Middle-East Journal of Scientific Research 25 (7): 1553-1558, 2017
ISSN 1990-9233

DOI: 10.5829/idosi.mejsr.2017.1553.1558

SIBEDE Approach For Total Graph of Path And Cycle Graphs

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Abstract: Let G (V(G), E(G)) be a graph with n vertices is said to be Binary Equivalent Decimal Edge Graceful Labeling (BEDE) graph if the vertices are assigned distinct numbers from 0, 1, 2, …, (n-1) such that the labels induced on edges by the values obtained using binary equivalent decimal coding of end vertices for each edge which are distinct. A graph G is said to be Strong Incident Binary Equivalent Decimal Edge Graceful Labeling (SIBEDE) if the vertices of G are labeled with distinct positive integers from 0, 1, 2, ... , (n-1) such that the label induced on the edges by Binary equivalent decimal coding are distinct from the vertex labeling. This paper is concerned with the SIBEDE approach of total graph of path and cycle.

Key words: BEDE · Binary · Graceful · IBEDE · Incident · Labeling · SIBEDE · Total graph

INTRODUCTION

Graph Labeling is a most vital area of Research in Graph theory. A Labeling of graph is where the vertices or edges or both are assigned real values by following some specific rule. Labeling plays an important role in Communication network addressing the system to determine optimal circuit design, Data base management, crystallographic etc, . Open problems in Labeling graph are discussed on Graph labeling by J.A. Gallian (2015) can be found in [1]. To any Graph G there corresponds a $v \times \epsilon$ matrix called incident matrix of G [2]. Let us denote the vertices of G by v_1, v_2, ... v_v and edges by e_1, e_2,...e_ϵ. Then the incident matrix of G is the matrix B(G) = $[b_{ij}]$ where b_{ij} is the number of times that v_i and e_j are incident. The Total Graph $T(G)$ [3] [4] of graph G is a graph with vertex set $V(G) \cup E(G)$ and two vertices x, y in $T(G)$ are adjacent if either (i) x, y are in $V(G)$ and x is adjacent to y in G or (ii) x, y are in $E(G)$ and x, y are adjacent in G or (iii)x is in $V(G)$, y is in $E(G)$ and x, y are incident in G.

Strong Binary Equivalent Decimal Edge Graceful Labeling

Strong Incident Binary Equivalent Decimal Edge Graceful Labeling

Definition: A graph G = (V(G), E(G)) be a graph with n vertices is said to be Incident Binary Equivalent Decimal Edge (IBEDE) Graceful labeling [5], if there corresponds a $V \times E$ matrix called incident matrix and f is a bijective mapping from vertices to the set of integers $\{0,1,2, ... , (n-1)\}$ such that the induced map f^* from edge set to integers which is defined as;

f: V(G) → $\{0,1,2, ..., (n-1)\}$
f^*: E(G) → $\{1,2,3,4,5, ..., m\}$ (m is finite) such that the edges are labeled with the values obtained from binary equivalent decimal coding. It is also equivalent to $e_k = (i, j) = 2^{n-i-1} + 2^{n-j-1}$ where $k = \{1,2,3, ..., q\}$ and i,j are finite positive integer labeled for end vertices of e_k, n is the number of vertices in G.

Definition: A graph G = (V(G), E(G)) be a graph with n vertices is said to be Strong Incident Binary Equivalent Decimal Edge Graceful Labeling (SIBEDE) [6], if the vertices of G are labeled with distinct positive integers from 0, 1, 2, ... , (n-1) such that the label induced on the edges by Binary equivalent decimal coding are distinct from the vertex labeling.

Example

(i) consider Total Graph $T(P_5)$ (for n odd)

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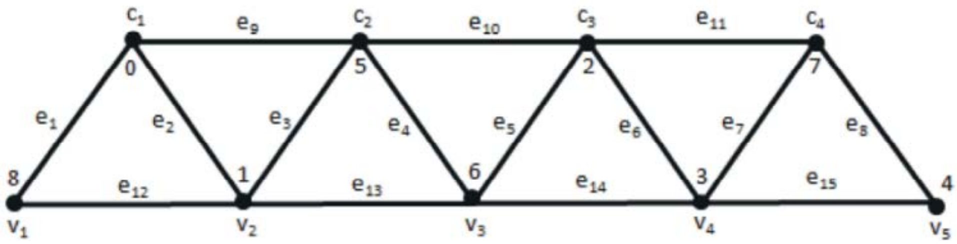


Fig. 1: Total Graph $T(P_5)$

Binary Equivalent Decimal Edge Labeling for Total Graph $T(P_5)$

	0	1	2	3	4	5	6	7	8	
e_1	1	0	0	0	0	0	0	0	1	257
e_2	1	1	0	0	0	0	0	0	0	384
e_3	0	1	0	0	0	1	0	0	0	136
e_4	0	0	0	0	0	1	1	0	0	12
e_5	0	0	1	0	0	0	1	0	0	68
e_6	0	0	1	1	0	0	0	0	0	96
e_7	0	0	0	1	0	0	0	1	0	34
e_8	0	0	0	0	1	0	0	1	0	18
e_9	1	0	0	0	0	1	0	0	0	264
e_{10}	0	0	1	0	0	1	0	0	0	72
e_{11}	0	0	1	0	0	0	0	1	0	66
e_{12}	0	1	0	0	0	0	0	0	1	129
e_{13}	0	1	0	0	0	0	1	0	0	132
e_{14}	0	0	0	1	0	0	1	0	0	36
e_{15}	0	0	0	1	1	0	0	0	0	48

(ii) consider Total Graph $T(P_6)$ (for n even)

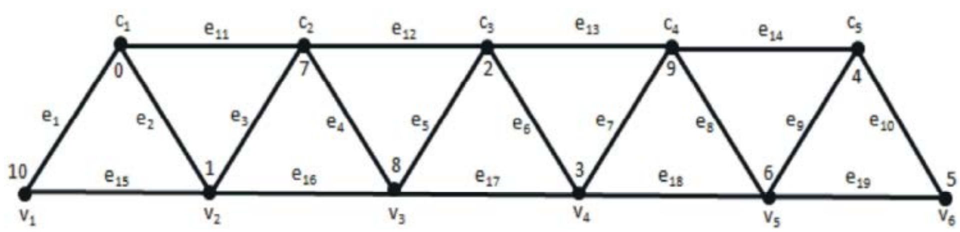


Fig. 2: Total Graph $T(P_6)$

Binary Equivalent Decimal Edge Labeling for Total Graph $T(P_6)$

	0	1	2	3	4	5	6	7	8	9	10	
e_1	1	0	0	0	0	0	0	0	0	0	1	1025
e_2	1	1	0	0	0	0	0	0	0	0	0	1536
e_3	0	1	0	0	0	0	0	1	0	0	0	520
e_4	0	0	0	0	0	0	0	1	1	0	0	12
e_5	0	0	1	0	0	0	0	0	1	0	0	260
e_6	0	0	1	1	0	0	0	0	0	0	0	384
e_7	0	0	0	1	0	0	0	0	0	1	0	130
e_8	0	0	0	0	0	0	1	0	0	1	0	18
e_9	0	0	0	0	1	0	1	0	0	0	0	80
e_{10}	0	0	0	0	1	1	0	0	0	0	0	96
e_{11}	1	0	0	0	0	0	0	1	0	0	0	1032
e_{12}	0	0	1	0	0	0	0	1	0	0	0	264
e_{13}	0	0	1	0	0	0	0	0	0	1	0	258
e_{14}	0	0	0	0	1	0	0	0	0	1	0	66
e_{15}	0	1	0	0	0	0	0	0	0	0	1	513
e_{16}	0	1	0	0	0	0	0	0	1	0	0	516
e_{17}	0	0	0	1	0	0	0	0	1	0	0	132
e_{18}	0	0	0	1	0	0	1	0	0	0	0	144
e_{19}	0	0	0	0	0	1	1	0	0	0	0	48

Theorem
For all $n \geq 3$ the Total Graph $T(P_n)$ of a path graph P_n with n points is SIBEDE graceful labeling graph.

Proof:
Let $V = \{v_1, v_2, v_3, ..., v_n, c_1, c_2, ..., c_{(n-1)}\}$ be the vertex set and $E = E_1 \cup E_2 \cup E_3 \cup E_4$ be the edge sets of Total graph $T(P_n)$ where
$E_1 = v_i \; v_{i+1} 1 \leq i \leq (n-1)$, $E_2 = c_i v_i, 1 \leq i \leq (n-1)$, $E_3 = c_i v_{i+1}, 1 \leq i \leq (n-1)$,
$E_4 = c_i c_{i+1}, 1 \leq i \leq (n-2)$

Let the total number of vertices of total graph $T(P_n)$ be $(2n–1)$.
Define a bijective mapping f: $V(T(P_n)) – \{0,1,2,..., 2(n–1)\}$

Case (i) n is Odd

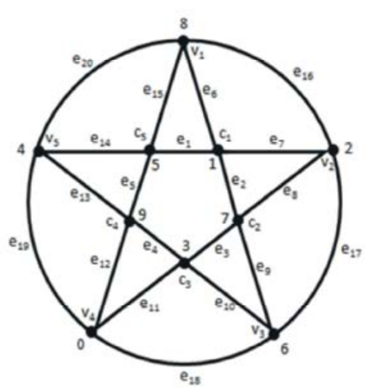
$$f(v_i) = 2n - i - 1 \; for \; i = 1, 3, 5, \ldots, n$$
$$f(v_i) = i - 1 \; for \; i = 2, 4, \ldots, (n-1)$$
$$f(c_2) = n$$
$$f(c_{j+2}) = f(c_j) + 2 \; for \; j = 2, 4, \ldots, (n-1)$$
$$f(c_j) = j - 1, for \; j = 1, 3, \ldots, (n-2)$$

Case (ii) n is Even

$$f(v_i) = 2n - i - 1 \; for \; i = 1, 3, 5, \ldots, (n-1)$$
$$f(v_i) = i - 1 \; for \; i = 2, 4, \ldots, n$$
$$f(c_j) = n - 1 + j \; for \; j = 2, 4, \ldots, (n-2)$$
$$f(c_j) = j - 1, for \; j = 1, 3, \ldots, (n-1)$$

Now we define an induced function f^*: $E(T(P_n)) \to \{1,2,...,m\}$ (m is finite) such that the edges are labeled with the values obtained from binary equivalent decimal coding or using $e_k = (i, j) = 2^{2n-i-2} + 2^{2n-j-2}$
where $k = \{1,2,3, ...(4n – 5)\}$ and i,j are finite positive integers labeled for the edge e_k.
Hence the induced map f^* gives the distinct labels for edges which is distinct from vertex labeling.
So the Total Graph $T(P_n)$ of a path graph P_n is SIBEDE graceful labeling graph.

Example 2:
(I) Consider the Total Graph $T(C_5)$ (for n odd)

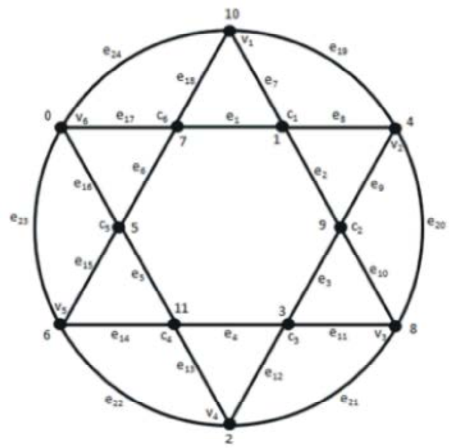


Binary Equivalent Decimal Edge Labeling

	0	1	2	3	4	5	6	7	8	9	
e_1	0	1	0	0	0	1	0	0	0	0	272
e_2	0	1	0	0	0	0	0	1	0	0	260
e_3	0	0	0	1	0	0	0	1	0	0	68
e_4	0	0	0	1	0	0	0	0	0	1	65
e_5	0	0	0	0	0	0	0	0	0	1	17
e_6	0	1	0	0	0	0	0	0	1	0	258
e_7	0	1	1	0	0	0	0	0	0	0	384
e_8	0	0	1	0	0	0	0	1	0	0	132
e_9	0	0	0	0	0	0	1	1	0	0	12
e_{10}	0	0	0	1	0	0	1	0	0	0	72
e_{11}	1	0	0	1	0	0	0	0	0	0	576
e_{12}	1	0	0	0	0	0	0	0	0	1	513
e_{13}	0	0	0	0	1	0	0	0	0	1	33
e_{14}	0	0	0	0	1	1	0	0	0	0	48
e_{15}	0	0	0	0	0	1	0	0	1	0	18
e_{16}	0	0	1	0	0	0	0	0	1	0	130
e_{17}	0	0	1	0	0	0	1	0	0	0	136
e_{18}	1	0	0	0	0	0	1	0	0	0	520
e_{19}	1	0	0	0	1	0	0	0	0	0	544
e_{20}	0	0	0	0	1	0	0	0	1	0	34

Fig. 3: Total Graph $T(C_5)$

(ii) Consider the Total Graph $T(C_6)$ (for n even)



Binary Equivalent Decimal Edge Labeling

	0	1	2	3	4	5	6	7	8	9	10	11	
e_1	0	1	0	0	0	0	0	1	0	0	0	0	1040
e_2	0	1	0	0	0	0	0	0	0	1	0	0	1028
e_3	0	0	0	1	0	0	0	0	0	1	0	0	260
e_4	0	0	0	1	0	0	0	0	0	0	0	1	257
e_5	0	0	0	0	0	1	0	0	0	0	0	1	65
e_6	0	0	0	0	0	1	0	1	0	0	0	0	80
e_7	0	1	0	0	0	0	0	0	0	0	1	0	1026
e_8	0	1	0	0	1	0	0	0	0	0	0	0	1152
e_9	0	0	0	0	1	0	0	0	0	1	0	0	132
e_{10}	0	0	0	0	0	0	0	0	1	1	0	0	12
e_{11}	0	0	0	1	0	0	0	0	1	0	0	0	264
e_{12}	0	0	1	1	0	0	0	0	0	0	0	0	768
e_{13}	0	0	1	0	0	0	0	0	0	0	0	1	513
e_{14}	0	0	0	0	0	0	1	0	0	0	0	1	33
e_{15}	0	0	0	0	0	1	1	0	0	0	0	0	96
e_{16}	1	0	0	0	0	1	0	0	0	0	0	0	2112
e_{17}	1	0	0	0	0	0	0	1	0	0	0	0	2064
e_{18}	0	0	0	0	0	0	0	1	0	0	1	0	18
e_{19}	0	0	0	0	1	0	0	0	0	0	1	0	130
e_{20}	0	0	0	0	1	0	0	0	1	0	0	0	136
e_{21}	0	0	1	0	0	0	0	0	1	0	0	0	520
e_{22}	0	0	1	0	0	0	1	0	0	0	0	0	544
e_{23}	1	0	0	0	0	0	1	0	0	0	0	0	2080
e_{24}	1	0	0	0	0	0	0	0	0	0	1	0	2050

Fig. 4: Total Graph $T(C_6)$

Theorem

For $n \geq 5$ the Total Graph $T(C_n)$ of Cycle graph C_n with n points is SIBEDE graceful Labeling Graph.

Proof:

Let $V = \{v_1, v_2, v_3, \ldots, v_n, c_1, c_2, \ldots, c_{(n-1)}\}$ be the vertex set and $E = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5$ be the edge sets of Total graph $T(C_n)$ where

$E_1 = c_i v_{i+1} 1 \leq i \leq n,\ E_2 = c_i v_i, 1 \leq i \leq (n-1),\ E_3 = c_i c_{i+1}, 1 \leq i \leq (n-1),$

$E_4 = v_i v_{i+1}, 1 \leq i \leq (n-1)$ and $E_5 = c_n v_1, c_n c_1, v_n v_1.$

Let the total number of vertices of total graph $T(C_n)$ be 2n.

Define a bijective mapping f: $V(T(C_n)) - \{0,1,2,\ldots,(2n-1)\}$

Case (i) n is Odd

$$f(v_1) = 2(n-1)$$
$$f(v_i) = f(v_{i-2}) - 2 \text{ for } i = 3,5,\ldots,n$$
$$f(v_i) = n - 1 - i \text{ for } i = 2,4,\ldots,(n-1)$$
$$f(c_j) = j \text{ for } j = 1,3,\ldots,n$$
$$f(c_j) = n + j \text{ for } j = 2,4,\ldots,(n-1)$$

Case (ii) n is Even

$$f(v_1) = 2(n-1)$$
$$f(v_i) = f(v_{i-2}) - 2 \text{ for } i = 3,5,\ldots,n-1$$
$$f(v_i) = n - i \text{ for } i = 2,4,\ldots,n$$
$$f(c_j) = j \text{ for } j = 1,3,\ldots,(n-1)$$
$$f(c_j) = n + j + 1 \text{ for } j = 2,4,\ldots,(n-2)$$
$$f(c_n) = n + 1$$

Now we define an induced function $f^*: E(T(C_n)) \rightarrow \{1,2,...,m\}$ (m is finite) such that the edges are labeled with the values obtained from binary equivalent decimal coding or using $e_k = (i,j) = 2^{2n-i-1} + 2^{2n-j-1}$ where $k = \{1,2,3,...,4n\}$ and i,j are finite positive integers labeled for the edge e_k.

Hence the induced map f^* gives the distinct labels for edges which is distinct from vertex labeling.

So the Total Graph $T(C_n)$ of Cycle graph C_n is SIBEDE graceful Labeling Graph.

Preposition: Total graph $T(C_3)$ and $T(C_4)$ are not SIBEDE graceful graphs since degree of each vertex is 4.

Preposition: Every total graph $T(C_n)$ is SIBEDE graceful graphs only if the degree of each vertex is less than 'n'

Observation:

- Total Graph $T(P_n)$ of Path P_n is planar. (It is explained in Example 1)
- Total Graph $T(C_n)$ of Cycle C_n is planar. (It is explained in Example 2)
- Total Graph of connected graph need not be planar. (It is explained in Example 3)

Example 3:

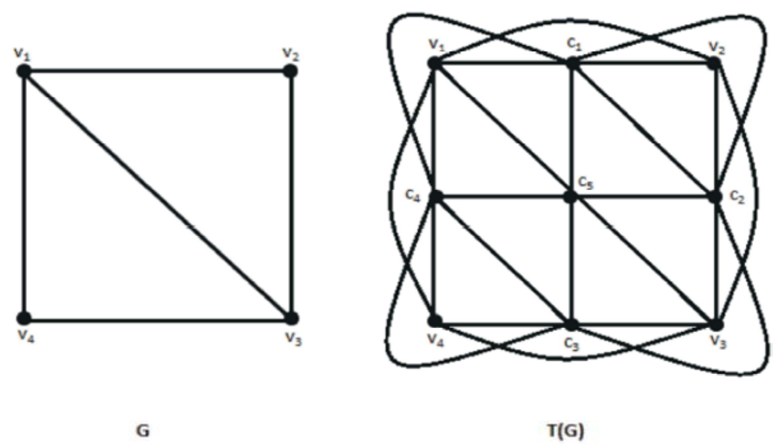


Fig. 5: Connected Graph G

CONCLUSION

In this Paper the Total graph of path and cycle are proved as strong Incident Binary Equivalent Decimal Edge (SIBEDE) graceful labeling and the Planarity of total graph of path and cycle are discussed.

ACKNOWLEDGEMENT

The authors would like to thank Dr. Ponnammal Natarajan, Former Director – Research, Anna University- Chennai, India and currently an Advisor, (Research and Development), Rajalakshmi Engineering College , Dr. E. Sampath Kumar Acharya & Dr. L. Pushpalatha, University of Mysore, Mysore, for their initiative ideas and fruitful discussions with respect to the paper's contribution.

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