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Optimizing the Nigeria Electricity Supply Chain Network: A Multi Start Value Approach

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Abstract: In this paper, we propose an equilibrium condition of the Nigeria Electricity Supply Chain Network and Demand Market by introducing an additional tier in the Variational inequality framework. The Power Retailers as an additional tier is not captured in the present restructuring of the Nigeria Electricity Supply Chain Network for effective service delivery of electricity to end time consumers. In paper we proposed an additional tier that is, the power retailers which the present restructuring did not capture. The paper also seek to discuss the effect of a de-monopolize electricity distribution network as against the present monopoly enjoyed by the Power Distribution Companies (DISCOS).The model covers the power generators (GENCOS), power transmitters (TRANSCOS), power distributors (DISCOS), power retailers (RETCOS) and the demand markets (CONSUMERS). We employed a multi-start value approach for the determination of equilibrium prices and power outputs as well as their respective Lagrange multipliers based on three different tariff customers. The result of the analysis showed that the least price of power a customer in R2T will be willing to pay is ₦ 242.62, for customers in R2S the least price they will be willing to pay is ₦238.57, while for R1 customer the least price is ₦ 236.01.

Key words: Consumers · Distribution Companies · Distribution Network · Electricity Supply · Power Generator

INTRODUCTION

Efficient power distribution has been identified as an important part of any electricity supply chain across the globe. The prevailing situation in Nigeria where small scale industries are finding it very difficult to survive with epileptic power supply and distribution is a major challenge facing the sector amongst other sectors of the Nigerian economy.

With the present electricity supply chain in Nigeria, consistency in power generation, effective transmission of power and a more efficient distribution of power to demand customers remains the only hope to position Nigeria among the industrialized nations.

Authors in Chinwuko *et al.* [1] examines the problem of extreme electricity shortage in Nigeria especially in Awka the capital of Anambra State, noted that the deficiency in electricity supply in Nigeria is multidimensional and are attributed to inadequate infrastructure, inadequate funding of the power sector and energy losses from generation to distribution to the demand customers. Based on their findings, they suggested that new substations should be built to accommodate the proposed new load demand and that the implementation will ensure constant and adequate power supply in Awka and its environs. As a result, the Nigeria government has added new substations across the 6 geo-political zones in the country but demand customers always complain of low supply and over billing from the distribution companies.

In Olugbenga, Jumah and Phillips [2], Olugbenga *et al.* opined that in other to achieve a sustainable electricity market which will ensure evidence based economic growth, effort should be made to monitor the Power Reform process, criticize, encourage and praise where necessary rather than folding arms and calling it a government affair.

Ohajianya *et al.* [3] opined that regardless of the general belief that the erratic power supply problem of Nigeria is as a result of the low power generation capacity

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of the Country, the major cause of this problem comprises of energy wastage by consumers occasioned by the estimated billing or crazy billing system adopted by power distribution companies. Also, they suggested the need for upgrading of power distribution and transmission equipment; and engagement / recruitment of competent and qualified work force by the electric power companies especially the distribution companies who have direct contact with the demand customers. The industrial and local demand for electricity in Nigeria has outmatched the inconsistent generation and epileptic distribution. Electricity situation like such as the present state in Nigeria will not hesitate to crumble economic growth and development of any nation. Even with the availability of vast natural resources in the country, Nigeria has been struggling to shape its economy.

In Sambo [4] noted that the estimated total investments required to meet the demand for the Optimistic Growth Scenario in the electricity industry in Nigeria is about US$ 484.62 billion. This is a huge investment which the Federal Government cannot fund alone. Hence, the urgent need for state governments, private sector and foreign investors to engage in small funding of electricity projects. According to Agboola [5], the electricity problem in Nigeria will be resolved only when Independent Power Producers become key players in the industry. This is because the huge financial investment required to revive the industry will be sorted outside government resources.

Speaking on monopoly in the electricity industry, in Leibenstein [6] Leibenstein opined that having a de-monopolized industry will bring about efficiency in the electricity market. [7], in support explained that absence of competition and poor service culture has severely constrained the much-desired adequate electricity generation capacity and effective delivery in Nigeria.

In Nigeria an estimated 40 percent of the total population have access to electricity and majority that constitute this 40% are concentrated in urban areas. Also, the inconsistency generation of power and poor distribution has forced the distribution companies to use the unpopular so called unannounced load shedding approach, which is unacceptable and intermittent power outages which has caused lots of loss and damages to the demand customers. Before the privatization of the electricity industry in Nigeria, government representatives from the Federal and State governments claim that privatization of the power sector will enhance the quality of service delivery in the industry. But after about seven years of privatization of the electricity industry in Nigeria, the situation tends to worsen with numerous cases of unprecedented outages.

It is no longer news that the electricity supply chain network records the highest complain at the distribution point. This is obvious from the National Electricity Regulatory Commission (NERC) records of petition against the sector across the country. In Nagurney [7], Nagurney defined supply chain network as a critical infrastructure for the production, distributions and consumption of goods as well as services in the present day globalized network economy. In their contribution, Chun *et al*. [8] explained that the supply chain is often viewed as a complicated network which has made the precise definition of this concept to be difficult. In Braido, Borenstein and Casalinho [9], Braido *et al.,* noted that to solve the design problem of a supply chain network, the network is broken down into sub problems. This growing interest in the design of a supply chain network begins with the identification of interesting sites that may support the skills needed for new installations. Hence, this paper seek to propose a de-monopolize electricity distribution network as against monopoly of electricity distribution network.

MATERIALS AND METHODS

This section deals with the research materials and methods for the electric supply chain network equilibrium (ESCNE) model. The model proposed in this paper were represented and solved by the Variational Inequality problem approach.

Algorithms Method for Variational Inequalities: The algorithms for solving variational inequalities {VI(S, F)} are usually classified into several categories depending upon formulation of method exploits. The algorithms for solving variational inequalities can also be categorized based on the sub-problems that are solved in each iteration. A general approach to solving VI(S, F) consists of creating a sequence $\{x^k\} \subset S$ where, S is a closed convex set such that each,

x^{k+1} solves $VI(S, F^k)$ so that

$$\left\langle F^k\left(x^{k+1}\right), y - x^{k+1}\right\rangle \geq 0 \quad \forall y \in S \tag{1}$$

where, $F^k(*)$ is some approximation to $F(x)$ which can be linear or nonlinear as described by [11].

Electric Power Supply Chain Network Equilibrium Models: We now discuss electric power supply chain network equilibrium models.

Variational Inequality of Power Generator: Suppose we assume that a typical power generator g is a profit – maximizer. Let p^{t}_{1gd} denote the price that a power generator g charges a power supplier d per unit of electricity through the transmission service provider, t. There is tendency that the power generator to set different prices for different power distributors. Hence, the optimization problem of the power generator g can be expressed as follows;

$$\text{maximize } U_g(q^t_{gd}) =$$

$$\sum_{d=1}^{D}\sum_{t=1}^{T} p^{t*}_{1gd}\, q^t_{gd} - f_g(Q^1) - \sum_{d=1}^{D}\sum_{t=1}^{T} C^t_{1gd}(q^t_{gd}) \tag{2}$$

subjet to $q^t_{gd} \geq 0$, $\forall$ d=(1, ..., D), t=(1, ..., T)

The optimality conditions of all power generators g; g = (1,....., G), simultaneously, under the above assumptions, can be compactly expressed as :

$$\text{Determine } Q^* \in R_+^{GTD} \text{ Satisfying :}$$

$$\sum_{g=1}^{G}\sum_{d=1}^{D}\sum_{t=1}^{T}\left[\frac{\partial f_g(Q^{1*})}{\partial q^t_{gd}} + \frac{\partial C^t_{gd}(q^t_{gd})}{\partial q^t_{gd}} - \rho^{t*}_{1gd}\right]\times\left[q^t_{gd} - q^{t*}_{gd}\right] \geq 0,\ \forall\, Q^1 \in R_+^{GTD} \tag{3}$$

The first half of formula (3) shows that the optimality, there is a positive flow of electric power between a generator/ distributor pair and so the price charged is equal to the sum of the marginal production cost plus the marginal transaction cost. On the other hand, if the sum exceeds the price, then there will be no electric power flow between the pair.

Variational Inequality of Power Distributors: The term power distributor refers to power marketers, traders and brokers, who serve as load-serving entities. They play a fundamental role in our model since they are responsible for acquiring electricity from power generators through transmission service provider and delivering it to the power retailers. A power distributor d is faced with certain expenses, which may include, for example, the cost licensing and the costs of maintenance.

If we assume that a typical power distributor *d* is a profit-maximizer, one can express the optimization problem of power distributor *d* as follows:

$$\text{maximize } U_d(q_{dr}) = \sum_{r=1}^{R} \rho^*_{2dr}\, q_{dr} - C_d(Q^1, Q^2)$$

$$\sum_{g=1}^{G}\sum_{t=1}^{T} \rho^{t*}_{1gd}\, q^t_{dr} - \sum_{g=1}^{G}\sum_{t=1}^{T} \hat{C}^t_{gd}\, q^t_{dr} - \sum_{r=1}^{R} C_{dr}\, q_{dr} \tag{4}$$

subject to, constraint (12)

$$q^t_{dr} \geq 0,\ \forall\ \text{g}=(1,\cdots,\text{G}),\ \forall\ \text{t}=(1,\cdots,\text{T}) \tag{5}$$

$$q_{dr} \geq 0,\ \forall\ r = (1,\cdots,\text{R}) \tag{6}$$

As noted above, it is assumed that each power distributor seeks to maximize his profit. Hence the optimality conditions of all power distributors $d;d = (1,...,D)$ simultaneously, under the above assumptions, can be compactly expressed as:

Determine, $(Q^{1*}, Q^{2*}, \lambda^{*}) \in R_{+}^{D(GT+R+1)}$ satisfying:

$$\sum_{t=1}^{T}\sum_{d=1}^{D}\sum_{g=1}^{G}\left[\frac{\partial \hat{C}_{gd}^{t*}(q_{gd}^{t*})}{\partial q_{gd}^{t}}+\frac{\partial C_{d}^{t}(Q^{1*}, Q^{2*})}{\partial q_{gd}^{t}}+\rho_{gd}^{t*}-\lambda_{d}^{*}\right]\times\left[q_{gd}^{t}-q_{gd}^{t*}\right]$$
$$+\sum_{d=1}^{D}\sum_{r=1}^{R}\left[\frac{\partial C_{dr}(q_{dr}^{*})}{\partial q_{dr}}+\frac{\partial C_{d}(Q^{1*}, Q^{2*})}{\partial q_{dr}}+\lambda_{d}^{*}-\rho_{2dr}^{*}\right]\times\left[q_{dr}-q_{dr}^{*}\right]+$$
$$\sum_{d=1}^{D}\left[\sum_{g=1}^{G}\sum_{t=1}^{T}q_{gd}^{t*}-\sum_{r=1}^{R}q_{dr}^{*}\right]\times\left[\lambda_{d}-\lambda_{d}^{*}\right]\geq 0,\ \forall\,(Q^{1}, Q^{2}, \lambda)\in R_{+}^{D(GT+R+1)} \tag{7}$$

where λ_d^* is the optimal Lagrange multiplier associated with constraint and λ is the corresponding D – dimensional vector of Lagrange multipliers, $\lambda : \lambda = (\lambda_1,...,\lambda_1,...,\lambda_1)$.

Variational Inequality of Power Retailers: In this subsection, we discuss the description of the behavior of energy retailers. Electricity retailing involves the supply of electricity to residential, small commercial and industrial customers. Retailer r should simultaneously face with the power distributors and the demand consumers in the process of transmitting the product. Nevertheless, the quantity of product sold by power retailer r does not exceed the total products obtained from all of the power distributors, namely:

$$\sum_{k=1}^{K} q_{rk} \leq \sum_{d=1}^{D} q_{dr},\ \forall\, r = (1,\cdots,R) \tag{8}$$

$$C_r = C_r(Q^1, Q^2, Q^3),\ \forall\, r = (1,\cdots,R) \tag{9}$$

Let function ρ_{3rk} denote the price associated with transmitting power from retailer r to end – consumer, k, with 3 sublevels. Let function C_{rk} denote the transaction cost associated with power retailer r transmitting electric power to end – consumer k, where;

$$C_{rk} = C_{rk}(Q^3),\ \forall\, r = (1,\cdots, R),\ k = (1,\cdots, K) \tag{10}$$

The purpose of the power retailer r is to maximize its profit, which can be modeled as an optimization problem:

$$\text{maximize} \sum_{k=1}^{K} \rho_{3rk}^{*}\, q_{rk} - \sum_{k=1}^{K} C_{rk}(Q^3) - C_r(Q^1, Q^2, Q^3) - \sum_{d=1}^{D} \rho_{2dr}^{*}\, q_{dr} - \sum_{d} \hat{C}_{dr}(q_{dr}) \tag{11}$$

$$\text{subject to: } \sum_{k=1}^{K} q_{rk} \leq \sum_{d=1}^{D} q_{dr} \tag{12}$$

$$q_{dr} \geq 0, \forall d = (1,...,D)) \tag{13}$$

$$q_{rk} \geq 0, \forall k = (1,...,K) \tag{14}$$

Suppose that all retailers compete in a non-cooperative manner in the retailing market of the product and that the transaction cost function for each retailer is continuously differentiable and convex. The Nash equilibrium solution for the retailers is equivalent to solving the following variational inequality.

Suppose we are expected to find a vector $(Q^{1*}, Q^{2*}, Q^{3*}, \beta^*) \in R_+^{(GTD+DR+RK+R)}$ Such that.

$$\sum_{r=1}^{R}\sum_{k=1}^{K}\left[\frac{\partial C_{rk}(Q^3)}{\partial q_{rk}} + \frac{\partial C_r(Q^{1*}, Q^{2*}, Q^{3*})}{\partial q_{rk}} + \beta_r^* - \rho_{3rk}^*\right] \times \left[q_{rk} - q_{rk}^*\right]$$
$$+ \sum_{t=1}^{T}\sum_{d=1}^{D}\sum_{g=1}^{G}\left[\frac{\partial C_r(Q^{1*}, Q^{2*}, Q^{3*})}{\partial q_{gd}^t}\right] \times \left[q_{gd}^t - q_{gd}^{t*}\right] +$$
$$\sum_{r=1}^{R}\sum_{d=1}^{D}\left[\frac{\partial C_r(Q^{1*}, Q^{2*}, Q^{3*})}{\partial q_{dr}} + \rho_{2dr}^* + \frac{\partial \hat{C}_{dr}(q_{dr}^*)}{\partial q_{rk}} - \beta_r^*\right] \times \left[q_{dr} - q_{dr}^*\right]$$
$$+ \sum_{r=1}^{R}\left[\sum_{d=1}^{D} q_{dr}^* - \sum_{k=1}^{K} q_{rk}^*\right] \times \left[\beta_r - \beta_r^*\right] \geq 0, \forall (Q^1, Q^2, Q^3, \beta) \in R_+^{GTD+R(D+K+1)} \tag{15}$$

Equilibrium Condition for the Demand Market: Considering the demand market k, the demand consumers' consumption behavior for the product is assumed to be governed by deterministic demand function d_k (ρ_3) where the K– dimensional row vector ρ_3 - (ρ_{31}, ρ_{32},...,ρ_{3k}) in which ρ_{3k} denotes unit price of the power output that the demand consumers in demand market $K(k = 1,...,K)$ are willing to pay.

Suppose we let q_{rk} be the quantity of electricity bought from power retailer r by end – consumers in demand market k. Let function $\hat{C}_{rk}$ (Q_3) denote unit transaction cost between power retailer r and demand market k. The equilibrium conditions for end – consumers located at all demand markets in the electric power supply chain. Thus can be governed by the following VI.

Find a vector $(Q^{3*}, \rho_3) \in R_+^{K(R+1)}$ Such that;

$$\sum_{k=1}^{K}\sum_{r=1}^{R}\left[\rho_{3rk}^* + \hat{C}_{rk}(Q^{3*}) - \rho_{3k}^*\right] \times \left[q_{rk} - q_{rk}^*\right] + \sum_{k=1}^{K}\left[\sum_{r=1}^{R} q_{rk}^* - d_r(\rho_3^*)\right] \times \left[\rho_{3k} - \rho_{3k}^*\right] \geq 0, \tag{16}$$

$$\forall (Q^3, \rho_3) \in R_+^{K(R+1)}$$

Hence, the electric power supply chain will involve five kinds of decision makers: power generators, transmission service provider, power distributors, power retailers and demand consumers and they are interacted and highly correlated in the electric power supply chain of the product, respectively. Nagurney *et al*. (2002) proposed a novel equilibrium concept from the point of view of entire supply chain network. The SCNE model can be formulated by the following variational inequality formulation:

Determine a vector $(Q^{1*}, Q^{2*}, Q^{3*}, \lambda^*, \beta^*, \rho_3^*) \in R_+^{GTD+DR+RK+D+R+K}$ such thatl;

$$\sum_{t=1}^{T}\sum_{g=1}^{G}\sum_{d=1}^{D}\left[\frac{\partial f_g(Q^{1*})}{\partial q_{gd}^t}+\frac{\partial C_{gd}^t(q_{gd}^{t*})}{\partial q_{gd}^t}+\frac{\partial C_d(Q^{1*},Q^{2*})}{\partial q_{gd}^t}+\frac{\partial \hat{C}_{gd}^t(q_{gd}^{t*})}{\partial q_{gd}^t}-\lambda_d^*+\frac{\partial C_r(Q^{1*},Q^{2*},Q^{3*})}{\partial q_{gd}^t}\right]$$
$$\times\left[q_{gd}^t-q_{gd}^{t*}\right]+\sum_{d=1}^{D}\sum_{r=1}^{R}\left[\frac{\partial C_r(Q^{1*},Q^{2*},Q^{3*})}{\partial q_{dr}}+\frac{\partial C_{dr}(q_{dr}^*)}{\partial q_{dr}}+\frac{\partial C_d(Q^{1*},Q^{2*})}{\partial q_{dr}}+\frac{\partial \hat{C}_{dr}(q_{dr}^*)}{\partial q_{dr}}+\lambda_d^*-\beta_r^*\right]$$
$$\times\left[q_{dr}-q_{dr}^*\right]+\sum_{r=1}^{R}\sum_{k=1}^{K}\left[\frac{\partial C_r(Q^{1*},Q^{2*},Q^{3*})}{\partial q_{rk}}+\frac{\partial C_{rk}(Q^{3*})}{\partial q_{rk}}+\hat{C}_{rk}(Q^{3*})+\beta_r^*-\rho_{3k}^*\right]\times\left[q_{rk}-q_{rk}^*\right]+$$
$$\sum_{d=1}^{D}\left[\sum_{g=1}^{G}\sum_{t=1}^{T}q_{gd}^{t*}-\sum_{r=1}^{R}q_{dr}^*\right]\times\left[\lambda_d-\lambda_d^*\right]+\sum_{r=1}^{R}\left[\sum_{d=1}^{D}q_{dr}^*-\sum_{k=1}^{K}q_{rk}^*\right]\times\left[\beta_r-\beta_r^*\right]+$$
$$\sum_{k=1}^{K}\left[\sum_{r=1}^{R}q_{rk}^*-d_k(\rho_3^*)\right]\times\left[\rho_{3k}-\rho_{3k}^*\right]\geq 0,\ \forall\,(Q^1,Q^2,Q^3,\lambda,\beta,\rho_3)\in R_+^{GTD+DR+RK+D+R+K}$$
$$(17)$$

where $R_+^{GTD+DR+RK+D+R+K}$ is the nonnegative in the $GTD+DR+RK+D+R+K-$ dimensional real space $R^{GTD+DR+RK+D+R+K}$.

Having obtained the solution for the VI (17), the relevant equilibrium prices for power output can be identified by the formulae below:

$$\rho_{1gd}^{t*}=\frac{\partial f(Q^{1*})}{\partial q_{gd}^t}+\frac{\partial C_{gd}^t(q_{gd}^{t*})}{\partial q_{gd}^t},\ \text{if } q_{gd}^{t*}>0 \qquad (18)$$

$$\rho_{2dr}^*=\frac{\partial C_{dr}(q_{dr}^*)}{\partial q_{dr}}+\frac{\partial C_{dr}(Q^{1*},Q^{2*})}{\partial q_{dr}}+\lambda_d^*,\ \text{if } q_{dr}^*>0 \qquad (19)$$

$$\rho_{3rk}^*=\frac{\partial C_{rk}(Q^{2*})}{\partial q_{rk}}+\frac{\partial C_{dr}(Q^{1*},Q^{2*},Q^{3*})}{\partial q_{rk}}+\beta_r^*,\ \text{if } q_{rk}^*>0 \qquad (20)$$

Multi-Start Optimization Method: In this paper, the Multi-start values optimization method will be used to optimize the variational inequalities. The method strategically samples the solution space of an optimization problem in Ribeiro and Resende [10]. The most successful of the method of multi start values has two phases that are alternated for a certain number of global iterations. First phase generates a solution and the second seeks to improve the outcome. Each global iteration give a solution that is typically a local optimum and the best overall solution is the output of the algorithm. The interaction between the two phases creates a balance between search structural variation and search improvement, to yield an e?ective means for generating high-quality solutions.

Data Analysis and Results: Apart from the inconsistency in the power generation rate of Nigeria, the major concern has always been power getting to the demand customer. In this regard, we shall consider the Nigeria power distribution network for a single power generating station (G), a single transmission supplier (T), three distribution supplier (D), two retailers (R) and two single demand customer (K). (Recast the above sentence in red) Hence, G = 1, T = 1, D = 3, R = 2 and K = 2.

The data are as follows, the power generating cost function is given as;

$$f_1(q) = 2.5\left(q_1^1\right)^2 + 2q_1^1 \quad (21)$$

The transaction cost functions were given as:

$$C_{11}^1\left(q_{11}^1\right) = 0.25\left(q_{11}^1\right)^2 + 3.5\,q_{11}^1 \quad (22)$$

$$C_{12}^1\left(q_{12}^1\right) = 0.25\left(q_{12}^1\right)^2 + 3.5\,q_{12}^1 \quad (23)$$

$$C_{13}^1\left(q_{13}^1\right) = 0.25\left(q_{13}^1\right)^2 + 3.5\,q_{13}^1 \quad (24)$$

The operating cost functions were given as:

$$C_1^1\left(Q^1, Q^2\right) = \left(q_{11}^1 + q_{11} + q_{12}\right)^2 \quad (25)$$

$$C_2^1\left(Q^1, Q^2\right) = \left(q_{12}^1 + q_{21} + q_{22}\right)^2 \quad (26)$$

$$C_3^1\left(Q^1, Q^2\right) = \left(q_{13}^1 + q_{31} + q_{32}\right)^2 \quad (27)$$

Other transaction cost function are :

$$\hat{C}_{11}^1\left(q_{11}^1\right) = 0.01\left(q_{11}^1\right)^2 + 0.01\,q_{11}^1 \quad (28)$$

$$\hat{C}_{12}^1\left(q_{12}^1\right) = 0.01\left(q_{12}^1\right)^2 + 0.01\,q_{12}^1 \quad (29)$$

$$\hat{C}_{13}^1\left(q_{13}^1\right) = 0.01\left(q_{13}^1\right)^2 + 0.01\,q_{13}^1 \quad (30)$$

$$C_{11}\left(Q^2\right) = 0.25q_{11}^2 \quad (31)$$

$$C_{12}\left(Q^2\right) = 0.25q_{12}^2 \quad (32)$$

$$C_{21}\left(Q^2\right) = 0.25q_{21}^2 \quad (33)$$

$$C_{22}\left(Q^2\right) = 0.25q_{22}^2 \quad (34)$$

$$C_{31}\left(Q^2\right) = 0.25q_{31}^2 \quad (35)$$

$$C_{32}\left(Q^2\right) = 0.25q_{32}^2 \quad (36)$$

$$\hat{C}_{11}\left(q_{11}\right) = 0.25q_{11}^2 \quad (37)$$

$$\hat{C}_{12}\left(q_{12}\right) = 0.25q_{12}^2 \quad (38)$$

$$\hat{C}_{21}\left(q_{21}\right) = 0.25q_{21}^2 \quad (39)$$

$$\hat{C}_{22}\left(q_{22}\right) = 0.25q_{22}^2 \quad (40)$$

$$\hat{C}_{31}\left(q_{31}\right) = 0.25q_{31}^2 \quad (41)$$

$$\hat{C}_{32}\left(q_{32}\right) = 0.25q_{32}^2 \quad (42)$$

$$C_{11}\left(Q^3\right) = q_{11} + 5 \quad (43)$$

$$C_{12}\left(Q^3\right) = q_{12} + 5 \quad (44)$$

$$C_{21}\left(Q^3\right) = q_{21} + 5 \quad (45)$$

$$C_{22}\left(Q^3\right) = q_{22} + 5 \quad (46)$$

$$\hat{C}_{11}\left(Q^3\right) = q_{11} + 5 \quad (47)$$

$$\hat{C}_{12}\left(Q^3\right) = q_{12} + 5 \quad (48)$$

$$\hat{C}_{21}\left(Q^3\right) = q_{21} + 5 \quad (49)$$

$$\hat{C}_{22}\left(Q^3\right) = q_{22} + 5 \quad (50)$$

$$C_1\left(Q^1, Q^2, Q^3\right) = 0.5\left(\sum_{i=1}^{3} q_{i1}\right)^2 \tag{51}$$

$$C_2\left(Q^1, Q^2, Q^3\right) = 0.5\left(\sum_{i=1}^{3} q_{i2}\right)^2 \tag{52}$$

The demand functions for the end customers at the demand market are:

$$d_1\left(\rho_3\right) = -2\rho_{31} - 1.5\rho_{32} + 1100 \tag{53}$$

$$d_1\left(\rho_3\right) = -1.5\rho_{32} - 2\rho_{31} + 1100 \tag{54}$$

Inputting the above data into the variational inequality (17) to determine the following, q^*_{11}, q^*_{12}, q^*_{21}, q^*_{22}, q^*_{31}, q^*_{32}, q^{1*}_{11}, q^{1*}_{12}, q^{1*}_{13}, ρ^*_{31}, ρ^*_{31}, λ^*_1, λ^*_2, λ^*_3, ρ^*_{31}, ρ^*_{32}, β_1^* and β_2^*, all nonnegative and satisfying:

$$\begin{bmatrix}
(7.5\, q^{1*}_{11} + 5.51 + 2q^*_{11} + 2\, q^*_{12} - \lambda^*_1)\,(q^1_{11} - q^{1*}_{11}) + \\
+ (7.5\, q^{1*}_{12} + 5.51 + 2q^*_{21} + 2\, q^*_{22} - \lambda^*_2)\,(q^1_{12} - q^{1*}_{12}) + \\
(7.5\, q^{1*}_{13} + 5.51 + 2q^*_{31} + 2\, q^*_{32} - \lambda^*_3)\,(q^1_{13} - q^{1*}_{13}) \\
(4\, q^*_{11} + 2\, q^*_{31} + 6 + \lambda^*_1 - \rho^*_{31})(q_{11} - q^*_{11}) + \\
(3\, q^*_{12} + q^*_{22} + q^*_{32} + 6 + \lambda^*_1 + \beta^*_1 - \beta^*_2 - \rho^*_{31})(q_{12} - q^*_{12}) + \\
(3\, q^*_{21} + q^*_{11} + q^*_{31} + 6 + \lambda^*_2 + \beta^*_2 - \beta^*_1 - \rho^*_{31})(q_{21} - q^*_{21}) + \\
(4\, q^*_{22} + 2\, q^*_{12} + 2q^*_{32} + 6 + \lambda^*_2 - \rho^*_{32})(q_{22} - q^*_{22}) + \\
(q^*_{11} + q^*_{21} + 2q^*_{31} + \lambda^*_3 - \beta^*_1)(q_{31} - q^*_{31}) + \\
(q^*_{12} + q^*_{22} + 2\, q^*_{32} + \lambda^*_3 - \beta^*_2)(q_{32} - q^*_{32}) + \\
(q^{1*}_{11} - q^*_{11} - q^*_{12})(\lambda_1 - \lambda^*_1) + (q^{1*}_{12} - q^*_{21} - q^*_{22})(\lambda_2 - \lambda^*_2) + \\
(q^{1*}_{13} - q^*_{31} - q^*_{32})(\lambda_3 - \lambda^*_3) + (q^*_{21} + q^*_{31} - q^*_{12})(\beta_1 - \beta^*_1) + \\
(q^*_{12} + q^*_{32} - q^*_{21})(\beta_2 - \beta^*_2) + \\
(q^*_{11} + q^*_{21} + 2\rho^*_{31} + 1.5\rho^*_{32} - 1100)(\rho_{31} - \rho^*_{31}) + \\
(q^*_{12} + q^*_{22} + 1.5\rho^*_{31} + 2\rho^*_{32} - 1100)(\rho_{32} - \rho^*_{32})
\end{bmatrix} \geq 0 \tag{55}$$

Table 1: Summary of parameters for the three categories of customer

Category of Demand Customers	R1	R2S	R2T
Tariff ₦	4	30.93	34.28
q^{1*}_{11} GWH	159.41	163.66	161.12
q^{1*}_{12} GWH	137.43	144.45	142.30
q^{1*}_{13} GWH	111.05	127.28	127.10
q^{1*}_{11} GWH	83.05	83.80	83.65
q^{*}_{12} GWH	76.36	79.86	77.42
q^{*}_{21} GWH	47.43	54.40	53.90
q^{*}_{22} GWH	90.00	90.05	88.40
q^{*}_{31} GWH	28.93	25.46	23.57
q^{*}_{32} GWH	90.54	83.67	79.46
q^{*}_{31} ₦	307.75	301.97	299.26
ρ^{*}_{32} ₦	236.01	238.57	242.62
λ_1^* ₦	1523.09	1563.55	1533.86
λ_2^*₦	1313.84	1380.67	1360.21
λ_3^*₦	1079.59	1180.90	1167.36
β^*_1 ₦	1267.93	1370.02	1352.05
β^*_2 ₦	1.56	12.93	1167.36

Equation (55) was used to solve for three tariff category of demand customers (R1: which is the residential with single phase supply with single meter with consumption on 50KWH and below; R2S: which is single phase supply with single phase meter with consumption above 50KWH; and R2T: which is three phase supply with three phase meter with consumption below 45KVA).

The Multistart optimization method was used to solve the inequality using specified start up value obtained from the Nigerian Electricity Regulatory Commission (NERC). For R1 tariff demand customers, the startup value employed were the average daily of power distributors receive from power generators (q_{11} = 68.93 GWH) and the unit cost of power by distributors to R1 customers (ρ_{31} = ρ_{32} = 4 Naira). For R2S tariff demand customers, the startup value employed were the average daily of power distributors receive from power generators (q_{11} = 68.93 GWH) and the unit cost of power by distributors to R1 customers (ρ_{31} = ρ_{32} = 30.93 Naira). For R2T tariff demand customers, the startup value employed were the average daily of power distributors receive from power generators (q_{11} = 68.93 GWH) and the unit cost of power by distributors to R1 customers (ρ_{31} = ρ_{32} = 34.28 Naira).

Table1 represent the summary of the result obtained from the three categories of customers.

CONCLUSION

The paper proposed a new model of electric power supply chain networks in the Nigeria situation, which allows for multiple power generators (GENCOS), transmission (TCN) and distribution (DISCOS), retailers or retailing (RETCOS) and demand customers. The supply chain network introduces retailing of power (unbundling of DISCOS to smaller DISCOS called RETCOS or POWER RETAILERS OR RETAILING) from distributors to demand customers. We derived the optimality conditions of the decision-makers and proved that the governing equilibrium problem conditions satisfy a variational inequality problem. The variational inequality problem for a single power generator, single transmission, three power distribution suppliers, two power retailers and two demand customers were used to illustrate the method. A simple scenario is to use single power generator (GENCO), single transmission (TCN), SINGLE power distribution (DISCO) (supplier), four power retailers (RETCOS) and FOUR or more demand customers. The multi start optimization method was used to solve the inequality using specified start value obtained from the Nigeria Regulatory Commission (NERC). The result of the analysis showed that the least price of power a customer in R2T will be willing to pay is ₦ 242.62, for customers in R2S the least price they will be willing to pay is ₦ 238.57, while for R1 customer the least price is ₦ 236.01.

However, this work has succeeded in clarifying the use of variational inequality as a tool in electric supply chain network for finding equilibrium condition in line with Nash concept.

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