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On Cordial Labeling of Vertex Switching of Cycles

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Abstract: The *Cordial labeling* of graph G is an injection f: $V(G) \rightarrow \{0,1\}$ such that each edge uv in G is assigned the label $|f(u) - f(v)|$ with the property $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(i)$ for $i = 0, 1$ denote the number of vertices with label i and $e_f(i)$ for $i = 0, 1$ denote the number of edges with label i. The graph which admits cordial labeling is called the *Cordial graph*. In this paper, we prove that the cycle of n copies of the class G of graphs is cordial, where the class G denotes the vertex switching of cycles.

Key words: Cordial labeling · Vertex switching · Cycle of graphs

INTRODUCTION

Graph labeling methods trace their origin to the graceful labeling introduced by Rosa [7] in 1967. For the past five decades variations in labeling methods have evolved. One such labeling method is the cordial labeling introduced by Cahit [4] in 1987. The *Cordial labeling*of graph G is an injection f: $V(G) \rightarrow \{0,1\}$ such that each edge uv in G is assigned the label $|f(u) - f(v)|$ with the property $|v_f(0) - v_f(1) \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$, where $v_f(i)$ for $i = 0, 1$ denote the number of vertices with label i and $e_f(i)$ for $i = 0, 1$ denote the number of edges with label i. The graph which admits cordial labeling is called the *Cordial graph*.Various graphs are shown to be cordial. Andar *et al*. [1, 2, 3] have proved that the helms, closed helms, flowers, gears and sunflower graphs and multiple shells are cordial. Again in [1, 2, 3] the one point union of helms, flowers, gears, sunflower graphs are shown to be cordial. Cahit [5] has proved that every tree is cordial. In [5] Cahit has shown that all fans are cordial, the wheel W_n when $n \not\equiv 3(mod\ 4)$ is cordial, the complete graph K_n is cordial if and only if $n \leq 3$, the bipartite graph $K_{m,n}$ is cordial for allm and n, the friendship graph $C_3^{(t)}$ is cordial if and only if $t \not\equiv 2(mod\ 4)$. An extensive survey of cordial labeling methods is available in [6] by Gallian. In this paper, we prove that the cycle of graphs $C(n°H)$ is cordial.

Main Results: In this section we first recall the definition for cycle, vertex switching of graph [8], cycle of graphs. Later we prove that the cycle of graphs $C(n°H)$ is graceful.

Definition 1: A sequence of vertices $[v_0, v_1, v_2,...,v_n, v_0]$ is a cycle of length n+1 if $v_{i+1}v_i \in E$, i=1, 2, 3, ... , n and $v_n\ v_0 \in$ iE. A cycle of length n is denoted by C_n.

Definition 2: A vertex switching [8] of a graph G is the graph G_v which is obtained by taking a vertex v of$G$, removing all the edges incident to v and adding edges joining v to every other vertex which are not adjacent to v in G. We call v as the *switching vertex*.

Definition 3: A *cycle of graphs* is the graph obtained by replacing each vertex of the cycle C_n by connected graphs $G_1, G_2,...,G_n$. The cycle of graphs thus obtained is denoted by $C(G_1,G_2,...,G_n)$. Note that if $G_1, G_2,...,G_n = G$ and then the cycle of graphs is denoted by $C(n°G)$.

Theorem: Let G be the cycle C_m and let H be the vertex switching graph of G. Then the cycle of graphs $C(n°H)$ is cordial.

Proof: Let$G$ be a cycle C_m with m vertices that are denoted as $v_1, v_2,...,v_m$ in the anticlockwise direction. Let H be the vertex switching graph of the graph G with $v_1 \in G$ as the switching vertex. The graphs G and H are shown in Figure 1.

Let $C(n°H)$ be the cycle of graphs that has been obtained by considering another cycle C_n whose vertices are denoted as $u_1, u_2,...,u_n$ in anticlockwise direction and replacing each vertex of C_n by the graph H at the switching vertex v_1. The graph $C(n°H)$ is shown in

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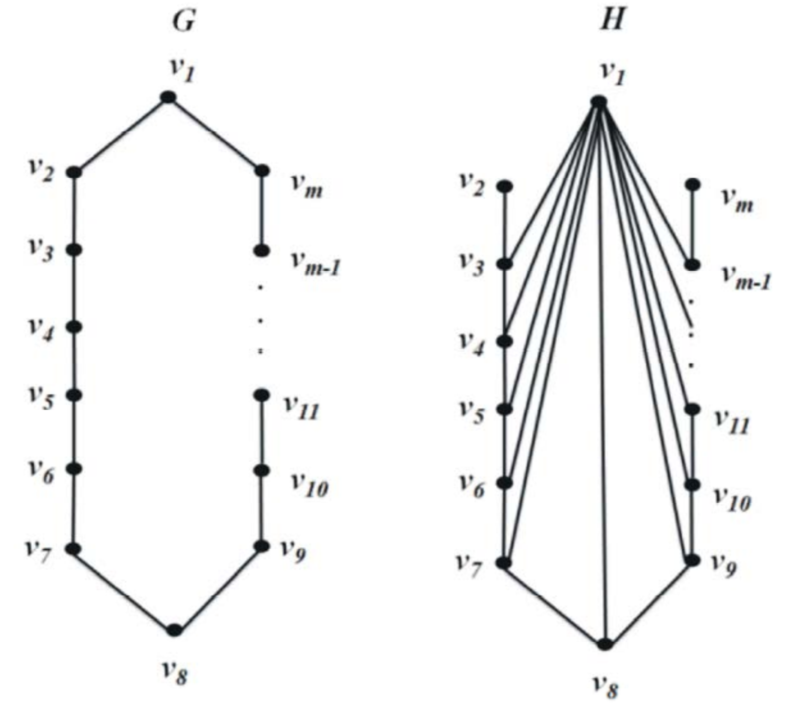

Fig. 1: The graphs G and H

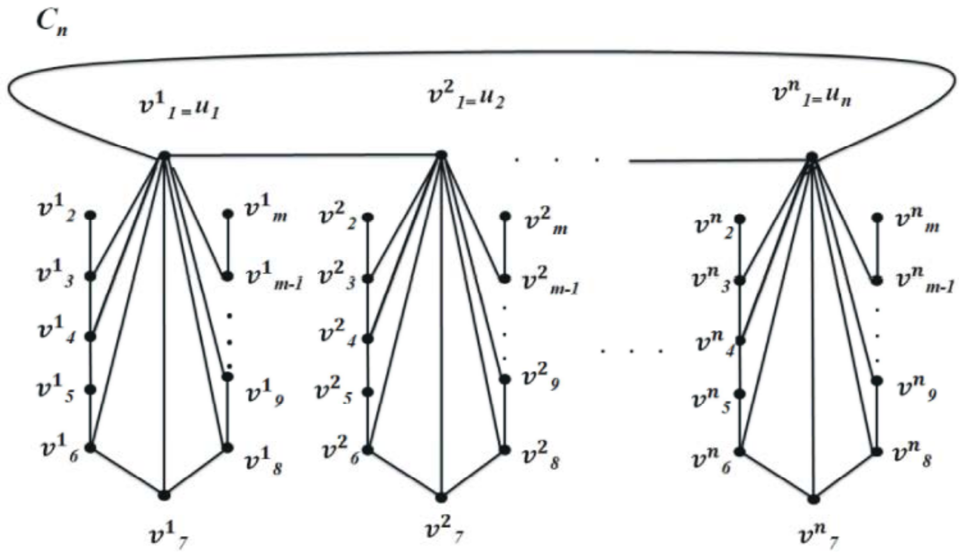

Fig. 2: The graph $C(n°H)$

Figure 2. In other words, each vertex u_i, $1\leq i\leq n$ of the cycle C_n is identified with the switching vertex v_1 of H. As n copies of H are attached to the cycle C_n by the identification of u_i's of C_n with v_1 of each of the n copies of H, we shall denote these identified vertices as $v_1^1 = u_1$ for the first copy of H, $v_1^2 = u_2$ for the second copy of H, $v^n{}_1 = u_n$ for the n[th] copy of H. In general, $v^i{}_1 = u_i$, $1 \leq i\leq n$ for the i[th] copy of H. Now we rename the other vertices in the first copy of H in $C(n°H)$ as $v^1{}_2, v^1{}_3, \ldots v_m{}^1$. The vertices in the second copy of H are renamed as $v^2{}_2, v^2{}_3, \ldots v_m{}^2$. This continues and the vertices in the last copy (that is n[th] copy) of H are renamed as $v^n{}_2, v^n{}_3, \ldots v_m{}^n$. Thus v_j^i represents the vertices in the i[th] copy of H for $1 \leq i \leq n$, $2 \leq j \leq m$,

Note that if$p$ denotes number of vertices in $C(n°H)$ then $p=mn$ and if q denotes the number of edges in $C(n°H)$ then $q=2n(m-2)$. Also note that the theorem is proved for $m \geq 5$ and $n \equiv 0(mod\ 2)$.

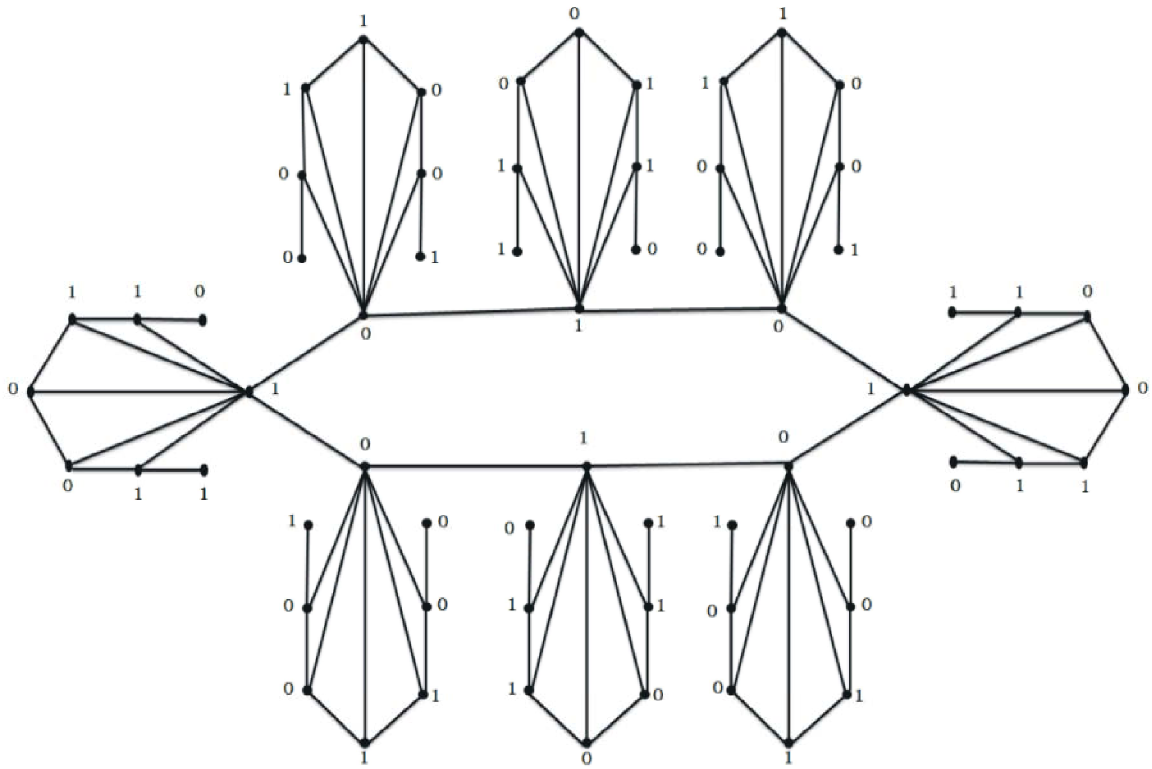

Fig. 3: Cordial labeling of $C\,(8 \circ C_8)$

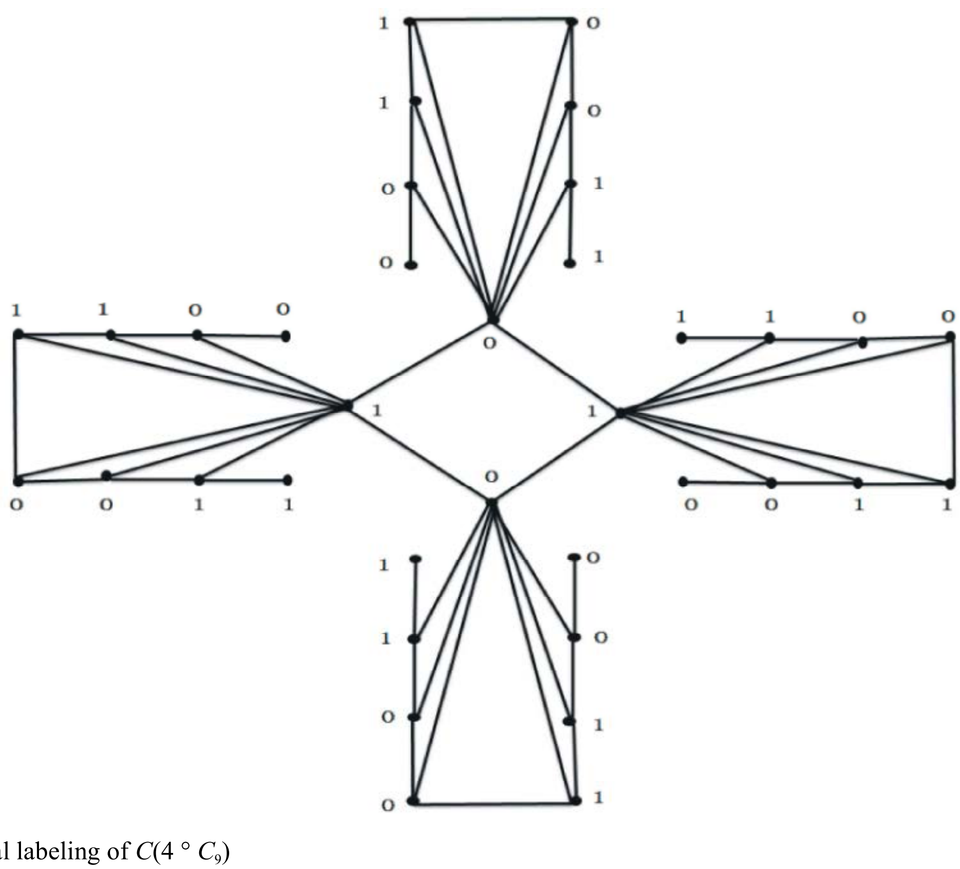

Fig. 4: Cordial labeling of $C(4 \circ C_9)$

The vertices of $C(n°H)$ are labeled as follows

Case 1: $m \equiv 1(mod\ 2)$

For $1 \leq i \leq n$ where $i \equiv 1(mod\ 2)$

$$f(v_j^i) = \begin{cases} 0 & 1 \leq j \leq m \quad j \equiv 0,1(\text{mod } 4) \\ 1 & 1 \leq j \leq m \quad j \equiv 2,3(\text{mod } 4) \end{cases}$$

For $1 \leq i \leq n$ where $i \equiv 0(mod\ 2)$

$$f(v_j^i) = \begin{cases} 1 & 1 \leq j \leq m \quad j \equiv 0,1(\text{mod } 4) \\ 0 & 1 \leq j \leq m \quad j \equiv 2,3(\text{mod } 4) \end{cases}$$

Case 2: $m \equiv 0(mod\ 2)$

$$f(v_j^i) = \begin{cases} 0 & 1 \leq i \leq n \quad i \equiv 1(\text{mod } 2) \\ 1 & 1 \leq i \leq n \quad i \equiv 0(\text{mod } 2) \end{cases}$$

For $1 \leq i \leq n$ where $i \equiv 1(mod\ 2)$

$$f(v_j^i) = \begin{cases} 0 & 2 \leq j \leq m \quad j \equiv 0,3(\text{mod } 4) \\ 1 & 2 \leq j \leq m \quad j \equiv 1,2(\text{mod } 4) \end{cases}$$

For $1 \leq i \leq n$ where $i \equiv 0(mod\ 2)$

$$f(v_j^i) = \begin{cases} 1 & 2 \leq j \leq m \quad j \equiv 0,3(\text{mod } 4) \\ 0 & 2 \leq j \leq m \quad j \equiv 1,2(\text{mod } 4) \end{cases}$$

From the above definition it is clear that $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. Therefore the graph $C(n°H)$ is cordial. The above two cases are illustrated in Figure 3 and Figure 4.

CONCLUSION

In this paper we have proved that the graph $C(n°H)$ is cordial. Further we intend to prove that the graph $C,(G_1, G_2,...,G_n)$ where each $G_i = C_m$ with arbitrary m, is cordial. Also we intend to prove C (G_1, G_2,...,G_n is cordial for any other connected graphs G_i $1 \leq i \leq n$.

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