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Between Closed Sets and ω-Closed Sets in Topological Spaces

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Abstract: Sheik John (= Veera Kumar) introduced the notion of ω-closed sets (= $\hat{g}$ -closed sets). Many variations of ω-closed sets were introduced and investigated. In this paper, we introduce the notion of mω-closed sets and obtain the unified characterizations for certain families of subsets between closed sets and ω-closed sets.

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INTRODUCTION

In 1970, Levine [1] introduced the notion of generalized closed (g-closed) sets in topological spaces. Recently, many variations of g-closed sets are introduced and investigated. In this paper, we introduce the notion of mω-closed sets and obtain the basic properties, characterizations and preservation properties. In the last section, we define several new subsets which lie between closed sets and mω-closed sets.

Preliminaries: Let (X, τ) be a topological space and Aa subset of X. The closure of A and the interior of A are denoted by cl(A) and int(A), respectively. A subset A of a space (X, τ) is an α-open [2] (resp. preopen [3]) set if A⊆int(cl(int(A))) (resp. A⊆int(cl(A))). The family of all α-open sets in (X, τ), denoted by τ , is a topology on X finer than τ. The closure of a subset A in (X, τ^α) is denoted by cl_α(A).

Definition 2.1: A subset A of a topological space (X, τ) is said to be

- Semiopen [4] if A⊆cl(int (A)).
- Semipreopen [5] if A⊆cl(int(cl (A))).

The complement of semi-open (resp. semipreopen) set is said to be semiclosed (resp. semi-preclosed). The family of all semiopen (resp. semipreopen) sets in X is denoted by SO(X) (resp .SPO(X)). The semiclosure of A [3] (resp. semipreclosure of A [5]), denoted by scl(A) (resp. spcl(A)), is defined by;

scl(A) = ∩ {F: A⊆F, X – F ∈ SO(X)},
spcl(A) = ∩{ F: A⊆F, X – F ∈ SPO(X)}.

Definition 2.2: A subset A of a topological space (X, τ) is said to be g-closed set [1] if cl(A)⊆U whenever A⊆U and U is open in X.

Definition 2.3: A subset A of a topological space (X, τ) is said to beω-closed set [6] (or $\hat{g}$ -closed set [7]) if cl(A)⊆U whenever A⊆U and U is semi-open in X. The complement of ω-closed set is said to be ω-open in X.

Definition 2.4: A subset A of a topological space (X, τ) is said to be*g-closed set [8] if cl(A)⊆U whenever A⊆U and U is ω-open in X. The complement of *g-closed set is said to be *g-open in X.

Definition 2.5: A subset A of a topological space (X, τ) is said to be#gs-closed set [9] if scl(A)⊆U whenever A⊆U and U is *g-open in X. The complement of #gs-closed set is said to be #gs-open in X.

Definition 2.6: A subset A of a topological space (X, τ) is said to be#pg-closed set [10] if spcl(A)⊆U whenever A⊆U and U is semi-open in X.

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Definition 2.7: A subset A of a topological space (X, τ) is said to be $\tilde{g}$ -closed set [11] if cl(A)⊆U whenever A⊆U and U is #gs-open in X.

Definition 2.8: A subset A of a topological space (X, τ) is said to be $\tilde{g}$ s-closed set [12] if scl(A)⊆U whenever A⊆U and U is #gs-open in X.

Definition 2.9: A subset A of a topological space (X, τ) is said to besg-closed set [2] if scl(A)⊆U whenever A⊆U and U is semi-open in X.

Definition 2.10: A subset A of a topological space (X, τ) is said to be gs-closed set [1] if scl(A)⊆U whenever A⊆U and U is open in X.

Definition 2.11: A subset A of a topological space (X, τ) is said to be gsp-closed set [5] if spcl(A)⊆U whenever A⊆U and U is open in X.

Throughout the present paper (X, τ) and (Y, σ) always denote topological spaces and f : (X, τ) →(Y, σ) presents a function.

3.m-Structures:

Definition 3.1: A subfamily m_x⊆P (X) is said to be a minimal structure [13](briefly*m*-structure) on X if ϕ, X∈m_x. The pair (X, m_x) is called a minimal space (*m* -space). Each member of$m_x$is said to be *m*-open and the complement of an *m*-open set is said to be *m*-closed.

Remark 3.2: Let (X, τ) be a topological space. Then m_x=τ, SO(X) and SPO(X) are minimal structures on X.

Definition 3.3: Let (X, m_x) be an *m*-space. For a subset A of X, the m_x-closure of A and the m_x-interior of A are defined in [14] as follows:

- $m-cl(A)=\cap\left\{\mathrm{F}:\mathrm{A}\subset\mathrm{F},\mathrm{F}^{c}\in m_{x}\right\}$
- $m-\operatorname{int}(A)=\cup\left\{\mathrm{U}:\mathrm{U}\subset\mathrm{A},\mathrm{U}\in m_{x}\right\}$

4.mω-Closed Sets: In this section, let (X, τ) be a topological space and $m_x$an*m*-structure on X. We obtain several basic properties of *m*ω-closed sets.

Definition 4.1: Let (X, τ) be a topological space and m_x an *m*-structure on X. A subset A of X is said to be *m*-semi open [14] if A⊆*m*-cl(*m*-int(A)). The family of all *m*-semi open sets in X is denoted by *m*SO(X). The complement of *m*-semi open set is said to be *m*-semiclosed.

Definition 4.2: Let (X, τ) be a topological space and m_x an *m*-structure on X. For a subset A of X, the *m*-semiclosure of A [14] and the *m*-semiinterior of A, denoted by *m*-scl(A) and *m*-sint(A), respectively are defined as follows:

- *m*-scl(A) = ∪{F:A⊂F, F *is* *m*-semi closed in X},
- *m*-sint(A) = ∩{U :U⊂A, Uis*m*-semi open in X}.

Definition 4.3: Let (X, τ) be a topological space and m_x an *m*-structure on X. A subset A of X is said to be *m*-space. A subset A of X is said to be;

- mω-closed if cl(A)⊆U whenever A⊆U and U is *m*-semi-open,
- *m*ω-open if its complement is *m*-ω-closed.

Remark 4.4: Let (X, τ) be a topological space and Aa subset of X. If *m*SO(X) = SO(X) (resp. τ) and A is *m*ω-closed , then A isω-closed (g-closed).

Theorem 4.5: Let (X, *m*SO(X)) be an *m*-space and Aa subset of X. Then x ∈*m*-scl(A) if and only if U∩A ≠ ϕ for every *m*-semi open set U containing x.

Proof: Suppose there exists *m*-semi open set U containing x such that U ∩ A = ϕ. Then A⊆X –U and X– (X–U) = U ∈ *m*SO(X). Then by definition 4.2, *m*-scl(A)⊆ X –U. Since x ∈ U, we have x ∉ *m*-scl(A). Conversely, suppose that x ∉ *m*-scl(A). There exists a subset F of X such that X – F ∈ *m*SO(X), A⊆F and x ∉ F. Then there exists *m*-semi open set X– F containing x such that (X– F) ∩ A = ϕ.

Definition 4.6: An *m*-structure m_x on a nonempty set X is said to have property C [13] if the union of any family of subsets belonging to m_x belongs to m_x.

Example 4.7: Let X = {a, b, c, d}, m_x = {ϕ, X, {a, b}, {a, c}, {b, d}}, τ={ϕ, X, {a}, {d}, {a, d}, {a, b, d}}. Then *m*ω -open sets are ϕ, X, {a}, {b}, {d}, {a, b}, {a, d} and {a, b, d}}. It is shown that *m*ωO(X) does not have property C.

Remark 4.8: Let (X, τ) be a topological space. Then the families SO(X) andτare all *m*-structure with property C.

Lemma 4.9: Let X be a nonempty set and*m*SO(X)an *m*-structure on X satisfying property C. For a subset A of X, the following properties hold:

- A ∈ *m*SO(X) if and only if *m*-sint(A) =A,
- A is *m*-semi closed if and only if *m*-scl(A) =A,
- *m*-sint(A) ∈ *m*SO(X) and *m*-scl(A) is *m*- semi closed.

Proposition 4.10: Let SO(X)⊆*m*SO(X). Then the following implications hold:

Closed → *m*ω-closed → ω-closed.

Proof: It is obvious that every closed set is *m*ω-closed. Suppose that A is an *m*ω-closed set. Let A⊆U and U∈ SO(X). Since SO(X)⊆*m*SO(X), cl(A)⊆U and hence A is ω-closed.

Example 4.11: Let X = {a, b, c }, m_x = {ϕ, X, {c}} and τ ={ϕ, X, {b}, {a, c}}. Then ω-closed sets are the power sets of X: *m*ω-closed are ϕ, X, {a}, {b}, {a, b} and {a, c} and closed sets are ϕ, X, {b} and {a, c}. It is clear that {b, c} is ω-closed but it is not *m*ω-closed and {a, b} is *m*ω-closed but it is not closed.

Proposition 4.12: If A and B are *m*ω-closed then A∪B is *m*ω-closed.

Proof: Let A∪B⊆U and U∈*m*SO(X), Then A⊆U and B⊆U. Since A and B are *m*ω-closed, we have cl(A∪B) = cl(A) ∪ cl(B) ⊆ U. Therefore, A∪B is *m*ω-closed.

Proposition 4.13: If A is *m*ω-closed and *m*-semi open, then A is closed.

Proposition 4.14: If A is *m*ω-closed and A⊆B⊆cl(A), then B is *m*ω-closed.

Proof: Let B⊆U and U∈*m*SO(X). Then A⊆U and A is*m*ω-closed. Hence cl(B)⊆cl(A)⊆U and B is *m*ω-closed.

Definition 4.15: Let (X, *m*SO(X)) be an *m*-space and Aa subset of X. Then *m*Semi-Frontier of A, *m*S-Fr(A), is defined as follows: *m*S-Fr(A) = *m*-scl(A)∩ *m*-scl(X– A).

Proposition 4.16: If A is *m*ω-closed and A⊆ U∈*m*SO(X), then *m*S-Fr(U)⊆int(X–A).

Proof: Let A be *m*ω-closed and A⊆ U∈*m*SO(X). Then cl(A)⊆U. Suppose that x∈*m*S-Fr(U). Since U∈*m*SO(X), *m*S-Fr(U) = *m*-scl(U)∩ *m*-scl(X– U)= *m*-scl(U)∩ (X–U)=*m*-scl(U) – U. Therefore, x∉U and x∉cl(A). This shows that x∈int(X–A) and hence *m*S-Fr(U)⊆int(X– A).

Proposition 4.17: A subset A of X is *m*ω-open if and only if F⊆int(A) whenever F⊆A and A is *m*-semi closed.

Proof: Suppose that A is *m*ω-open. Let F⊆A and F be *m*-semi closed. Then X–A⊆X– F∈*m*SO(X) and X –A is *m*ω-closed. Therefore, we have X – int(A) = cl(X– A)⊆X– F and hence F⊆int(A). Conversely, let X –A⊆G and G∈*m*SO(X). Then X – G⊆A and X – G is *m*-semi closed. By hypothesis, we have X – G⊆int(A) and hence cl(X– A) = X –int(A)⊆G. Therefore, X –A is *m*ω-closed and A is *m*ω-open.

Corollary 4.18: Let SO(X)⊆*m*SO(X). Then the following properties hold:

- Every open set is *m*ω-open and every *m*ω-open set is ω-open,
- If A and B are *m*ω-open, then A∩B is *m*ω-open,
- If A is *m*ω-open and *m*-semi closed, then A is open,
- If A is *m*ω-open and int(A)⊆B⊆A, then B is *m*ω-open.

Proof: This follows from propositions 4.10, 4.12, 4.13 and 4.14.

Characterizations of *m*ω-Closed Sets: In this section, let (X, τ) be a topological space and m_x an *m*-structure on X. We obtain some characterizations of *m*ω-closed sets.

Theorem 5.1: A subset A of X is *m*ω-closed if and only if cl(A) ∩ F=ϕ whenever A∩F = ϕ and F is *m*-semi closed.

Proof: Suppose that A is *m*ω-closed. Let A ∩ F = ϕ and F be *m*-semi closed. Then A⊆X– F∈ *m*SO(X) and cl(A) ⊆X– F. Therefore, we have cl(A) ∩ F = ϕ. Conversely, let A⊆U and U ∈ *m*SO(X). Then A∩ (X– U) = ϕ and X– U is *m*-semi closed. By the hypothesis, cl(A)∩ (X– U) = ϕ and hence cl(A)⊆U. Therefore, A is *m*ω-closed.

Theorem 5.2: Let SO(X)⊆*m*SO(X) and *m*SO(X) have property C. A subset A of X is *m*ω-closed if and only if cl(A) –A contains no nonempty *m*-semi closed.

Proof: Suppose that A is *m*ω-closed. Let F⊆cl(A) –A and F be *m*-semi closed. Then F⊆cl(A) and F ⊄ A and so A⊆X– F ∈ *m*SO(X) and hence cl(A)⊆X – F. Therefore, we have F⊆X –cl(A). Hence F = ϕ. Conversely, suppose that A is not *m*ω-closed. Then by Theorem 5.1, ϕ ≠ cl(A) –U for some U∈*m*SO(X) containing A. Since τ ⊆ SO(X) ⊆ *m*SO(X) and *m*SO(X)has property C, cl(A)–U is *m*-semi closed. Moreover, we have cl(A) –U ⊆cl(A) –A, a contradiction. Hence A is *m*ω-closed.

Theorem 5.3: Let SO(X)⊆*m*SO(X) and *m*SO(X) have property C. A subset A of X is *m*ω-closed if and only if cl(A) –A is *m*ω-open.

Proof: Suppose that A is $m\omega$-closed. Let F⊆cl(A) –A and F be m-semi closed. By Theorem 5.2, we have F = ϕ and F⊆ It follows from proposition 4.16, cl(A) –A is $m\omega$-open. Conversely, let A⊆U and U∈mSO(X). Then cl(A) ∩ (X– U)⊆ cl(A) –A and cl(A) –A is $m\omega$-open. Since τ ⊆ SO(X)⊆mSO(X) and mSO(X) has property C, cl(A) ∩ (X– U) is m-semi closed and by proposition 4.17, cl(A) ∩ (X– U)⊆int(cl(A) – A). Now int(cl(A) – A)= int(cl(A)) ∩ int(X– A) ⊆cl(A) ∩ int(X– A) = cl(A) ∩ (X– cl(A)) = ϕ Therefore, we have cl(A) ∩ (X– U)= ϕ and hence cl(A)⊆U. This shows that A is $m\omega$-closed.

Theorem 5.4: Let (X, mSO(X)) be an m-structure with property C. A subset A of X is $m\omega$-closed if and only if m-scl({x}) ∩ A ≠ ϕ for each x ∈ cl(A).

Proof: Suppose that A is $m\omega$-closed and m-scl({x}) ∩ A = ϕ for some x ∈ cl(A). By lemma 4.9, m-scl({x}) is m-semi closed and A⊆X–(m-scl({x})) ∈ mSO(X). Since A is $m\omega$-closed, cl(A)⊆ X–(m-scl({x}))⊆X– {x}, a contradiction, since x ∈ cl(A). Conversely, suppose that A is not $m\omega$-closed. Then by Theorem 5.1, ϕ ≠ cl(A) –U for some U ∈ mSO(X) containing A. There exists x ∈ cl(A) –U. Since x ∉ U, by Theorem 4.5, m-scl({x}) ∩ U = ϕ and hence m-scl({x}) ∩ A ⊂ m-scl({x}) ∩ U = ϕ. This shows that m-scl({x}) ∩ A = ϕ for some x ∈ cl(A). Hence A is $m\omega$-closed.

Corollary 5.5: Let SO(X)⊆mSO(X) and mSO(X) have property C. For a subset A of X, the following properties are equivalent:

- A is$m\omega$-open,
- A– int(A) contains no nonempty m-semi closed set,
- A– int(A) is $m\omega$-open,
- m-scl({x}) ∩ (X-A) ≠ ϕ for each x ∈ A-int(A).

Proof: This follows from Theorems 5.2, 5.3 and 5.4.

Preservation theorems

Definition 6.1: A function f: (X, m_x) → (Y, m_y) is said to be

- M-semi continuous if f^{-1}(V) is m-semi closed in (X, m_x) for every m-semi closed V in (Y, m_y),
- M-semi closed if for each m-semi closed set F of (X, m_x), f(F) is m-semi closed in (Y, m_y).

Theorem 6.2: Let$m$SO(X) be an m-structure with property C. Let f: (X, m_x) → (Y, m_y) be a function from a minimal space (X, m_x) into a minimal space (Y, m_y). Then the following are equivalent:

- f is M-semi continuous,
- f^{-1}(V) ∈ mSO(X) for every V ∈ mSO(Y).

Proof: Assume that f: (X, m_x) → (Y, m_y) is M-semi continuous. Let V ∈ mSO(Y). Then V^c is$m$-semi closed in (Y, m_y). Since f is M-semi continuous, $f^{-1}(V^c)$ is m-semi closed in (X, m_x). But $f^{-1}(V^c)$ = X – f^{-1}(V). Thus X– f^{-1}(V) is m-semi closed in (X, m_x) and so f^{-1}(V) is m-semi open in (X, m_x). Conversely, let for each V ∈ mSO(Y), f^{-1}(V) ∈ mSO(X). Let F be any m-semi closed in (Y, m_y). By assumption, $f^{-1}(F^c)$ is$m$-semi open in (X, m_x). But $f^{-1}(F^c)$= X– f^{-1}(F). Thus X– f^{-1}(F) is m-semi open in (X, m_x) and so f^{-1}(F) is m-semi closed in (X, m_x), Hence f is M-semi continuous.

Lemma 6.3: A function f: (X, m_x) → (Y, m_y) is M-semi closed if and only if for each subset B of Y and each U ∈ mSO(X) containing f^{-1}(B), there exists V ∈ mSO(Y) such that B⊆V and f^{-1}(V)⊆U.

Proof: Suppose that f is M-semi closed. Let B⊆Y and U ∈ mSO(X) containing f^{-1}(B). Put V= Y – f(X –U). Then V is m-semi open in (Y, m_y) and f^{-1}(V)⊆ f^{-1}(Y)–(X– U)=X– (X– U)=U.Also, since f^{-1}(B)⊆U, then X– U⊆f^{-1}(Y– B) which implies f(X– U)⊆Y–B and hence B⊆V. Hence we obtain V ∈ mSO(Y) such that B⊆V and f^{-1}(V)⊆U. Conversely, let F be any m-semi closed of (X, m_x). Set f(F) = B, then F⊆ f^{-1}(B) and f^{-1}(Y– B)⊆X– F ∈ mSO(X). By the hypothesis, there exists V ∈ mSO(Y) such that Y– B⊆V and f^{-1}(V)⊆X–F and so F ⊂ f^{-1}(Y– V). Therefore f(F)⊆ Y– V. Hence, we obtain Y– V⊆B = f(F)⊆Y– V. Therefore f(F) = Y– V is m-semi closed in (Y, m_y). Hence f is M-semi closed.

Theorem 6.4: Iff: (X, m_x) → (Y, m_y) is closed and f: (X, m_x) → (Y, m_y) is M-semi continuous, where mSO(X) has property C, then f(A) is $m\omega$-closed in (Y, m_y) for each $m\omega$-closed set A of (X, m_x).

Proof: Let A be any $m\omega$-closed set of (X, m_x) and f(A) ⊆ V ∈ mSO(Y). Then, by Theorem 6.2, A⊆ f^{-1}(V) ∈ mSO(X). Since A is $m\omega$-closed, cl(A)⊆f^{-1}(V) and f(cl(A))⊆V. Since f is closed, cl(f(A))⊆ f(cl(A))⊆V. Hence f(A) is $m\omega$-closed in (Y, m_y).

Theorem 6.5: Iff: (X, m_x) → (Y, m_y) is continuous and f: (X, m_x) → (Y, m_y) is M-semi closed, then f^{-1}(B) is $m\omega$-closed in (X, m_x) for each $m\omega$-closed set B of (Y, m_y).

Proof: Let B beany mω-closed set of (Y, m_y) and $f^{-1}(B) \subseteq U \in mSO(X)$. Since f is M-semi closed, by Lemma 6.3, there exists $V \in mSO(Y)$ such that $B \subseteq V$ and $f^{-1}(V) \subseteq U$. Since B is *m*ω-closed, $cl(B) \subseteq V$ and since f is continuous, $cl(f^{-1}(B)) \subseteq f^{-1}(cl(B)) \subseteq f^{-1}(V) \subseteq U$. Hence $f^{-1}(B)$ is *m*ω-closed in (X, m_x).

New Forms of Closed Sets in Topological Spaces

Definition 6.1: A subset A of X is called a $\tilde{g}$ -semi preclosed set ($\tilde{g}$ sp-closed set) if spcl (A) ⊆U whenever A⊆U and U is #gs-open in X.

By SO(X) (resp. $\tilde{G}$ SO(X), SGO(X), GSO(X), SPO(X), $\tilde{G}$ SPO(X), SPGO(X), GSPO(X)) we denote the collection of all semi open (resp. #gs-open, sg-open, gs-open, semi preopen, $\tilde{g}$ sp-open, spg-open, gsp-open) setsof topological space (X, τ). If $m_x = \tau$, these collection are minimal structures on X.

By the definitions, we obtain the following diagram:

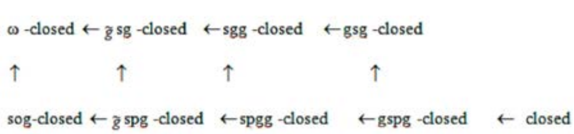
Diagram I

For subsets of a topological space (X, τ), we can define new types of closed sets as follows:

Definition 6.2: A subset A of a topological space (X, τ) is said to be ω-closed (resp. $\tilde{g}$ sg -closed, sgg -closed, gsg -closed, sog -closed, $\tilde{g}$ spg -closed, spgg -closed, gspg -closed) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open (resp. $\tilde{g}$ s-open, sg-open, gs-open, semi-preopen, $\tilde{g}$ sp -open, spg-open, gsp-open) in (X, τ).

By Diagram I and Definition 6.2, we have the following diagram:

Diagram II



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