Middle-East Journal of Scientific Research 28 (5): 391-394, 2020
ISSN 1990-9233

DOI: 10.5829/idosi.mejsr.2020.391.394

Comparison of Newton's Interpolation and Aitken's Method with Runge-Kutta Method for Solving First Order Differential Equation

Ide Nasr Al Din

Department of Mathematics, Faculty of Science, Aleppo University, Syria

Abstract: There has been greater attempt to solving differential equations by analytic methodsand numerical methods. Most of researchers treated numerical approach to solve first order ordinary differential equations. These methods such as RungeKutta method, Taylor series method and Euler's method, etc. Faith Chelimo Kosgeistudied this problem by combined the newton's interpolation and Lagrange method. In this study we will treat the combined Newton's Interpolation and Aitken's Method with Runge-Kutta Method for Solving First Order Differential Equation.

Key words: Differential equation · Analytic method · Numerical method · Newton's interpolation method · Aitken's method · Runge-Kutta Method

INTRODUCTION

In real life many problems can be formulated in theform of ordinary differential equation, especiallythe equations of first order, hence we need to study and solve the differential equations. Numerical method is used to solve these problems. The differential equation problem [1-7], consists of at least one differential equation and at least one additional equation such that the system together have one and only one solution called the analytic or exact solution to distinguish it from the approximate numerical solutions that we shall consider. In thispaper, to find the solution of differential equation of first order, Faith C. K [1] studied this problem by using combination of newton's interpolation and Lagrange method. In this study we will Compared of Newton's Interpolation and Aitken's Method with Runge-Kutta Method for Solving First Order Differential Equation [2-4, 6, 8]. Finally we verified on a number of examples and numerical results obtained show the efficiency of these methods.Let's consider the following first order differential equation or initial value problem:

$$Y' = f(\mathrm{x}, \mathrm{y}), \ \mathrm{y}(\mathrm{x_o}) = \mathrm{y_o} \tag{1}$$

where f (x, y) is a known function and the values in the initialconditions are also known numbers.

Combined Newton's Interpolation and Lagrange Method [1]: This study combine both Newton's interpolation methodand Lagrange method .it used newton's interpolation method tofind the second two terms then use the three values for y toform a quadratic equation using Lagrange interpolation method as follows;

Newton's Interpolation Method [1, 6]:

$$f_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + ... + a_n(x - x_0)(x - x_1)...a_2(x - x_{n-1}) \tag{2}$$

where

$$a_0 = y_0,\ a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)},\ a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} \tag{3}$$

etc

Lagrang Interpolation Method [1, 6]:

$$y_n = \frac{(x - x_1) - (x - x_2)}{(x_0 - x_1)(x_0 - x_2)} y_0 + \frac{(x - x_0) - (x - x_2)}{(x_1 - x_0)(x_1 - x_2)} y_1 + \frac{(x - x_0) - (x - x_1)}{(x_2 - x_0)(x_2 - x_1)} y_2 \tag{4}$$

Corresponding Author: Dr. Ide Nasr Al Din, Department of Mathematics, Faculty of Science, Aleppo University, Syria.

Description of the Method: This method will combine both Newton's interpolation methodand Aitken method .it used newton's interpolation method tofind the second two terms then use the three values for y toform a linear or quadratic equations using Aitken interpolation method as follows;

$$f_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + ... + a_n(x - x_0)(x - x_1)...a_2(x - x_{n-1}) \quad (5)$$

where

$$a_0 = y_0,\ a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)},\ a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} \quad (6)$$

etc

$$y_1 = a_0 + a_1(x - x_0) \quad (7)$$

$$y_2 = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) \quad (8)$$

Note: We can use Newton's Forward Interpolation Formula instead of Newton's divided Interpolation method in (2.1).

Aitken Interpolation Method [6]:

$$P_{o,k}(x) = \frac{1}{x_k - x_o}\begin{vmatrix} y_o & x_o - x \\ y_k & x_k - x \end{vmatrix} \quad (9)$$

$$P_{o,1,2}(x) = \frac{1}{x_2 - x_1}\begin{vmatrix} P_{o,1}(x) & x_1 - x \\ P_{o,2}(x) & x_2 - x \end{vmatrix} \quad (10)$$

$$y_n = P_{o,1,2,...,n}(x) = \frac{1}{x_n - x_{n-1}}\begin{vmatrix} P_{o,1,...,(n-1)}(x) & x_{n-1} - x \\ P_{o,1,...,(n-2),n}(x) & x_n - x \end{vmatrix} \quad (11)$$

Runge-Kutta Method [8]: For the equation y' = f(x, y) and the initial condition $y(x_o) = y_o$

$$y(x+h) \sim y(x) + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) \quad (12)$$

$$K_1 = h.f(x, y)$$
$$K_2 = h.f\left(x + \frac{h}{2}, y + \frac{K_1}{2}\right)$$
$$K_3 = h.f\left(x + \frac{h}{2}, y + \frac{K_2}{2}\right)$$
$$K_4 = h.f(x + h, y + K_3) \quad (13)$$

Examples: In this section, we will check the effectiveness of thepresent technique (3). First numerical comparison for the following test examples taken in [1].

Example 1:

$$\text{Solve} = \frac{dy}{dx} = 1 - y, \quad y(0) = 0$$

By taking the step h=0.01
First by using Newton's interpolation, we have

$$a_0 = 0 = y_0$$
$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = [\frac{dy}{dx}]_{0,0} = 1$$
$$y_1 = 0 + 1(0.01 - 0) = 0.01$$
$$a_2 = \frac{\frac{f(x_2) - f(x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} = \frac{[\frac{dy}{dx}]_{0.01,0.01} - [\frac{dy}{dx}]_{0,0}}{0.02 - 0} = -0.05$$
$$y_2 = 0 + a1(0.02 - 0) - 0.5(0.02 - 0)(0.02 - 0.01) = 0.0199$$

Now, forming linear and quadratic using Aitken Method

$$P_{0,1}(x) = x$$
$$P_{0,2}(x) = 0.995x$$
$$P_{0,1,2}(x) = -0.5x^2 + 1.005x$$

Hence, we can take the approximation solutions of linear and quadratic using Aitken Method and Runge-Kutta method with the exact solution, Table 1.

Example 2:

$$\text{Solve} = \frac{dy}{dx} = x^2 - y, \quad y(0) = 1$$

By taking the step h=0.01
First by using Newton's interpolation, we have

$$a_0 = 1 = y_0$$

Table 1: Solution of $\frac{dy}{dx} = 1 - y, \quad y(0) = 0$

x	Combined Newton's Interpolation and Aitken	*Exact Values*	*Runge Kutta Values*	*abs* (*Runge Kutta – exact*)	*abs* (*Combined* Newton's Interpolation and Aitken – *exact*)
0	0	0	0	0	0
0.01	0.0100	0.009950166	0.01	0.000049834	0.000049834
0.02	0.0199	0.019801326	0.019900000	0.000098674	0.000096740
0.03	0.0297	0.029554466	0.029701000	0.000146534	0.000145534
0.04	0.0394	0.039210560	0.039403990	0.000193430	0.000189440
0.05	0.0490	0.048770575	0.049009950	0.000239375	0.000229425
0.06	0.0585	0.058235466	0.581985059	0.000036960	0.000264534
0.07	0.0679	0.067606180	0.067934652	0.000328472	0.000293820
0.08	0.0772	0.076883653	0.077255305	0.000371653	0.000316347
0.09	0.0864	0.086068814	0.086482752	0.000413939	0.000331186
0.1	0.0955	0.095162581	0.095617924	0.000455344	0.000337419

Table 2: Solution of $\frac{dy}{dx} = x^2 - y, \quad y(0) = 1$

x	Combined Newton's Interpolation and Aitken	*Exact Values*	*Runge Kutta Values*	*abs* (*Runge Kutta – exact*)	*abs* (*Combined* Newton's Interpolation and Aitken – *exact*)
0	1	1	1	0	0
0.01	0.990000000	0.990050166	0.990000000	0.000050168	0.000050168
0.02	0.980101000	0.980201326	0.980101000	0.000100326	0.000100326
0.03	0.970303000	0.970454466	0.970303990	0.000150476	0.000150476
0.04	0.960606000	0.96081056	0.960609950	0.000200610	0.000204560
0.05	0.950101000	0.951270575	0.951019851	0.000250724	0.001169575
0.06	0.941515000	0.941835466	0.941517169	0.0000318297	0.000320466
0.07	0.932121000	0.93250618	0.932137997	0.000368183	0.000385180
0.08	0.922828000	0.923283653	0.922865617	0.000418036	0.000455653
0.09	0.913636000	0.914168814	0.913700691	0.000467449	0.000532814
0.1	0.904545000	0.905162582	0.904644951	0.000517631	0.000617582

$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = [\frac{dy}{dx}]_{0,1} = -1$$

$$y_1 = 1 - 1(0.01 - 0) = 0.99$$

$$a_2 = \frac{\frac{f(x_2) - (x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} = \frac{[\frac{dy}{dx}]_{0.01,0.99} - [\frac{dy}{dx}]_{0,1}}{0.02 - 0.01} = 0.505$$

$$y_2 = 1 - 1(0.02 - 0) + 0.505(0.02 - 0)(0.02 - 0.01) = 0.980101$$

Now, forming linear and quadratic using Aitken Method

$$P_{0,1}(x) = 1 - x$$

$$P_{0,2}(x) = 1 - 0.99495x$$

$$P_{0,1,2}(x) = 0.505x^2 - 1.00505x + 1$$

Hence, we can take the approximation solutions of linear and quadratic using Aitken Method and Runge-Kutta method with the exact solution, Table 2.

Example 3:

Solve = $\frac{dy}{dx} = y - x, \quad y(0) = 0.5$

By taking the step h=0.01

First by using Newton's interpolation, we have

$$a_0 = 0.5 = y_0$$

$$a_1 = \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} = [\frac{dy}{dx}]_{0,0.5} = 0.5$$

$$y_1 = 0.5 + 0.5(0.01 - 0) = 0.505$$

$$a_2 = \frac{\frac{f(x_2) - (x_1)}{(x_2 - x_1)} - \frac{f(x_1) - f(x_0)}{(x_1 - x_0)}}{(x_2 - x_0)} = \frac{[\frac{dy}{dx}]_{0.01,0.505} - [\frac{dy}{dx}]_{0,0.5}}{0.02 - 0} = -0.25$$

$$y_2 = 0.5 + 0.5(0.02 - 0) - 0.25(0.02 - 0)(0.02 - 0.01) = 0.50995$$

Now, forming linear and quadratic using Aitken Method

Table 3: Solution of $\frac{dy}{dx} = y - x$, $y(0) = 0.5$

x	Combined Newton's Interpolation and Aitken	*Exact Values*	*Runge Kutta Values*	*abs* (*Runge Kutta – exact*)	*abs* (*Combined* Newton's Interpolation and Aitken – *exact*)
0	0.5	0.5	0.5	0	0
0.01	0.505000000	0.504974916	0.505000000	0.000025084	0.000025084
0.02	0.509950000	0.50999933	0.509950000	0.000049330	0.000049330
0.03	0.514850000	0.514772733	0.514849500	0.000076767	0.000072670
0.04	0.519700000	0.519594612	0.519697995	0.000103383	0.000105388
0.05	0.524500000	0.524364451	0.524494975	0.000130524	0.000135549
0.06	0.529250000	0.529081726	0.529239925	0.000158199	0.000168274
0.07	0.533950000	0.533745909	0.533932324	0.000186420	0.000204091
0.08	0.538600000	0.538356466	0.543157364	0.000215394	0.000243534
0.09	0.543200000	0.542912858	0.547697961	0.000244506	0.000287142
0.1	0.547750000	0.547414541	0.552174941	0.000283420	0.000335459

$$P_{0,1}(x) = 0.5 - 0.5x$$

$$P_{0,2}(x) = 0.5 - 0.4975x$$

$$P_{0,1,2}(x) = -0.25x^2 + 0.5025x + 0.5$$

Hence, we can take the approximation solutions of linear and quadratic using Aitken Method and Runge-Kutta method with the exact solution, Table 3.

CONCLUSIONS

In this work, we havebeen comparedRunge-Kutta method andNewton's interpolation and Aitken's method with the exact solution to solve first order differential equation, we find the same order of error for these two methods.

REFERENCES

1. Faith, C.K., Solution of First Order Differential Equation Using Numerical Newton's Interpolation and Lagrange Method, International Journal of Development Research, ISSN: 2230-9926, Vol. 08, Issue, 02: 18973-18976, February, 2018.
2. Dr. CuneytSert Numerical Methods Interpolation Mechanical Engineering Department Middle East Technical University Ankara, Turkey 2004. 3. Isaacson E., Keller H.B., Analysis of numerical methods, Second edition, Dover, Mineola, NY, 1994.
4. Stoer, J. and R. Burlisch, 1993. Introduction to numerical analysis, Second edition, Springer-Verlag, New York, NY, 1993.
5. Islam, S., Y. Khan, N. Faraz and F. Austin, 2010. Numerical solution of logistic differential equations by using the Laplace decomposition method, World Appl. Sci. J., 8: 1100-1105.
6. Numerical Analysis, Richard L.Burden, J.Douglas Faires, 1989, PWS-KENT Publishing Company, Boston. 7. The Numerical Solution of ordinary and partialdifferential equations, Granville Sewell 1992, Academic Press, Inc, New York.
8. Davis, P.J., 1975. Interpolation and approximation, Second edition, Dover, New York, NY, 1975.