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Homomorphism and Anti Homomorphism on a Bipolar Anti L - Fuzzy Sub ℓ - HX Group

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Abstract: In this paper, we introduce the concept of an anti image and anti pre-image of a bipolar L - fuzzy sub ℓ - HX group of a group G and discuss the properties of bipolar anti L - fuzzy sub ℓ - HX group under ℓ - HX group homomorphism and ℓ - HX group anti homomorphism.

Key words: Bipolar L - fuzzy ℓ - HX group · Bipolar anti L - fuzzy ℓ - HX group · ℓ - HX group homomorphism · ℓ - HX group anti homomorphism · Anti image and anti pre-image of bipolar L - fuzzy subgroup

INTRODUCTION

The concept of fuzzy sets was initiated by Zadeh [1]. Then it has become a vigorous area of research in engineering, medical science, social science, graph theory etc. Rosenfeld [2] gave the idea of fuzzy subgroups. In fuzzy sets the membership degree of elements range over the interval [0, 1]. The membership degree expresses the degree of belongingness of elements to a fuzzy set. The membership degree 1 indicates that an element completely belongs to its corresponding fuzzy set and membership degree 0 indicates that an element does not belong to fuzzy set. The membership degrees on the interval (0, 1) indicate the partial membership to the fuzzy set. Sometimes, the membership degree means the satisfaction degree of elements to some property or constraint corresponding to a fuzzy set. Li Hongxing [3] introduced the concept of HX group and the authors Luo Chengzhong, Mi Honghai, Li Hongxing [4] introduced the concept of fuzzy HX group. The author W.R.Zhang [5] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. K.M. Lee [6] introduced Bipolar-valued fuzzy sets and their operations. In case of Bipolar-valued fuzzy sets membership degree range is enlarged from the interval [0, 1] to [-1, 1]. In a bipolar-valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the membership degree (0,1] indicates that elements somewhat satisfy the property and the membership degree [-1,0) indicates that elements somewhat satisfy the implicit counter-property. G.S.V.Satya Saibaba [7] initiated the study of L - fuzzy lattice ordered groups and introduced the notions of L - fuzzy sub ℓ - HX group. J.A Goguen [8] replaced the valuation set [0, 1] by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L - Fuzzy sets. R.Muthuraj, M.Sridharan [9] introduced Homomorphism and anti-homomorphism on a bipolar anti fuzzy sub HX groups. R.Muthuraj, T.Rakesh kumar [10] defined some characterization of L – fuzzy ℓ - HX group. In this paper we define the concept of an anti image and anti pre-image of a bipolar L – fuzzy sub ℓ - HX group and study some of their related properties.

Preliminaries: In this section, we site the fundamental definitions that will be used in the sequel. Throughout this paper, G = (G, .) is a group, e is the identity element of G and xy, we mean x. y

Definition 2.1: A Bipolar L - fuzzy set μ in G is a bipolar L - fuzzy subgroup of G if for all x,y∈G.

- $\mu^+(xy) \geq \mu^+(x) \wedge \mu^+(y)$
- $\mu^-(xy) \leq \mu^-(x) \vee \mu^-(y)$
- $\mu^+(x^{-1}) = \mu^+(x)$, $\mu^-(x^{-1}) = \mu^-(x)$.

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Definition 2.2: A Bipolar anti L - fuzzy set μ in G is a bipolar anti L - fuzzy subgroup of G if for all $x,y \in G$.

- $\mu^+(xy) \le \mu^+(x) \vee \mu^+(y)$
- $\mu^-(xy) \ge \mu^-(x) \wedge \mu^-(y)$
- $\mu^+(x^{-1}) = \mu^+(x)$, $\mu^-(x^{-1}) = \mu^-(x)$.

Definition 2.3: Let μ be a bipolar L - fuzzy subset defined on G. Let $\vartheta \subset 2^G - \{\phi\}$ be a ℓ - HX group on G. A bipolar L - fuzzy set λ^μ defined on ϑ is said to be a bipolar ℓ - fuzzy sub ℓ - HX group on ϑ if for all $A,B \in \vartheta$.

- $(\lambda^\mu)^+(AB) \ge (\lambda^\mu)^+(A) \wedge (\lambda^\mu)^+(B)$
- $(\lambda^\mu)^-(AB) \le (\lambda^\mu)^-(A) \vee (\lambda^\mu)^-(B)$
- $(\lambda^\mu)^+(A) = (\lambda^\mu)^+(A^{-1})$
- $(\lambda^\mu)^-(A) = (\lambda^\mu)^-(A^{-1})$
- $(\lambda^\mu)^+(A \vee B) \ge (\lambda^\mu)^+(A) \wedge (\lambda^\mu)^+(B)$
- $(\lambda^\mu)^-(A \vee B) \le (\lambda^\mu)^-(A) \vee (\lambda^\mu)^-(B)$
- $(\lambda^\mu)^+(A \wedge B) \ge (\lambda^\mu)^+(A) \wedge (\lambda^\mu)^+(B)$
- $(\lambda^\mu)^-(A \wedge B) \le (\lambda^\mu)^-(A) \vee (\lambda^\mu)^-(B)$

where $(\lambda^\mu)^+(A) = \max\{\mu^+(x) \text{ / for all } x \in A \subseteq G\}$
and
$(\lambda^\mu)^-(A) = \min\{\mu^-(x) \text{ / for all } x \in A \subseteq G\}$

Example 2.1: Let $G=\{Z_5 - \{0\}, \cdot_5\}$ be a group and define a bipolar L- fuzzy set μ on G as $\mu^+(1) = 0.7$, $\mu^+(2) = 0.6$, $\mu^+(3) = 0.6$, $\mu^+(4) = 0.6$ and $\mu^-(1) = -0.8$, $\mu^-(2) = -0.5$, $\mu^-(3) = -0.5$, $\mu^-(4) = -0.5$.

By routine computations, it is easy to see that μ is a bipolar L-fuzzy sub group of G.

Let $\vartheta = \{\{1, 4\}, \{2, 3\}\}$ be a ℓ - HX group of G.
Let us consider $A = \{1, 4\}$, $B = \{2, 3\}$.

$\cdot_5$	A	B	$\wedge$	A	B	$\vee$	A	B
A	A	B	A	A	A	A	A	B
B	B	A	B	A	B	B	B	B

Define $(\lambda^\mu)^+(A) = \max\{\mu^+(x) \text{ / for all } x \in A \subseteq G\}$
and
$(\lambda^\mu)^-(A) = \min\{\mu^-(x) \text{ / for all } x \in A \subseteq G$

Now

$(\lambda^\mu)^+(A)=(\lambda^\mu)^+(\{1,4\})=\max\{\mu^+(1),\mu^+(4)\}= \max\{0.7,0.6\} = 0.7$

$(\lambda^\mu)^+(B)=(\lambda^\mu)^+(\{2,3\})=\max\{\mu^+(2),\mu^+(3)\}= \max\{0.6,0.6\} = 0.6$

$(\lambda^\mu)^+(AB) = (\lambda^\mu)^+(B) = 0.6$

$(\lambda^\mu)^+(A \wedge B) = (\lambda^\mu)^+(A) = 0.7$

$(\lambda^\mu)^+(A \vee B) = (\lambda^\mu)^+(B) = 0.6$

$(\lambda^\mu)^-(A)=(\lambda^\mu)^-(\{1,4\})=\min\{\mu^-(1),\mu^-(4)\}=\min\{-0.8,-0.5\}= -0.8$

$(\lambda^\mu)^-(B)=(\lambda^\mu)^-(\{2,3\})=\min\{\mu^-(2),\mu^-(3)\}=\min\{-0.5,-0.5\}= -0.5$

$(\lambda^\mu)^-(AB) = (\lambda^\mu)^-(B) = -0.5$

$(\lambda^\mu)^-(A \wedge B) = (\lambda^\mu)^-(A) = -0.8$

$(\lambda^\mu)^-(A \vee B) = (\lambda^\mu)^-(B) = -0.5$

By routine computations, it is easy to see that λ^μ is a bipolar L-fuzzy sub ℓ - HX group of ϑ.

Definition 2.4: Let μ be a bipolar L - fuzzy subset defined on G. Let $\vartheta \subset 2^G - \{\phi\}$ be a ℓ - HX group on G. A bipolar L - fuzzy set λ^μ defined on ϑ is said to be a bipolar anti L - fuzzy sub ℓ - HX group on ϑ if for all $A,B \in \vartheta$.

- $(\lambda^\mu)^+(AB) \le (\lambda^\mu)^+(A) \vee (\lambda^\mu)^+(B)$
- $(\lambda^\mu)^-(AB) \ge (\lambda^\mu)^-(A) \wedge (\lambda^\mu)^-(B)$
- $(\lambda^\mu)^+(A) = (\lambda^\mu)^+(A^{-1})$
- $(\lambda^\mu)^-(A) = (\lambda^\mu)^-(A^{-1})$
- $(\lambda^\mu)^+(A \vee B) \le (\lambda^\mu)^+(A) \vee (\lambda^\mu)^+(B)$
- $(\lambda^\mu)^-(A \vee B) \ge (\lambda^\mu)^-(A) \wedge (\lambda^\mu)^-(B)$
- $(\lambda^\mu)^+(A \wedge B) \le (\lambda^\mu)^+(A) \vee (\lambda^\mu)^+(B)$
- $(\lambda^\mu)^-(A \wedge B) \ge (\lambda^\mu)^-(A) \wedge (\lambda^\mu)^-(B)$

where $(\lambda^\mu)^+(A) = \min\{\mu^+(x) \text{ / for all } x \in A \subseteq G\}$
and
$(\lambda^\mu)^-(A) = \max\{\mu^-(x) \text{ / for all } x \in A \subseteq G\}$

Example 2.2: Let $G=\{Z_5 - \{0\}, \cdot_5\}$ be a group and define a bipolar L- fuzzy set μ on G as $\mu^+(1) = 0.3$, $\mu^+(2) = 0.7$, $\mu^+(3) = 0.7$, $\mu^+(4) = 0.7$ and $\mu^-(1) = -0.4$, $\mu^-(2) = -0.6$, $\mu^-(3) = -0.6$, $\mu^-(4) = -0.6$

By routine computations, it is easy to see that μ is a bipolar anti L-fuzzy subgroup of G.

Let $\vartheta = \{\{1, 4\}, \{2, 3\}\}$ be a ℓ - HX group of G.
Let us consider $A = \{1, 4\}$, $B = \{2, 3\}$.

$\cdot_5$	A	B	$\wedge$	A	B	$\vee$	A	B
A	A	B	A	A	A	A	A	B
B	B	A	B	A	B	B	B	B

Define $(\lambda^\mu)^+(A) = \min\{\mu^+(x) \text{ / for all } x \in A \subseteq G\}$
and
$(\lambda^\mu)^-(A) = \max\{\mu^-(x) \text{ / for all } x \in A \subseteq G\}$

Now

$(\lambda^\mu)^+(A)=(\lambda^\mu)^+(\{1,4\})=\min\{\mu^+(1),\mu^+(4)\}=\min\{0.3,0.7\}= 0.3$

$(\lambda^\mu)^+(B)=(\lambda^\mu)^+(\{2,3\})=\min\{\mu^+(2),\mu^+(3)\}=\min\{0.7,0.7\}= 0.7$

$(\lambda^\mu)^+(AB) = (\lambda^\mu)^+(B) = 0.7$

$(\lambda^\mu)^+(A \wedge B) = (\lambda^\mu)^+(A) = 0.3$

$(\lambda^\mu)^+(A \vee B) = (\lambda^\mu)^+(B) = 0.7$

$(\lambda^\mu)^-(A)=(\lambda^\mu)^-(\{1,4\})=\max\{\mu^-(1),\mu^-(4)\}=\max\{-0.4,-0.6\}=- 0.4$

$(\lambda^\mu)^-(B)=(\lambda^\mu)^-(\{2,3\})=\max\{\mu^-(2),\mu^-(3)\}=\max\{-0.6,-0.6\}=- 0.6$

$(\lambda^\mu)^-(AB) = (\lambda^\mu)^-(B) = -0.6$

$(\lambda^\mu)^-(A \wedge B) = (\lambda^\mu)^-(A) = -0.4$

$(\lambda^\mu)^-(A \vee B) = (\lambda^\mu)^-(B) = -0.6$

By routine computations, it is easy to see that λ^μ is a bipolar anti L-fuzzy sub ℓ - HX group of ϑ.

Definition 2.5: Let ϑ_1 and ϑ_2 be any two ℓ - HX groups on G_1 and G_2 respectively. The function f: $\vartheta_1 \to \vartheta_2$ is called an ℓ - HX group homomorphism if for all A,B in ϑ_1.

- f(AB) = f(A) f(B),
- $f(A\vee B) = f(A) \vee f(B)$,
- $f(A\wedge B) = f(A) \wedge f(B)$.

Definition 2.6: Let ϑ_1 and ϑ_2 be any two l - HX groups on G and G respectively. The function f: $\vartheta_1 \to \vartheta_2$ is called an ℓ - HX group anti homomorphism if for all A,B in ϑ_1.

- f(AB) = f(B) f(A),
- $f(A \vee B) = f(A) \vee f(B)$,
- $f(A\wedge B) = f(A) \wedge f(B)$.

Definition 2.7: Let G_1 and G_2 be any two groups. Let $\vartheta_1 \subset 2^{G_1} - \{\phi\}$ and $\vartheta_2 \subset 2^{G_2} - \{\phi\}$ be any two l - HX groups defined on G_1 and G_2 respectively. Let $\mu = (\mu^+, \mu^-)$ and $\alpha = (\alpha^+, \alpha^-)$ are bipolar L - fuzzy subsets in G_1 and G_2 respectively. Let $\lambda^\mu = ((\lambda^\mu)^+, (\lambda^\mu)^-)$ and $\eta^\alpha = ((\eta^\alpha)^+, (\eta^\alpha)^-)$ are bipolar L - fuzzy subsets in ϑ_1 and ϑ_2 respectively. Let f: $\vartheta_1 \to \vartheta_2$ be a mapping then the anti image $f_a(\lambda^\mu)$ of λ^μ is the bipolar L - fuzzy subset

$f_a(\lambda^\mu) = ((f_a(\lambda^\mu))^+, (f_a(\lambda^\mu))^-)$ of ϑ_2 defined as for each U $\in \vartheta_2$

$$(f_a\ (\lambda^\mu))^+\ (U) = \begin{cases} \min\{(\lambda^\mu)^+(X) : X \in f^{-1}(U)\}, & \text{if } f^{-1}(U) \neq \phi \\ 1, & \text{otherwise} \end{cases}$$

and

$$(f_a\ (\lambda^\mu))^-\ (U) = \begin{cases} \min\{(\lambda^\mu)^+(X) : X \in f^{-1}(U)\}, & \text{if } f^{-1}(U) \neq \phi \\ -1 & \text{otherwise} \end{cases}$$

and the anti pre-image $f_a^{-1}(\eta^\alpha)$ of η^α under f is the bipolar L - fuzzy subset of ϑ_1 defined as for each $X \in \vartheta_1$, $((f_a^{-1}(\eta^\alpha))^+ (X) = (\eta^\alpha)^+ (f_a(X)), (f_a^{-1}(\eta^\alpha))^- (X) = (\eta^\alpha)^- (f_a(X))$.

Properties of a Bipolar anti L - fuzzy sub ℓ - HX group under ℓ - HX group homomorphism and ℓ - HX group anti homomorphism: In this section, we define the notion of an anti image and anti pre-image of bipolar anti L - fuzzy sub ℓ - HX group of a ℓ - HX group under ℓ - HX group homomorphism and l - HX group anti homomorphism.

Theorem 3.1: Let f be a homomrphism from a ℓ - HX group ϑ_1 in to a ℓ - HX group ϑ_2. If $\eta^\alpha = ((\eta^\alpha)^+, (\eta^\alpha)^-)$ is a bipolar L - fuzzy subset of ϑ_2 then $f^{-1}((\eta^\alpha)^c) = [f^{-1}(\eta^\alpha)]^c$

Proof: Let $\eta^\alpha = ((\eta^\alpha)^+, (\eta^\alpha)^-)$ be a bipolar L-fuzzy subset of ϑ_2 then for each X $\in \vartheta_1$

- $[f^{-1}((\eta^\alpha)^c)]^+ (X) = ((\eta^\alpha)^c)^+ (f(X))$
$= 1- (\eta^\alpha)^+ (f(X))$
$= 1-f^{-1}((\eta^\alpha)^+) (X)$
$= [f^{-1}((\eta^\alpha)^+)]^c (X)$
$[f^{-1}((\eta^\alpha)^c)]^+ = [f^{-1}((\eta^\alpha)^+)]^c$
- $[f^{-1}((\eta^\alpha)^c)]^- (X) = ((\eta^\alpha)^c)^- (f(X))$
$= -1- (\eta^\alpha)^- (f(X))$
$= -1-f^{-1}((\eta^\alpha)^-) (X)$
$= [f^{-1}((\eta^\alpha)^-)]^c (X)$
$[f^{-1}((\eta^\alpha)^c)]^- = [f^{-1}((\eta^\alpha)^-)]^c$

Hence, $f^{-1}((\eta^\alpha)^c) = [f^{-1}(\eta^\alpha)]^c$

Theorem 3.2: Let f be a homomorphism from a ℓ - HX group ϑ_1 in to a ℓ - HX group ϑ_2. If $\lambda^\mu = ((\lambda^\mu)^+, (\lambda^\mu)^-)$ is a bipolar L-fuzzy subset of ϑ_1 then

- $f((\lambda^\mu)^c) = (f_a(\lambda^\mu))^c$
- $f_a((\lambda^\mu)^c) = (f(\lambda^\mu))^c$

Proof: Let $\lambda^\mu = ((\lambda^\mu)^+, (\lambda^\mu)^-)$ be a bipolar L-fuzzy subset of ϑ_1 then for each X $\in \vartheta_1$, Let f(X) = U $\in \vartheta_2$

- $[f((\lambda^\mu)^c)]^+ (U) = \max\{((\lambda^\mu)^c)^+(X) : X\in f^{-1}(U)\}$
$= \max\{1- (\lambda^\mu)^+(X): X\in f^{-1}(U)\}$
$= 1-\min\{(\lambda^\mu)^+(X): X\in f^{-1}(U)\}$
$= 1-f_a ((\lambda^\mu)^+) (U)$
$= [f_a ((\lambda^\mu)^+)]^c (U)$
$[f((\lambda^\mu)^c)]^+ = [f_a ((\lambda^\mu)^+)]^c$
$[f((\lambda^\mu)^c)]^- (U) = \max\{((\lambda^\mu)^c)^-(X) : X\in f^{-1}(U)\}$
$= \max\{-1- (\lambda^\mu)^-(X): X\in f^{-1}(U)\}$
$= -1-\min\{(\lambda^\mu)^-(X): X\in f^{-1}(U)\}$
$= -1-f_a ((\lambda^\mu)^-) (U)$
$= [f_a ((\lambda^\mu)^-)]^c (U)$
$[f((\lambda^\mu)^c)]^- = [f_a ((\lambda^\mu)^-)]^c$

Hence, $f((\lambda^\mu)^c) = (f_a(\lambda^\mu))^c$

- $[f_a((\lambda^\mu)^c)]^+ (U) = \min\{((\lambda^\mu)^c)^+(X) : X\in f^{-1}(U)\}$
$= \min\{1- (\lambda^\mu)^+(X): X\in f^{-1}(U)\}$
$= 1-\max\{(\lambda^\mu)^+(X): X\in f^{-1}(U)\}$
$= 1-f((\lambda^\mu)^+) (U)$
$= [f ((\lambda^\mu)^+)]^c (U)$
$[f_a((\lambda^\mu)^c)]^+ = [f ((\lambda^\mu)^+)]^c$
$[f_a((\lambda^\mu)^c)]^- (U) = \min\{((\lambda^\mu)^c)^-(X) : X\in f^{-1}(U)\}$
$= \min\{-1- (\lambda^\mu)^-(X): X\in f^{-1}(U$
$= -1-\max\{(\lambda^\mu)^-(X): X\in f^{-1}(U)\}$
$= -1-f ((\lambda^\mu)^-) (U)$
$= [f ((\lambda^\mu)^-)]^c (U)$
$[f_a((\lambda^\mu)^c)]^- = [f ((\lambda^\mu)^-)]^c$

Hence, $f_a((\lambda^\mu)^c) = (f(\lambda^\mu))^c$

Theorem 3.3: Let f be a homomorphism from a ℓ - HX group ϑ_1 in to a ℓ - HX group ϑ_2. If $\lambda^\mu=((\lambda^\mu)^+, (\lambda^\mu)^-)$ is a bipolar anti L - fuzzy sub ℓ - HX group of ϑ_1 then the anti image $f_a(\lambda^\mu)$ of λ^μ under f is a bipolar anti L - fuzzy sub ℓ - HX group of ϑ_2.

Proof: Let $\lambda^\mu=((\lambda^\mu)^+, (\lambda^\mu)^-)$ be a bipolar anti L - fuzzy sub ℓ - HX group of a ℓ - HX group ϑ_1. For all A,B in ϑ_1.

- $(f_a(\lambda^\mu))^+((f(A)f(B)) = (f_a(\lambda^\mu))^+(f(AB))$
  $= (\lambda^\mu)^+(AB)$
  $\leq (\lambda^\mu)^+(A) \vee (\lambda^\mu)^+(B)$
  $\leq (f_a(\lambda^\mu))^+(f(A)) \vee (f_a(\lambda^\mu))^+(f(B))$
  $f_a(\lambda^\mu))^+((f(A)f(B)) \leq (f_a(\lambda^\mu))^+(f(A)) \vee (f_a(\lambda^\mu))^+(f(B))$
- $(f_a(\lambda^\mu))^-((f(A)f(B)) = (f_a(\lambda^\mu))^-(f(AB))$
  $= (\lambda^\mu)^-(AB)$
  $\geq (\lambda^\mu)^-(A) \wedge (\lambda^\mu)^-(B)$
  $\geq (f_a(\lambda^\mu))^-(f(A)) \wedge (f_a(\lambda^\mu))^-(f(B))$
  $(f_a(\lambda^\mu))^-((f(A)f(B)) \geq (f_a(\lambda^\mu))^-(f(A)) \wedge (f_a(\lambda^\mu))^-(f(B))$
- $(f_a(\lambda^\mu))^+(f(A)^{-1}) = (f_a(\lambda^\mu))^+(f(A^{-1}))$
  $= (\lambda^\mu)^+(A^{-1})$
  $= (\lambda^\mu)^+(A)$
  $= (f_a(\lambda^\mu))^+(f(A)$
  $(f_a(\lambda^\mu))^+(f(A)^{-1}) = (f_a(\lambda^\mu))^+(f(A))$
- $(f_a(\lambda^\mu))^-(f(A)^{-1}) = (f_a(\lambda^\mu))^-(f(A^{-1}))$
  $= (\lambda^\mu)^-(A^{-1})$
  $= (\lambda^\mu)^-(A)$
  $= (f_a(\lambda^\mu))^-(f(A)$
  $(f_a(\lambda^\mu))^-(f(A)^{-1}) = (f_a(\lambda^\mu))^-(f(A))$
- $(f_a(\lambda^\mu))^+((f(A)\vee f(B)) = (f_a(\lambda^\mu))^+(f(A \vee B))$
  $= (\lambda^\mu)^+(A \vee B)$
  $\leq (\lambda^\mu)^+(A) \vee (\lambda^\mu)^+(B)$
  $\leq (f_a(\lambda^\mu))^+(f(A)) \vee (f_a(\lambda^\mu))^+(f(B))$
  $(f_a(\lambda^\mu))^+((f(A)\vee f(B)) \leq (f_a(\lambda^\mu))^+(f(A)) \vee (f_a(\lambda^\mu))^+(f(B))$
- $(f_a(\lambda^\mu))^-((f(A)\vee f(B)) = (f_a(\lambda^\mu))^-(f(A\vee B))$
  $= (\lambda^\mu)^-(A\vee B)$
  $\geq (\lambda^\mu)^-(A) \wedge (\lambda^\mu)^-(B)$
  $\geq (f_a(\lambda^\mu))^-(f(A)) \wedge (f_a(\lambda^\mu))^-(f(B))$
  $(f_a(\lambda^\mu))^-((f(A)\vee f(B)) \geq (f_a(\lambda^\mu))^-(f(A)) \wedge (f_a(\lambda^\mu))^-(f(B))$
- $(f_a(\lambda^\mu))^+((f(A)\wedge f(B)) = (f_a(\lambda^\mu))^+(f(A\wedge B))$
  $= (\lambda^\mu)^+(A\wedge B)$
  $\leq (\lambda^\mu)^+(A) \vee (\lambda^\mu)^+(B)$
  $\leq (f_a(\lambda^\mu))^+(f(A)) \vee (f_a(\lambda^\mu))^+(f(B))$
  $(f_a(\lambda^\mu))^+((f(A)\wedge f(B)) \leq (f_a(\lambda^\mu))^+(f(A)) \vee (f_a(\lambda^\mu))^+(f(B))$
- $(f_a(\lambda^\mu))^-((f(A)\wedge f(B)) = (f_a(\lambda^\mu))^-(f(A \vee B))$
  $= (\lambda^\mu)^-(A \vee B)$
  $\geq (\lambda^\mu)^-(A) \vee (\lambda^\mu)^-(B)$
  $\geq (f_a(\lambda^\mu))^-(f(A)) \vee (f_a(\lambda^\mu))^-(f(B))$

$(f_a(\lambda^\mu))^-((f(A) \wedge f(B)) \geq (f_a(\lambda^\mu))^-(f(A)) \vee (f_a(\lambda^\mu))^-(f(B))$

Hence, $f_a(\lambda^\mu)=((f_a(\lambda^\mu))^+, (f_a(\lambda^\mu))^-)$ is a bipolar anti L - fuzzy sub ℓ - HX group of a ℓ - HX group ϑ_2.

Theorem 3.4: The homomorphic anti pre-image of a bipolar anti

L - fuzzy sub ℓ - HX group $\eta^\alpha=((\eta^\alpha)^+, (\eta^\alpha)^-)$ of a l - HX group ϑ_2 is a bipolar anti ℓ - fuzzy sub ℓ - HX group of a ℓ - HX group ϑ_1.

Proof: Let $\eta^\alpha=((\eta^\alpha)^+, (\eta^\alpha)^-)$ be a bipolar anti L - fuzzy sub l - HX group of a l - HX group ϑ_2.

- $(f^{-1}(\eta^\alpha))^+(AB) = (\eta^\alpha)^+(f(AB))$
  $= (\eta^\alpha)^+(f(A)\, f(B))$
  $\leq (\eta^\alpha)^+(f(A)) \vee (\eta^\alpha)^+(f(B))$
  $\leq (f^{-1}(\eta^\alpha))^+(A) \vee (f^{-1}(\eta^\alpha))^+(B)$
  $(f^{-1}(\eta^\alpha))^+(AB) \leq (f^{-1}(\eta^\alpha))^+(A) \vee (f^{-1}(\eta^\alpha))^+(B)$
- $(f^{-1}(\eta^\alpha))^-(AB) = (\eta^\alpha)^-(f(AB))$
  $= (\eta^\alpha)^-(f(A)\, f(B))$
  $\geq (\eta^\alpha)^-(f(A)) \wedge (\eta^\alpha)^+(f(B))$
  $\geq (f^{-1}(\eta^\alpha))^-(A) \wedge (f^{-1}(\eta^\alpha))^-(B)$
  $(f^{-1}(\eta^\alpha))^-(AB) \geq (f^{-1}(\eta^\alpha))^-(A) \wedge (f^{-1}(\eta^\alpha))^-(B)$
- $(f^{-1}(\eta^\alpha))^+(A^{-1}) = (\eta^\alpha)^+(f(A^{-1}))$
  $= (\eta^\alpha)^+(f(A)^{-1})$
  $= (\eta^\alpha)^+(f(A))$
  $= (f^{-1}(\eta^\alpha))^+(A)$
  $(f^{-1}(\eta^\alpha))^+(A^{-1}) = (f^{-1}(\eta^\alpha))^+(A)$
- $(f^{-1}(\eta^\alpha))^-(A^{-1}) = (\eta^\alpha)^-(f(A^{-1}))$
  $= (\eta^\alpha)^-(f(A)^{-1})$
  $= (\eta^\alpha)^-(f(A))$
  $= (f^{-1}(\eta^\alpha))^-(A)$
  $(f^{-1}(\eta^\alpha))^-(A^{-1}) = (f^{-1}(\eta^\alpha))^-(A)$
- $(f^{-1}(\eta^\alpha))^+(A\vee B) = (\eta^\alpha)^+(f(A\vee B))$
  $= (\eta^\alpha)^+(f(A)\vee f(B))$
  $\leq (\eta^\alpha)^+(f(A)) \vee (\eta^\alpha)^+(f(B))$
  $\leq (f^{-1}(\eta^\alpha))^+(A) \vee (f^{-1}(\eta^\alpha))^+(B)$
  $(f^{-1}(\eta^\alpha))^+(A\vee B) \leq (f^{-1}(\eta^\alpha))^+(A) \vee (f^{-1}(\eta^\alpha))^+(B)$
- $(f^{-1}(\eta^\alpha))^-(A\vee B) = (\eta^\alpha)^-(f(A\vee B))$
  $= (\eta^\alpha)^-(f(A) \vee f(B))$
  $\geq (\eta^\alpha)^-(f(A)) \wedge (\eta^\alpha)^-(f(B))$
  $\geq (f^{-1}(\eta^\alpha))^-(A) \wedge f^{-1}(\eta^\alpha))^-(B)$
  $(f^{-1}(\eta^\alpha))^-(A\wedge B) \geq (f^{-1}(\eta^\alpha))^-(A) \wedge (f^{-1}(\eta^\alpha))^-(B)$
- $(f^{-1}(\eta^\alpha))^+(A\wedge B) = (\eta^\alpha)^+(f(A \wedge B))$
  $= (\eta^\alpha)^+(f(A) \wedge f(B))$
  $\leq (\eta^\alpha)^+(f(A)) \vee (\eta^\alpha)^+(f(B))$
  $\leq (f^{-1}(\eta^\alpha))^+(A) \vee (f^{-1}(\eta^\alpha))^+(B)$
  $(f^{-1}(\eta^\alpha))^+(A \wedge B) \leq (f^{-1}(\eta^\alpha))^+(A) \vee (f^{-1}(\eta^\alpha))^+(B)$
- $(f^{-1}(\eta^\alpha))^-(A \wedge B) = (\eta^\alpha)^-(f(A \wedge B))$
  $= (\eta^\alpha)^-(f(A) \wedge f(B))$
  $\geq (\eta^\alpha)^-(f(A)) \wedge (\eta^\alpha)^-(f(B))$
  $\geq (f^{-1}(\eta^\alpha))^-(A) \wedge (f^{-1}(\eta^\alpha))^-(B)$
  $(f^{-1}(\eta^\alpha))^-(A \wedge B) \geq (f^{-1}(\eta^\alpha))^-(A) \wedge (f^{-1}(\eta^\alpha))^-(B)$

Hence, $f^{-1}(\eta^{\alpha}) = ((f^{-1}(\eta^{\alpha}))^{+}, (f^{-1}(\eta^{\alpha})^{-})$ is a bipolar anti L - fuzzy sub ℓ - HX group of a ℓ - HX group ϑ_1.

Theorem 3.5: Let f be an anti homomorphism from a ℓ - HX group ϑ_1 in to a ℓ - HX group ϑ_2. If $\lambda^{\mu} =((\lambda^{\mu})^{+}, (\lambda^{\mu})^{-})$ is a bipolar anti L - fuzzy sub ℓ - HX group of ϑ_1 then the anti image $f_a(\lambda^{\mu})$ of λ^{μ} under f is a bipolar anti L - fuzzy sub ℓ - HX group of ϑ_2.

Proof: Let $\lambda^{\mu} =((\lambda^{\mu})^{+},(\lambda^{\mu})^{-})$ be a bipolar anti L - fuzzy sub ℓ - HX group of a ℓ - HX group ϑ_1 for all A,B in ϑ_1.

- $f_a(\lambda^{\mu})^{+}((f(A)\, f(B))$ $= (f_a(\lambda^{\mu}))^{+}(f(BA))$
  $= (\lambda^{\mu})^{+}(BA)$
  $\leq (\lambda^{\mu})^{+}(B) \vee (\lambda^{\mu})^{+}(A)$
  $\leq (\lambda^{\mu})^{+}(A) \vee (\lambda^{\mu})^{+}(B)$
  $\leq (f_a(\lambda^{\mu}))^{+}(f(A)) \vee (f_a(\lambda^{\mu}))^{+}(f(B))$
- $f_a(\lambda^{\mu})^{-}((f(A)\, f(B))$ $= (f_a(\lambda^{\mu}))^{-}(f(BA))$
  $= (\lambda^{\mu})^{-}(BA)$
  $\geq (\lambda^{\mu})^{-}(B) \wedge (\lambda^{\mu})^{-}(A)$
  $\geq (\lambda^{\mu})^{-}(A) \wedge (\lambda^{\mu})^{-}(B)$
  $\geq (f_a(\lambda^{\mu}))^{-}(f(A)) \wedge (f_a(\lambda^{\mu}))^{-} (f(B)$
- $(f_a(\lambda^{\mu}))^{+}(f(A)^{-1})$ $= (f_a(\lambda^{\mu}))^{+}(f(A^{-1}))$
  $= (\lambda^{\mu})^{+}(A^{-1})$
  $= (\lambda^{\mu})^{+}(A)$
  $= (f_a(\lambda^{\mu}))^{+}(f(A)$
  $(f_a(\lambda^{\mu}))^{+}(f(A)^{-1}$ $= (f_a(\lambda^{\mu}))^{+}(f(A))$
- $(f_a(\lambda^{\mu}))^{-}(f(A)^{-1})$ $= (f_a(\lambda^{\mu}))^{-}(f(A^{-1}))$
  $= (\lambda^{\mu})^{-}(A^{-1})$
  $= (\lambda^{\mu})^{-}(A)$
  $= (f_a(\lambda^{\mu}))^{-}(f(A)$
  $(f_a(\lambda^{\mu}))^{-}(f(A)^{-1})$ $= (f_a(\lambda^{\mu}))^{-}(f(A))$
- $(f_a(\lambda^{\mu}))^{+}((f(A) \vee f(B))$ $= (f_a(\lambda^{\mu}))^{+}(f(A \vee B))$
  $= (\lambda^{\mu})^{+}(A \vee B)$
  $\leq (\lambda^{\mu})^{+} (A) \vee (\lambda^{\mu+}(B)$
  $\leq (f_a(\lambda^{\mu}))^{+}(f(A)) \vee (f_a(\lambda^{\mu}))^{+}(f(B))$
  $(f_a(\lambda^{\mu}))^{+}((f(A) \vee f(B))$ $\leq (f_a(\lambda^{\mu}))^{+}(f(A)) \vee (f_a(\lambda^{\mu}))^{+}(f(B))$
- $(f_a(\lambda^{\mu}))^{-}((f(A) \vee f(B))$ $= (f_a(\lambda^{\mu}))^{-}(f(A \vee B))$
  $= (\lambda^{\mu})^{-}(A \vee B)$
  $\geq (\lambda^{\mu})^{-} (A) \wedge (\lambda^{\mu})^{-}(B)$
  $\geq f_a(\lambda^{\mu}))^{-}(f(A)) \wedge (f_a(\lambda^{\mu}))^{-}(f(B))$
  $(f_a(\lambda^{\mu}))^{-}((f(A) \vee f(B))$ $\geq (f_a(\lambda^{\mu}))^{-}(f(A)) \wedge (f_a(\lambda^{\mu}))^{-}(f(B))$
- $(f_a(\lambda^{\mu}))^{+}((f(A) \wedge f(B))$ $= (f_a(\lambda^{\mu}))^{+}(f(A \wedge B))$
  $= (\lambda^{\mu})^{+}(A \wedge B)$
  $\leq (\lambda^{\mu})^{+} (A) \vee (\lambda^{\mu})^{+}(B)$
  $\leq (f_a(\lambda^{\mu}))^{+}(f(A)) \vee (f_a(\lambda^{\mu}))^{+}(f(B))$
  $(f_a(\lambda^{\mu}))^{+}((f(A) \wedge f(B))$ $\leq (f_a(\lambda^{\mu}))^{+}(f(A)) \vee (f_a(\lambda^{\mu}))^{+}(f(B))$
- $(f_a(\lambda^{\mu}))^{-}((f(A) \wedge f(B))$ $= (f_a(\lambda^{\mu}))^{-}(f(A \wedge B))$
  $= (\lambda^{\mu})^{-}(A \wedge B)$
  $\geq (\lambda^{\mu})^{-} (A) \wedge (\lambda^{\mu})^{-}(B)$
  $\geq (f_a(\lambda^{\mu}))^{-}(f(A)) \wedge (f_a(\lambda^{\mu}))^{-}(f(B))$
  $(f_a(\lambda^{\mu}))^{-}((f(A) \wedge f(B))$ $\geq (f_a(\lambda^{\mu}))^{-}(f(A)) \wedge (f_a(\lambda^{\mu}))^{-}(f(B))$

Hence, $f_a(\lambda^{\mu})=((f_a(\lambda^{\mu}))^{+}, (f_a(\lambda^{\mu}))^{-})$ is a bipolar anti L - fuzzy sub ℓ - HX group of a ℓ - HX group ϑ_2.

Theorem 3.6: The anti homomorphic anti pre-image of a bipolar anti L - fuzzy sub ℓ - HX group $\eta^{\alpha} =((\eta^{\alpha})^{+}, (\eta^{\alpha})^{-})$ of a ℓ - HX group ϑ_2 is a bipolar anti L - fuzzy sub ℓ - HX group of a ℓ - HX group ϑ_1

Proof: Let $\eta^{\alpha} =((\eta^{\alpha})^{+}, (\eta^{\alpha})^{-})$ be a bipolar anti L - fuzzy sub ℓ - HX group of a ℓ - HX group ϑ_2.

- $(f^{-1}(\eta^{\alpha}))^{+}(AB)$ $= (\eta^{\alpha})^{+}(f(AB))$
  $= (\eta^{\alpha})^{+}(f(B)f(A))$
  $\leq (\eta^{\alpha})^{+}(f(B)) \vee (\eta^{\alpha})^{+}(f(A))$
  $\leq (\eta^{\alpha})^{+}(f(A)) \vee (\eta^{\alpha})^{+}(f(B))$
  $\leq (f^{-1}(\eta^{\alpha}))^{+}(A) \vee (f^{-1}(\eta^{\alpha}))^{+}(B)$
  $(f^{-1}(\eta^{\alpha}))^{+}(AB)$ $\leq (f^{-1}(\eta^{\alpha}))^{+}(A) \vee (f^{-1}(\eta^{\alpha}))^{+}(B)$
- $(f^{-1}(\eta^{\alpha}))^{-}(AB)$ $= (\eta^{\alpha})^{-}(f(AB))$
  $= (\eta^{\alpha})^{-}(f(B)f(A))$
  $\geq (\eta^{\alpha})^{-}(f(B)) \wedge (\eta^{\alpha})^{-}(f(A))$
  $\geq (\eta^{\alpha})^{-}(f(A)) \wedge (\eta^{\alpha})^{-}(f(B))$
  $\geq (f^{-1}(\eta^{\alpha}))^{-}(A) \wedge (f^{-1}(\eta^{\alpha}))^{-}(B)$
  $(f^{-1}(\eta^{\alpha}))^{-}(AB)$ $\geq (f^{-1}(\eta^{\alpha}))^{-}(A) \wedge (f^{-1}(\eta^{\alpha}))^{-}(B)$
- $(f^{-1}(\eta^{\alpha}))^{+}(A^{-1})$ $= (\eta^{\alpha})^{+}(f(A^{-1}))$
  $= (\eta^{\alpha})^{+}(f(A)^{-1})$
  $= (\eta^{\alpha})^{+}(f(A))$
  $= (f^{-1}(\eta^{\alpha}))^{+}(A)$
  $(f^{-1}(\eta^{\alpha}))^{+}(A^{-1})$ $= (f^{-1}(\eta^{\alpha}))^{+}(A)$
- $(f^{-1}(\eta^{\alpha}))^{-}(A^{-1})$ $= (\eta^{\alpha})^{-}(f(A^{-1}))$
  $= (\eta^{\alpha})^{-}(f(A)^{-1})$
  $= (\eta^{\alpha})^{-}(f(A))$
  $= (f^{-1}(\eta^{\alpha}))^{-}(A)$
  $(f^{-1}(\eta^{\alpha}))^{-}(A^{-1})$ $= (f^{-1}(\eta^{\alpha}))^{-}(A)$
- $(f^{-1}(\eta^{\alpha}))^{+}(A \vee B)$ $= (\eta^{\alpha})^{+}(f(A \vee B))$
  $= (\eta^{\alpha})^{+}(f(A) \vee f(B))$
  $\leq (\eta^{\alpha})^{+}(f(A)) \vee (\eta^{\alpha})^{+}(f(B))$
  $\leq (f^{-1}(\eta^{\alpha}))^{+}(A) \vee (f^{-1}(\eta^{\alpha}))^{+}(B)$
  $(f^{-1}(\eta^{\alpha}))^{+}(A \vee B)$ $\leq (f^{-1}(\eta^{\alpha}))^{+}(A) \vee (f^{-1}(\eta^{\alpha}))^{+}(B)$
- $(f^{-1}(\eta^{\alpha}))^{-}(A \vee B)$ $= (\eta^{\alpha})^{-}(f(A \vee B))$
  $= (\eta^{\alpha})^{-}(f(A) \vee f(B))$
  $\geq (\eta^{\alpha})^{-}(f(A)) \wedge (\eta^{\alpha})^{-}(f(B))$
  $\geq (f^{-1}(\eta^{\alpha}))^{-}(A) \wedge (f^{-1}(\eta^{\alpha}))^{-}(B)$
  $(f^{-1}(\eta^{\alpha}))^{-}(A \vee B)$ $\geq (f^{-1}(\eta^{\alpha}))^{-}(A) \wedge (f^{-1}(\eta^{\alpha}))^{-}(B)$
- $(f^{-1}(\eta^{\alpha}))^{+}(A \wedge B)$ $= (\eta^{\alpha})^{+}(f(A \wedge B))$
  $= (\eta^{\alpha})^{+}(f(A) \vee f(B))$
  $\leq (\eta^{\alpha})^{+}(f(A)) \vee (\eta^{\alpha})^{+}(f(B))$
  $\leq (f^{-1}(\eta^{\alpha}))^{+}(A) \vee (f^{-1}(\eta^{\alpha}))^{+}(B)$
  $f^{-1}(\eta^{\alpha}))^{+}(A \wedge B)$ $\leq (f^{-1}(\eta^{\alpha}))^{+}(A) \vee (f^{-1}(\eta^{\alpha}))^{+}(B)$
- $(f^{-1}(\eta^{\alpha}))^{-}(A \wedge B)$ $= (\eta^{\alpha})^{-}(f(A \wedge B))$
  $= (\eta^{\alpha})^{-}(f(A) \wedge f(B))$
  $\geq (\eta^{\alpha})^{-}(f(A)) \wedge (\eta^{\alpha})^{-}(f(B))$
  $\geq (f^{-1}(\eta^{\alpha}))^{-}(A) \wedge (f^{-1}(\eta^{\alpha}))^{-}(B)$
  $(f^{-1}(\eta^{\alpha}))^{-}(A \wedge B)$ $\geq (f^{-1}(\eta^{\alpha}))^{-}(A) \wedge (f^{-1}(\eta^{\alpha}))^{-}(B)$

Hence, $f^{-1}(\eta^{\alpha}) = ((f^{-1}(\eta^{\alpha}))^{+}, (f^{-1}(\eta^{\alpha})^{-})$ is a bipolar anti L - fuzzy sub ℓ - HX group of a ℓ - HX group ϑ_1.

CONCLUSION

In this paper we discuss the notion of an anti image and anti pre-image of bipolar anti L - fuzzy sub ℓ - HX group of a ℓ - HX group under ℓ - HX group homomorphism and ℓ - HX group anti homomorphism.

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