levels, particularly, their critical radii. In this review, a comparison of similar calculations y different studies made by using similar potential, solving Schrodinger's equation through different approaches, have been also discussed.

Key words: Missing

INTRODUCTION

A large group of physicists have studied thermodynamic properties of high temperature low density Plasma using Yukawa type (YT) potential, also called static screened Coulomb's potential (ssc). In this case static screened implies that the time period of an orbital particle ~ 10^{-15} sec, is considerably shorter than the life time of an ionic state. This is very much correct for plasma state. In high density low temperature case such time difference becomes considerably large. It has been shown that in a low density high temperature plasma [1-7], such potential describes the two body interaction through a chains of intermediary particle, just like among nucleons. This potential also called Debye-Huckle (DH) potential, is a sort of renormalized atom approach first used in the works [8, 9] for copper, using in Schrodinger's quantum wave equation. In this scheme one utilizes the free atom wave function truncated at the Wigner-Seitz's radius after having renormalized it within this radial sphere. It can be shown basing upon uncertainty principle ($\Delta x.\ \Delta p \sim$) that any attractive potential which approaches to zero faster than $\sim r^{-2}$,as r approaches to infinity can have, at most, a finite number of energy levels [5]. In this respect YT-potential certainly qualifies this condition as it gives potential with faster approach to zero than Coulomb's potential. The YT-potential also has the following advantages. The degeneracy w. r. t the Azimuthal quantum number ℓ (due to circular symmetry of Coulomb's potential), is removed and does not depend upon the magnetic quantum number. The oscillatory factor "exp (-r/D)" of the YT potential keeps on disturbing the equipotential circular orbits to remove degeneracy. Here D serves as a screening radius. In other words energy pressure non linearity is removed for thermodynamic properties. For any thermodynamic calculations where partition function for an isolated atom is used, Coulomb's potential can't serve the purpose because of its infinite energy states, as the screening radius becomes shorter i.e. $D \tilde{} D_c$, the highest bound states disappear [9]. This potential also describes the potential of an impurity in metals and in semiconductors. In this paper we shall use Dirac's relativistic quantum theory to account for spin –orbital effects for ionic energy levels. Since in the derivation of Dirac's equation spin-orbital interaction is accounted for in the form i.e. $L.\sigma \rightarrow J^2 - L^2 - \frac{\sigma 2}{4}$. As in plasma field, physicists are trying to include high-density quantum effects in their classic studies. Especially Cold Fusion plasma i.e. plasma due to high density, catalyzed by muon is becoming an interesting field for future energy resources. In such case fusion is carried out at low temperature that is why it is called Cold fusion catalyzed by muon. By the introduction of muon as an orbital particle, orbital radius is reduced by 206.8 times which is equivalent to high external compression or high atomic density. We want to see how this could improvise using Dirac's quantum theory, since Dirac's equation accounts for spin-orbital effects altogether. Further to calculate the thermodynamic parameters i.e. probability density, pressure, statistical sum and Helmholtz free energy, either using Maxwell's

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Table 1: $a_0 = 0.529\times10^{-8}$cm

Z = 1	n_r	x	D_i/a_0
$1S_{1/2}$	0	-1	3.5284
$2S_{1/2}$	1		9.0871
$2P_{1/2}$	1	1	10.0880
Z = 2	n_r	x	D_i a_0
$1S_{1/2}$	0	-1	1.7652
$2S_{1/2}$	1		4.5432
$2P_{1/2}$	1	1	5.0437

Table 2: Comparative analysis

Sources	$D_0(1s)^{za0}$
Hulthen and Laurikainen Variation method [14]	0.8399
Harris by variation method	0.8700
Roger-Graboske [3]	0.8398
Present results based on spin-orbital model	3.5284

distribution for ideal gas particles or Fermi-Dirac's distribution for the fermionic gas, one needs energy eigen values. To calculate pressure either one can use hard-ball approach or fluctuated volume approach. For fluctuated volume approach one can use Gibbs distribution function [12]. We end this introduction once again with remarks that we are only after spin- orbital contribution to the already calculated results as given in Table (1) and (2). Since spin time period of an orbital particle is shorter than the orbital time period. The static effects predominate over dynamic especially for heavy electrons.

Theory and Mathematical Analysis: As in such dealings most of the authors call ssc potential as Debye-Huckle (DH) potential rather to Yukawa potential therefore for onward discussion we shall also call it DH potential. The introduced DH-potential can be found in any fundamental book on plasma and is given by [3]

$$V(r) = \frac{ze^2}{r}\exp(-r/D), \quad r < D \tag{1}$$

$$\text{Where } D = \left[\frac{4\pi}{KT}\sum_{i=1}^{N}\rho i Z_i^2 e^2\right]^{-1/2} \tag{2}$$

is known as Debye's ionic fields range or effective radius in plasma of electronic shell of an ion, ρi – represents density or number of particles per unit volume with Ze-charge. The variable r shows the radial distance with condition $R_0 \le r \le D$, where R_0-nuclear radius of the ion. If we look at the relation (2) we can see that $\rho i \propto T/D^2$. Even a small variation in D would bring effective change in results since in the said equation D appears in squared form and T shall be very low. Expression (1) for D = r, reduces to the ordinary Coulomb's potential of an isolated atom, only divided by a factor of $e^{-1} = 2, 17$. As a result potential would become faster than r^{-2} and consequently stronger screening would take place. The screening is a repulsive perturbation displacing the isolated energy levels upwards and eventually in to the continuum [8] as Debye length becomes smaller. For light elements Bohr's average radii of any energy level $a_o < D \sim 10^{-6}$ cm [5], the Debye length. There is a finite value of the screening length D_c (n, ℓ) for which the energy becomes zero. In other words bound system moves to the threshold of continuum. In general the DH-potential is applicable for partially ionized plasma. For this purposed different authors have used different approximation methods. For example (1) first order perturbation method; (2) variation method [13]; (3) even direct numerical calculations [3, 14]. Schrodinger wave equation does not account for the spin-orbital interaction and also the exclusion considerations which can be accounted only by Dirac's quantum equation. Therefore for this purpose assuming radial symmetry we shall solve Dirac's radial equation with DH-potential which can be given in $\lambda = \hbar/mc=1$, system of units (see appendix 1) as follow [15];

$$\begin{aligned}(rf(e))' - \frac{x}{r}(rf(r)) + (e - 1 - V(r))(rg(r)) = 0\\ (rg(r))' + \frac{x}{r}(rg(r)) - (e + 1 - V(r))(rf(r)) = 0,\end{aligned} \tag{3}$$

Where f(r) and g(r) are Dirac's radial functions, ε – kinetic energy; and total energy $mc^2 = 1$, as units being used in this treatise. x-relativistic quantum number defined as

$$\left.\begin{aligned} x &= -(\ell+1), \text{ ifj}=\ell+1/2\\ x &= \ell, \quad \text{ifj}=\ell-1/2\end{aligned}\right\} \begin{aligned}&\ell - \text{orbital quantum number,}\\ &\text{j} - \text{total quantum number,}\end{aligned}$$

These quantum conditions means both the spinners (the possible orbits of the orbital particle) are not moving in the same orbit. If the spin (+1/2) practically moves on ℓ, then spin (-1/2) must be moving on ℓ. Both will have different critical Debye radii. Both of them will not be ionized under the same pressure conditions. This is in accordance with Pauli's exclusion principle. This implies that for relativistic case the volume demand for both the orbitals would be more than that of Schrodinger's case. So we expect the Debye to Bohr's radii ratio greater than that of Schrodinger to Bohr's ratio. Using potential (1) in (3) and the system of units as mentioned earlier,

for which $e^2/4\pi=\alpha$ and the ionization condition $\varepsilon = mc^2 = 1$, we get a new set of Dirac's equation as follow,

$$F'(r) - \frac{x}{r}F(r) + \frac{az}{r} - e^{-\beta r}G(r) = 0, \quad \beta = 1/D$$

$$G'(r) + \frac{x}{r}G(r) - (2\varepsilon + \frac{az}{r}e^{-\beta r})F(r) = 0; \tag{4}$$

Where F(r) = rf(r) and G(r) = rg(r). For non-ionization level solution, we must keep the terms (ε -1) and (ε +1) to evaluate general energy expression. If we take finite nucleus atom, instead of point nucleus atom we have to assume some nuclear model. Generally most of the authors have used uniform distribution of nuclear charge over some spherical volume of radius R_0. For which case distribution of potential energy in side nuclear volume is given as

$$V(r) = \frac{-az}{R_0}[\frac{3}{2} - (\frac{r}{R_0})^2]; \tag{5}$$

For this purpose one will have to solve the Dirac's set of radial equation (3) both for nuclear as well as extra nuclear or ionic spaces. In this way found solutions are sewed at the nuclear surface r= R_0 in accordance with boundary condition [16].

$$f(r=R_0)_{in} = f(r=R_0)_{ext}, \tag{6}$$

Where $f(r)_{in}$ represent solution from $0 \le r \le R_0$ and $f(r)_{ext}$ in view of the Debye's critical length from $R_0 \le r \le D$ and it is assume that bound particle (electron) wave function does not penetrate in to the nuclear space. Due to sewing of the two types of wave function at r=R_0 and reducing the upper limits of volume radii from r= ∞ to r=D a change in principal quantum number " n " would take place and in our it case is denoted by Δn. Thus the new quantum number would be "n+Δn" with condition n $\ge$ Δn. The energy calculated on the basis of Dirac's theory considering point nucleus model with coulomb's potential is given by [15, 16].

$$e_n = m[1+(\frac{az}{n+r})^2]^{-1/2}, \tag{7}$$

With condition $? = \sqrt{x^2 - a^2z^2}$, n- principal quantum number. For $x < 0$. n= 0, 1…., called radial quantum number and for $x > 0$, n'= 1, 2… In general n'=n-1; where n=0, 1, 2…. for all cases. Since we as a first attempt, are interested mainly for spin–orbital contribution to the potential energy at critical level therefore we shall assume point model for our all coming considerations. Solving the first equation of (4) for G(r) we get

$$G(r) = \frac{x}{az}e^{\beta r}F(r) - \frac{r}{az}e^{\beta r}F'(r), \tag{a}$$

and then differentiating G(r) w.r.t r we can get

$$G'(r) = re^{\beta r}F''(r) - \frac{1}{az}[e^{\beta r} + r\beta e^{\beta r} + \frac{x}{az}]F'(r) + \frac{x\beta}{az}e^{\beta r}F(r), \tag{b}$$

Using these values of G (r) and G'(r) in the second equation of (4) we get.

$$F''(r) + (\frac{1+\beta r}{r})F'(r) + (\frac{2eaz}{r}e^{-\beta r} + \frac{a^2z^2}{r^2}e^{-2\beta r} - \frac{x\beta}{r} - \frac{X^2}{r^2})F(r) = 0, \tag{18}$$

Analytically it is not possible to solve such a differential equation. We may have approximated solution depending upon our requirement. For Schrodinger's equation the factor exp (-βr) can also be replaced considering Bohr's classical quantum condition [17] only with difference of units i.e. $e^2/4\pi = \alpha$

$$\exp(-\beta r) = \frac{n^2\hbar^2}{m\,azr(\beta r+1)} \tag{9}$$

To derive this condition Rawls and Schulz used Newton's Second Law i.e. centrifugal force around the Bohr's orbit was put equal to differential of DH-potential w.r.t r and after simple mathematical manipulation equating it with Bohr's quantized angular momentum but ultimately resorted to put βr =1 under the pretext of bound state energy condition which is equivalent to the use of first order perturbation in Coulomb's potential to eliminate the degeneracy. In fact the necessary and sufficient condition for bound state in classical quantum theory is that βr<1 or r<D. Here we shall also use the first order perturbation to make the problem easy i.e expanding the factor exp (-βr in Taylor series as follow.

$$\exp(-\beta r = [1 - \beta r + \ldots..], \tag{10}$$

Using first order bound plane wave perturbation given by (10), in (8) which is valid enough for our condition of critical Debye length as βr therefore $(-\beta r)^2 << 1$. Keeping only the first term i.e. $\exp(-\beta r)^{=} (1-\beta r)$ and substituting new variables in differential Equation (8) to transform it into a some standard form that is:

$F(r) = r^{-1/2}\exp(\frac{\beta r}{2})W(r)$ we get

$$W''(r) + [-\frac{-(\beta^2 + 4b)}{4} + \frac{(a - \beta/2)}{r} + \frac{1/4 - r^2}{r^2}]W(r) = 0, \quad (11)$$

Where a= $(2\epsilon\alpha z - 2\alpha^2 z^2 \beta - x\beta)$, b= $2\epsilon\alpha z\beta$ and

$\gamma = \sqrt{x^2 - a^2 z^2}$. Let ξ=[$(\sqrt{\beta^2 + 4b})$ r], then equation (11) can be further transformed exactly into the following equation

$$W''(\xi) + [-\frac{1}{4} + \frac{(2\alpha - \beta)}{2\sqrt{\beta^2 + 4b}}\ \frac{1}{\xi} + \frac{1/4 - \gamma^2}{\xi^2})W(\xi) = 0 \quad (12)$$

This form of the differential equation [18] is identical to that of the Whittaker's differential equation with $k = \frac{2\alpha - \beta}{2\sqrt{\beta^2 + 4b}} \approx$ - (x+1/2), which serves as principle quantum number. For ℓ =0 and x =1; k=3/2. This function satisfies the Whittaker's equation for values of k and γ and for all values of ξ except for –ve real values. If $W_{k,\tilde{a}}(\xi)$ is the solution of the said differential equation then $W_{-k,\tilde{a}}(-\xi)$ would also be solution of the same differential equation. Since simultaneous change of signs of (k, γ), the equation (12) remains unchanged or symmetric. These are the basic solution of the said equation. According to Maxwell's electromagnetic theory electric charges interact with each other either by emission or absorption of photons. If the under study ion is surrounded by relatively denser media its communication range to other ions must be reduced. That is why Coulomb's potential was replaced by Debye-Huckel's potential which accounts for density dependence of interaction range. Since we are studying Dirac's equation with condition $\varepsilon > mc^2$=1 i.e. at ionization level so our r=D_i, therefore our electron wave function has to oscillate between the boundaries $R_0 \le r \le D_i$ which in quantum language are known as turning points. In Dirac's equation spectrum characteristics are defined by the condition: if $\varepsilon > mc^2$ 'the energy spectrum would be continuous and for $\varepsilon < mc^2$, the spectrum would be discrete. A size criterion can also be used i.e. a state is said to be not bound if its average radius exceeds the average inter nuclear separation at a given density and temperature. The ionization is achieved at $\varepsilon = mc^2$ [15, 16]. This is what we have been following so for. For larger values of indices k we can have approximated solution as given [14].

$$Wk,?(\xi) \approx \frac{1}{\sqrt{\pi}}\Gamma(2\gamma + 1)k - \gamma - 1/4\cos(2\sqrt{\xi k} - \gamma\pi - \pi/4), \quad (13)$$

Quantity of special interest is the critical screening for the ground state of D (1s) of the two body system in the ssc type potential. This parameter has been computed by a large group of physicists through a variety of techniques. For our case function given by (13) must reduce to zero at r=D_i i.e.

$$\cos(2\sqrt{\xi k} - \gamma\pi - \frac{\pi}{4}) = 0, \quad (14)$$

This means our angle of equation (14) must fulfill the condition.

$$2\sqrt{\xi k} - \gamma\pi - \frac{\pi}{4}) = (n\pi + \frac{\pi}{2}) \quad (15)$$

Where n is an integer and after some simplification from (15) we can easily get

$$D_i = \frac{(n + \gamma + 3/4)\pi]^2 + 4x + 2}{8az} \quad (16)$$

To get the `relation (16) we first solved equation (15) for $\sqrt{\xi k}$ and then squaring both sides and again solving it under the condition when r=D_i and also ignoring the smaller terms. This relation gives the critical Debye's radii of nth orbital. For comparison of results with classical Quantum calculations we write equation (16) as follow:

$$\frac{D_t}{a_0} = \frac{[(n + \gamma + 3/4)\pi]^2 + 4x + 2}{8z} \quad (17)$$

To get this form we have divided by a_0 on both sides of equation (16) and putting αa_0 =1 as is admissible by the system of units $\lambda = \frac{\hbar}{mc} = 1$

Results and Comparative Analysis: Critical radius calculated using Coulomb's potential under the same conditions as mentioned above was given by [16] as follow

$$r_C = \frac{[(n + \gamma + 3/4)\pi]^2}{8az};\ n = \text{integer} \quad (18)$$

Comparing (16) and (18) we can easily get

$$D_i = r_C + \frac{2x + 1}{4az};\ \text{for } x < 0,\ D_n < r_n\ ;\ x > 0,\ D_n > r_n \quad (19)$$

Equation (19) shows that $D_i < r_c$ for all $x < 0$ and $D_i > r_c$ for all $x > 0$. This means that the critical radius for Debye's potential at total degeneracy (T=0) is lower than the corresponding critical radii computed using coulomb's potential. In other words since pressure is indirectly connected with volume radius therefore DH potential gives higher value of energy pressure as compared to coulomb potential. This is due to reduction in available volume space because of exclusion principle for fermions in full or partial plasma. Although on compression outer electrons interact more repulsively with the interior electrons which is out weighted largely by the increased nuclear attraction as radial distance between orbital electrons and the corresponding nucleus decreases

In Table (1) we have calculated values of D_i in terms of in $\frac{D_i}{a_0}$ accordance with the equation (17), where Z- element number a_0. Bohar's radius (a_0=0.529 x 10^{-8} cm). In this table we have provided calculations only for Z = (1 and 2) and $x = \pm 1$. In Table (2) we have provided the comparative results calculated by different authors, using different approaches to the same problem. The best results in classical theory are considered to be those of [3] " 0,8398" as given in Table (2) where as our result for $1s_{1/2}$ –state accounting for spin orbital interaction via Dirac's equation is ¯3,5284; which is in accordance with suggested by [3] in the free energy for a given composition of V and T using Maxwell's statics (classical). In high density plasma our results are almost four times higher than the already calculated as given in Table (2). This confirms the existence of the strongness of spin orbital interaction on relativistic quantum theory In term of, spin magnetic dipole moment, spin already have twice effectiveness than orbital effect in classical quantum theory, for example Einstein De Hass effect etc. The inclusion of spin interaction to improvise the dependence of energy on magnetic quantum number and consequently Coulomb's degeneracy is eliminated. From the Table (1) we can also see that energy gaps between $1S_{1/2}$ and $2S_{1/2}$ states for both Z= (1 & 2) is much larger than the gaps between $2S_{1/2}$ and $2P_{1/2}$ states. This means energy level is squeezed towards the bound states by the external pressure. Therefore a sort of energy surface tension is created. As is evident from eq(2) $D^2 xT/\rho_i$. This means for larger charges case D_i would be smaller which is confirmed by the comparison of values for D_i(Z= 1)> D_i(Z=2). Another quantity of interest is the maximum bound quantum number n of the number of states. Solving (16) for n we can get.

$$n_r = [\sqrt{\frac{8aZDi}{\pi^2}} - 4x - 2 - \gamma - \frac{3}{4} \qquad (20)$$

From equation (20) we can get the maximum umber of bound states corresponding to D_i which are inevitable to calculate the statistical sum. Since $D_i < r_c$ for all $x < 0$ this means that the number of bound states in Debye's potential is numerically lower than those of Coulomb's potential. The disappearance of the uppermost bound states under screening is by the reduction of separation between n levels toward its continuum, to make the density of states higher. Since spin-orbital interaction demands for the available energy states value smaller than the ordinary case because of exclusion principle, therefore the number of bound states is expected to be less than for any other approach.

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Appendix: In the system of unit in which Compton's wave length λ=1. One Compton unit is = 3,862x10^{-11} cm. Acceleration (a) $= \frac{mc^2}{h}$. Force (f) $= \frac{m^2c^2}{h}$, pressure (p) $= = \frac{m^4c^2}{h^4}$

electron charge $e^2 = \hbar c$ [20].

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