

Weak Q(a) Balance Edge-Magic Graphs of Prism and Anti prism graphs

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Abstract: A graph G is a (p,q) graph in which the edges are labeled by $1,2,3,\dots,q$ so that the vertex sum are constant, mod p , than G is called an edge-magic graph. A (p,q) -graph G in which the edges are labeled by $Q(a)$ so that vertex sums mod p is constant, is called $Q(a)$ -Balance Edge-Magic Graph (In short $Q(a)$ -BEM). In this article defines weak $Q(a)$ balance edge-magic of Triangular prism graphs, Cubical prism graphs, Pentagonal prism graphs, Hexagonal prism graphs, Heptagonal prism graph, Cube Antiprism graph, Square Antiprism graph, Antiprism graph.

Key words: Magic · Edge magic · $Q(a)$ balance edge magic · Prism · Antiprism

INTRODUCTION

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Number theory gives more conjunctures to find labelling graphs. In 1963, Sedláček introduced magic labelling. This Labeled graphs are useful to form family of Mathematical Models from a broad range of applications. An enormous body of literature has grown around labeling in about more than 1700 papers. Stewart [1, 2] defined magic graphs and super-magic complete graphs if there is a labelling of the edges with integers such that for each vertex v the sum of the labels of all edges incident with v is the same for all v . A semi-magic labelling where the edges are labeled with distinct positive integers. Labeling includes lots of vertices such as graceful, magic, antimagic, bimagic, vertex magic, edge magic, total magic, Harmonious, felicitous, elegant, cordial, prime labeling, $Q(a)$ Balance Edge-Magic graphs (BEM), $Q(a)$ Balance Super Edge-Magic (BSEM) graphs etc. All labeled graphs using in varies places like networking, neural networks, stochastic models, etc.

If G admits a magic labelling then such a graph is called a magic graph. In 1963, J. Sedláček defines on magic graphs in [3]. J.A. Gallian [4], survey to know about the numerous graph labelling methods in one by one. All graphs considered here are finite, simple and undirected. [1, 5-7] defined magic graphs and magic valuations. Magic valuations extended to finite graph and complete graphs by Kotzig and A. Rosa [8, 9]. In, [10-12] discussed edge-magic results. In 2007, introduced $Q(a)$ -Balance Edge-

Magic Graphs(BEM) by Sin-Min Lee and Thomas Wong, Sheng – Ping Bill Lo [13] verified for regular graph, pendent vertices, friendship graph, complete graph, complete bipartite graph, fan graph and wheel graphs. In 2009, $Q(a)$ Balance Edge-Magic extended the results to some conjuncture by Ping-Tsai Chung, Sin-Min Lee [10]. In [14, 15] extended some conjuncture results and some graphs. This article contains the research work of weak $Q(a)$ Balance Edge Magic graphs of prism graph family and antiprism graphs family.

Premenaries: G is called edge-magic if there exists a bijective function $f:V(G) \square E(G) \rightarrow \{1,2,3,\dots,|V(G)| \square |E(G)|\}$ such that $f(x) + f(xy) + f(y)$ is a constant for every edge $x,y \in E(G)$. Unless, A Graph (p,q) -graph G such that the edges are labeled $1,2,3,\dots,p$ so that the vertex sums are constant, mod p , is called edge-magic.

Note: A necessary condition for a (p, q) -graph to be edge-magic is $q(q+1) \equiv 0 \pmod{p}$.

A (p,q) -graph G in with the edges are labeled by $Q(a)$ so that the vertex sums mod p is a constant, is called $Q(a)$ -balance edge-magic (in short $Q(a)$ -BEM).

For $a \geq 1$, we denote

$\{\pm a, \dots, \pm(a-1+q/2)\}$, if q is even,

$Q(a) = \{$

$\{0, \pm a, \dots, \pm(a-1+(q-1)/2)\}$, if q is odd.

Cartesian product of cycle and path graph $(C_m \times P_n)$ are called prism graphs.

A triangular prism denoted by $Y_{(3,n)}$ is a prism composed of two triangular bases and three rectangular sides. It is pentagon.

A pentagonal prism is a prism having two pentagonal bases and five rectangular sides. It is heptagon.

A pentagonal prism is a prism having two pentagonal bases and five rectangular sides. It is heptagon.

A hexagonal prism is a prism composed of two hexagonal bases and six rectangular sides. It is octahedron.

An antiprism graph is a graph corresponding to the skeleton of an antiprism. Anti prism graphs are therefore polyhedral and planer. The n -antiprism graph has $2n$ vertices and $4n$ edges.

RESULTS AND DISCUSSION

Theorem 1: Triangular prism $Y_{(3,n)}$ graph is weak Q(a) BEM when $n = 12$

Proof:

Let vertices $n = 12$, edges $q = 2n - n$, i.e., $q = 21$

Triangular prism $Y_{(3,3)}$ is Q(a) balanced edge magic for $a = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$

By the definition of BEM and edge q is odd. Fig1, shows that Triangular prism $Y_{(3,3)}$ graph is Q(1) BEM.

A Q(2)-BEM labelling for Triangular prism $Y_{(3,3)}$ graph: $\{3, 9, -6, -7, 7, 4, 2, 8, 6, 10, -10, 9, -3, 0, 11, 2, -5, 5, -4, -11, -8\}$

A Q(3)-BEM labelling for Triangular prism $Y_{(3,3)}$ graph: $\{3, 9, -6, -7, 7, 4, 12, 8, 6, 10, -10, 9, -3, 0, 11, 12, -5, 5, -4, -11, -8\}$

A Q(4)-BEM labelling for Triangular prism $Y_{(3,3)}$ graph: $\{13, 9, -6, -7, 7, 4, 12, 8, 6, 10, -10, 9, -13, 0, 11, 12, -5, 5, -4, -11, -8\}$

A Q(5)-BEM labelling for Triangular prism $Y_{(3,3)}$ graph: $\{14, 13, 9, -6, -7, 7, -14, 12, 8, 6, 10, -10, 9, -13, 0, 11, 12, -9, -5, 5, -11, -8\}$

A Q(6)-BEM labelling for Triangular prism $Y_{(3,3)}$ graph: $\{14, 13, 9, -6, -7, 7, -14, 12, 8, 6, 10, -10, 9, -13, 0, 11, 12, -15, 15, -11, -8\}$

A Q(7)-BEM labelling for Triangular prism $Y_{(3,3)}$ graph: $\{16, 14, 13, 9, -16, -7, 7, -14, 12, 8, 10, -10, 9, -13, 0, 11, 12, -15, 15, -11, -8\}$

A Q(8)-BEM labelling for Triangular prism $Y_{(3,3)}$ graph: $\{17, -15, 16, 14, 13, 9, -16, -14, 12, 8, 10, -10, 9, -13, 0, 11, 12, -17, 15, -11, -8\}$

A Q(9)-BEM labelling for Triangular prism $Y_{(3,3)}$ graph: $\{13, 17, -15, 16, 14, 18, 9, -16, -14, 12, -18, 10, -10, 9, -13, 0, 11, 12, -17, 15, -11, -8\}$

A Q(10)-BEM labelling for Triangular prism $Y_{(3,3)}$ graph: $\{19, 13, -19, 17, -15, 16, 14, 18, -16, -14, 12, -18, 10, -10, -13, -11, 12, -17, 15, -11\}$

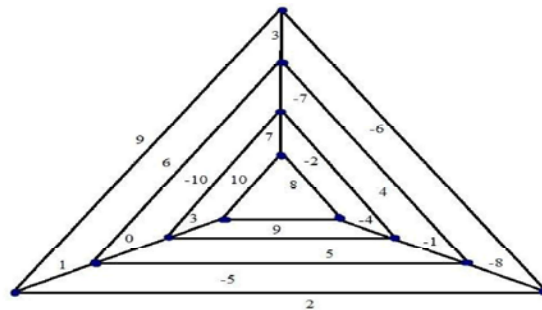


Fig. 1:

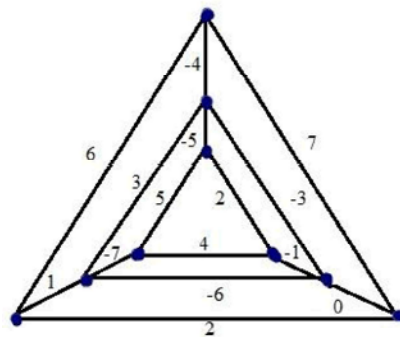


Fig 2:

A Q(11)-BEM labelling for Triangular prism $Y_{(3,3)}$ graph: $\{18, 19, 13, -19, 17, -15, 16, 14, 20, -16, -14, 12, -18, -13, 0, 11, 12, -17, 15, -11, -20\}$

A Q(12)-BEM labelling for Triangular prism $Y_{(3,3)}$ graph: $\{21, 18, -21, 19, 13, -19, 17, -15, 16, 14, 20, -16, -14, 12, -18, -13, 0, 12, -17, 15, -20\}$.

Example: Triangular prism $Y_{(3,2)}$ is weak Q(a)-BEM Generalization of Triangular prism is $2n - 3$

Then vertices $n = 9$, edges $q = 15$

Triangular prism $Y(3,2)$ is Q(a)-balanced edge magic for $a = 1, 2, 3, 4, 5, 6, 7, 8, 9$

By the definition of Q(a) BEM, for $a = 1$ and edges q is odd. By Fig 2, shows that Q(1) BEM labelling for Triangular prism $Y(3,2)$ graph

A Q(2)-BEM labelling for Triangular prism $Y(3,2)$ graph: $\{6, 7, -4, -3, 0, 8, -8, -6, 3, 5, -7, 2, 5, -5, -4, -2\}$

A Q(3)-BEM labelling for Triangular prism $Y(3,2)$ graph: $\{6, 7, -4, -3, 0, -6, 8, 3, 5, -7, -8, 5, -5, 9, -9, -4\}$

A Q(4)-BEM labelling for Triangular prism $Y(3,2)$ graph: $\{-9, 6, 7, -4, -10, 0, -6, 8, 10, 5, -7, -8, 5, -5, 9, 4\}$

A Q(5)-BEM labelling for Triangular prism $Y(3,2)$ graph: $\{6, 7, -9, 0, -6, 10, -10, 8, 9, 5, -7, -8, 5, -5, 11, -11\}$

A Q(6)-BEM labelling for Triangular prism $Y(3,2)$ graph: $\{6, 7, -9, 0, -6, 10, -10, 8, 9, 5, -7, -8, -12, 12, 11, -11\}$

A Q(7)-BEM labelling for Triangular prism $Y(3,2)$
graph: $\{7,-9,0,-13,10,-10,8,9,13,-7,-8,-12,12,11,-11\}$

A Q(8)-BEM labelling for Triangular prism $Y(3,2)$
graph: $\{14,-9,0,-13,10,-10,8,9,13,-14,-12,12,11,-11\}$

A Q(9)-BEM labelling for Triangular prism $Y(3,2)$
graph: $\{14,-9,0,-13,10,-10,15,9,13,-15,-14,-12,12,11,-11\}$

Theorem 2: Cubical prism $Y_{(4,n)}$ graph is weak Q(a) BEM for $n = 12$

Proof: Let $n = 16$, generalization of cubical prism graph is $2n-4, q = 28$

Cubical prism $Y_{(4,3)}$ graph is Q(a) balanced edge magic for $a = 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14$

Here q is even, By the definition of Q(a) BEM, Fig 3, shows that the Q(1) BEM of Cubical prism $Y_{(4,3)}$ graph

A Q(2)-BEM labelling for Cubical Prism $Y_{(4,3)}$ graph:
 $\{-3,-2,4,-7,15,11,-14,9,-4,-15,12,-12,-8,-9,8,-11,-6,5,7,-10,-13,14,2,10,-5,3,6,13\}$

A Q(3)-BEM labelling for Cubical Prism $Y_{(4,3)}$ graph: $\{-3,-16,4,-7,15,11,-14,9,-4,-15,12,-12,-8,-9,8,-11,-6,5,7,-10,-13,14,16,10,-5,3,6,13\}$

A Q(4)-BEM labelling for Cubical Prism $Y_{(4,3)}$ graph:
 $\{-17,17,4,-7,15,11,-14,9,-4,-15,12,-12,-8,-9,8,-11,-6,5,7,-10,-13,14,10,-5,6,13\}$

A Q(5)-BEM labelling for Cubical Prism $Y_{(4,3)}$ graph:
 $\{18,-17,17,-7,15,11,-14,9,-18,-15,12,-12,-8,-9,8,-11,-6,5,7,-10,-13,14,10,-5,6,13\}$

A Q(6)-BEM labelling for Cubical Prism $Y_{(4,3)}$ graph:
 $\{11,18,-17,17,-7,15,19,-14,9,-18,-15,12,-12,-8,-9,8,-11,-6,7,-10,-13,14,10,-19,6,13\}$

A Q(7)-BEM labelling for Cubical Prism $Y_{(4,3)}$ graph:
 $\{-20,11,18,-17,17,-7,15,19,-14,9,-18,-15,12,-12,-8,-9,8,-11,-20,7,-10,-13,14,10,-19,13\}$

A Q(8)-BEM labelling for Cubical Prism $Y_{(4,3)}$ graph:
 $\{17,-20,11,18,-17,21,15,19,-14,9,-18,-15,12,-12,-8,-9,8,-11,-20,-21,-10,-13,14,10,-19,13\}$

A Q(9)-BEM labelling for Cubical Prism $Y_{(4,3)}$ graph:
 $\{19,-20,11,18,-17,17,15,-14,9,-18,-15,12,-12,-22,-9,22,-11,-20,-10,-13,14,10,-19,13\}$

A Q(10)-BEM labelling for Cubical Prism $Y_{(4,3)}$ graph:
 $\{23,19,-20,11,18,-17,17,15,-14,-23,-18,-15,12,-12,-22,22,-11,-20,-10,-13,14,10,-19,13\}$

A Q(11)-BEM labelling for Cubical Prism $Y_{(4,3)}$ graph:
 $\{19,23,24,-20,11,18,-17,17,15,-14,-23,-18,-15,12,-12,-22,22,-11,-20,-13,14,-24,13\}$

A Q(12)-BEM labelling for Cubical Prism $Y_{(4,3)}$ graph:
 $\{25,19,23,24,-20,18,-17,17,15,-14,-23,-18,-15,12,-12,-22,22,-25,-20,-13,14,-24,13\}$

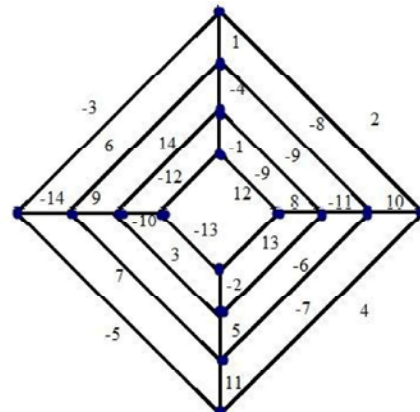


Fig. 3:

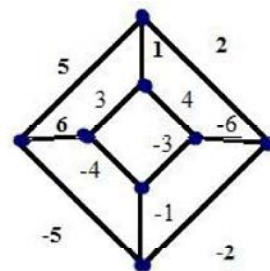


Fig. 4:

A Q(13)-BEM labelling for Cubical Prism $Y_{(4,3)}$ graph:
 $\{18,25,19,23,24,-20,26,-17,17,15,-14,-23,-18,-15,26,-22,-26,22,-25,-20,-13,14,-24,13\}$

A Q(14)-BEM labelling for Cubical Prism $Y_{(4,3)}$ graph:
 $\{27,18,25,19,23,24,-20,26,-17,17,15,-14,-23,-18,-15,26,-22,-26,22,-25,-20,-27,14,-24\}$

A Q(15)-BEM labelling for Cubical Prism $Y_{(4,3)}$ graph:
 $\{26,27,18,25,19,23,24,-20,28,-17,17,15,-23,-18,-15,26,-22,-26,22,-25,-20,-27,-28,-24\}$

A Q(16)-BEM labelling for Cubical Prism $Y_{(4,3)}$ graph:
 $\{29,26,27,18,25,19,23,24,-20,28,-17,17,-23,-18,-29,26,-22,-26,22,-25,-20,-27,-28,-24\}$

Example: Cubical prism $Y_{(4,1)}$ graph is weak Q(a) BEM when $a = 1, 2, 3, 4, 5, 6, 7, 8$.

Generalization of cubical prism $Y_{(4,n)}$ is $2n-4$. Let $n = 8$, $q = 12$

Cubical prism $Y_{(4,1)}$ is Q(a) balanced edge magic for $a = 1, 2, 3, 4, 5, 6, 7, 8$

By definition of Q(a) BEM, For $a = 1$ and q is even, Fig 4, shows that Cubical prism $Y_{(4,1)}$ -Q(1)-balanced edge magic

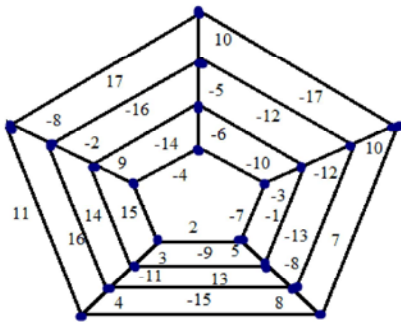


Fig. 5:

A Q(2)-BEM labelling for Cubical Prism $Y(4,1)$ graph:
 $\{5,2,7,4,-6,-3,3,-4,-7,-5,-2,6\}$

A Q(3)-BEM labelling for Cubical Prism $Y(4,1)$ graph:
 $\{-8,7,5,4,-6,-3,3,-4,-7,-5,8,6\}$

A Q(4)-BEM labelling for Cubical Prism $Y(4,1)$ graph:
 $\{7,5,9,4,-6,-4,-9,-7,-5,-8,6,8\}$

A Q(5)-BEM labelling for Cubical Prism $Y(4,1)$ graph:
 $\{10,7,5,9,-6,-10,-9,-7,-5,-8,6,8\}$

A Q(6)-BEM labelling for Cubical Prism $Y(4,1)$ graph:
 $\{-11,10,8,-10,9,-6,7,-9,-7,-5,-8,6,11\}$

A Q(7)-BEM labelling for Cubical Prism $Y(4,1)$ graph:
 $\{8,-11,10,12,-10,9,-12,7,-9,-7,-5,-8,11\}$

A Q(8)-BEM labelling for Cubical Prism $Y(4,1)$ graph:
 $\{-13,9,8,-11,10,12,-10,13,-12,-9,-8,11\}$.

Theorem 3: Pentagonal prism $Y_{(5,n)}$ graph is weak Q(a)-BEM when the vertices $n = 15$.

Proof:

Let $n = 20$, $q = 2n-5$,i.e., $q = 35$

Pentagonal prism $Y_{(5,3)}$ graph satisfies the Q(a)-balanced edge magic for $a = 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20$

By the definition of Q(a) BEM, For $a = 1$,

Here q is odd, Fig 5, shows that Q(1)-balanced edge magic for $Y_{(5,3)}$

A Q(2)-BEM labelling for Pentagonal prism $Y_{(5,3)}$ graph:
 $\{0,-17,17,7,-15,11,18,-8,-16,10,13,8,13,4,16,-14,-5,12,-12,-18,-8,-9,-11,14,-2,9,-6,-3,5,3,-4,2,-10,-7,15\}$

A Q(3)-BEM labelling for Pentagonal prism $Y_{(5,3)}$ graph:
 $\{0,-17,17,7,-15,11,18,-8,-16,10,13,8,13,4,16,-14,-5,12,12,-18,-8,-9,-11,14,-19,9,-6,-3,5,3,-4,19,-10,-7,15\}$

A Q(4)-BEM labelling for Pentagonal prism $Y_{(5,3)}$ graph:
 $\{20,-20,0,-17,17,7,-15,11,18,-8,-16,10,13,8,13,16,-14,-5,12,-12,-18,-8,-9,-11,14,-19,9,-6,5,4,-4,19,-10,-7,15\}$

A Q(5)-BEM labelling for Pentagonal prism $Y_{(5,3)}$ graph:
 $\{21,-21,20,-20,0,-17,17,7,-15,11,18,-8,-16,10,13,8,13,16,-14,-5,12,-12,-18,-8,-9,-11,14,-19,9,-6,5,19,-10,-7,15\}$

A Q(6)-BEM labelling for Pentagonal prism $Y_{(5,3)}$ graph:
 $\{21,-21,20,-20,0,-17,17,7,-15,11,18,-8,-16,10,13,8,13,16,-14,-22,12,-12,-18,-8,-9,-11,14,-19,9,-6,22,19,-10,-7,15\}$

A Q(7)-BEM labelling for Pentagonal prism $Y_{(5,3)}$ graph:
 $\{21,-21,20,-20,0,-17,17,22,-15,11,18,-8,-16,10,13,8,7,13,16,-14,-22,12,-12,-18,-8,-9,-11,14,-19,9,-7,22,19,-10,-22,15,23,-23\}$

A Q(8)-BEM labelling for Pentagonal prism $Y_{(5,3)}$ graph:
 $\{23,-23,21,-21,20,-20,0,-17,17,22,-15,11,18,-8,-16,10,13,8,13,16,-14,-22,12,-12,-18,-8,-9,-11,14,-19,9,22,19,-10,-22,15,24,-24\}$

A Q(9)-BEM labelling for Pentagonal prism $Y_{(5,3)}$ graph:
 $\{24,-24,23,-23,21,-21,20,-20,0,-17,17,22,-15,11,18,-16,10,13,25,13,16,-14,-22,12,-12,-18,-25,-9,-11,14,-19,9,-6,22,19,-10,-22,15\}$

A Q(10)-BEM labelling for Pentagonal prism $Y_{(5,3)}$ graph:
 $\{26,-26,23,-23,21,-21,20,-20,0,-17,17,22,15,11,18,-16,10,13,24,13,16,-14,-22,12,-12,-18,-24,-11,14,-19,22,19,-10,22,15,24,-24,25,-25\}$

A Q(11)-BEM labelling for Pentagonal prism $Y_{(5,3)}$ graph:
 $\{26,-26,23,-23,21,-21,20,-20,0,-17,17,22,-15,11,18,-16,27,13,24,13,16,-14,-22,12,-12,-18,-24,-11,14,-19,22,19,-27,22,15,24,-24,25,-25\}$

A Q(12)-BEM labelling for Pentagonal prism $Y_{(5,3)}$ graph:
 $\{26,-26,23,-23,21,-21,20,-20,0,-17,17,22,-15,28,18,-16,27,13,24,13,16,-14,-22,12,-12,-18,-24,-28,14,-19,22,19,-27,-22,15,24,-24,25,-25\}$

A Q(13)-BEM labelling for Pentagonal prism $Y_{(5,3)}$ graph:
 $\{26,-26,23,-23,21,-21,20,-20,0,-17,17,22,-15,28,18,-16,27,13,24,13,16,-14,-22,28,-28,-18,-24,-28,14,-19,22,19,-27,22,15,24,-24,25,-25\}$

A Q(14)-BEM labelling for Pentagonal prism $Y_{(5,3)}$ graph:
 $\{26,-26,23,-23,21,-21,20,-20,0,-17,17,22,-15,28,18,-16,27,29,24,-29,16,-14,-22,28,-28,-18,-24,-28,14,-19,22,19,27,-22,15,24,-24,25,-25\}$

A Q(15)-BEM labelling for Pentagonal prism $Y_{(5,3)}$ graph:
 $\{26,-26,23,-23,21,-21,20,-20,0,-17,17,22,-15,28,18,-6,27,29,24,-29,16,-30,-22,28,-28,-18,-24,-28,30,-9,22,19,-27,-22,15,24,-24,25,-25\}$

A Q(16)-BEM labelling for Pentagonal prism $Y_{(5,3)}$ graph:
 $\{26,-26,23,-23,21,-21,20,-20,0,-17,17,22,-31,28,18,-16,27,29,24,-29,16,-30,-22,28,-28,-18,-24,-28,30,-19,22,19,-27,-22,31,24,-24,25,-25\}$.

Example: Pentagonal prism $Y_{(5,1)}$ graphs is weak Q(a)-BEM for $a = 1,2,3,4,5,6,7,8,9,10$

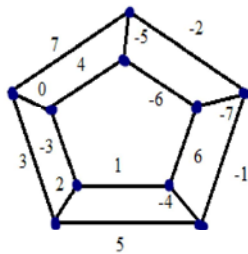


Fig. 6:

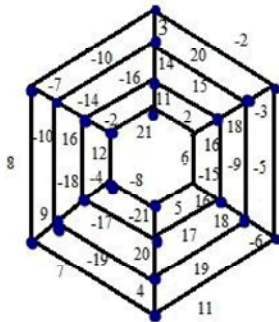


Fig. 7:

Given vertices $n = 10$, edges $q = 15$ ($q = 2n-5$)

Pentagonal prism $Y_{(5,1)}$ graph satisfies the $Q(a)$ -balanced edge magic for $a = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$

By the definition of $Q(a)$ BEM, For $a = 1$,

Here q is odd, Fig 6, shows that the $Q(1)$ -balanced edge magic graph of the pentagonal prism graph.

A $Q(2)$ -BEM labelling for Pentagonal prism $Y_{(5,1)}$ graph: $\{7, -2, -5, -7, 8, -8, 5, 6, -6, -4, 2, 3, 4, 0, -3\}$

A $Q(3)$ -BEM labelling for Pentagonal prism $Y_{(5,1)}$ graph: $\{9, -7, -5, -9, 7, 8, -8, 5, 6, -6, -4, 3, 4, 0, -3\}$

A $Q(4)$ -BEM labelling for Pentagonal prism $Y_{(5,1)}$ graph: $\{8, 10, 9, -7, -5, -9, 7, -10, -8, 5, 6, -6, 4, -4, 0\}$

A $Q(5)$ -BEM labelling for Pentagonal prism $Y_{(5,1)}$ graph: $\{11, 8, 10, 9, -7, -5, -9, 7, -10, -8, 5, 6, -6, -11, 0\}$

A $Q(6)$ -BEM labelling for Pentagonal prism $Y_{(5,1)}$ graph: $\{12, 11, 8, 10, 9, -7, -9, 7, -10, -8, 6, -6, -11, 0\}$

A $Q(7)$ -BEM labelling for Pentagonal prism $Y_{(5,1)}$ graph: $\{0, 12, 11, 8, 10, 9, -7, -9, 7, -10, -8, 13, -11, -13\}$

A $Q(8)$ -BEM labelling for Pentagonal prism $Y_{(5,1)}$ graph: $\{0, 12, 11, 8, 10, 9, -9, -10, -8, 13, -11, -13, 14, -14\}$

A $Q(9)$ -BEM labelling for Pentagonal prism $Y_{(5,1)}$ graph: $\{14, -14, 0, 12, 11, 10, 9, -9, -10, 13, -11, -13, 15, -15\}$

A $Q(10)$ -BEM labelling for Pentagonal prism $Y_{(5,1)}$ graph: $\{16, -16, 14, -14, 0, 12, 11, 10, -10, 13, -11, -13, 15, -15\}$

Theorem 4: Hexagonal prism $Y_{(6,n)}$ graph is weak $Q(a)$ -BEM for $n = 24$.

Proof:

Let vertices $n = 24$ and edges $q = 2n-6$ order, therefore $q = 42$

For Hexagonal prism $Y(6,3)$ graph satisfies the $Q(a)$ -balance edge magic for $a = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24$

By the definition of $Q(a)$ BEM, for $a = 1$ and q is even, Fig 7, shows that $Q(1)$ -balanced edge magic for $Y_{(6,3)}$.

A $Q(2)$ -BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: $\{-22, -2, -5, 11, 7, 8, -7, -20, 3, 20, -3, -9, -6, 19, 4, -19, 9, -10, -14, -16, 14, 15, -15, -11, 17, 20, -17, -18, 16, 18, -9, 22, -13, 13, -21, -4, 21, 2, 6, 5, -8, 12\}$

A $Q(3)$ -BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: $\{23, -23, -22, -5, 11, 7, 8, -7, -20, 3, 20, -3, -9, -6, 19, 4, -19, -10, -14, -16, 14, 15, -15, -11, 17, 20, -17, -18, 18, 16, -9, 22, -13, 13, -21, -4, 21, 6, 5, -8, 12, -12\}$

A $Q(4)$ -BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: $\{23, -23, -22, -5, 11, 7, 8, -7, -20, 24, 20, -24, -9, -6, 19, 4, -19, -10, -14, -16, 14, 15, -15, -11, 17, 20, -17, -18, 16, -9, 22, -13, 13, -21, -4, 21, 6, 5, -8, 12, -12, 18\}$

A $Q(5)$ -BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: $\{25, -25, 23, -23, -22, -5, 11, 7, 8, -7, -20, 24, 20, -24, -6, 19, -19, 9, -10, 14, -16, 14, 15, -15, -11, 17, 20, -17, -18, 16, -9, 22, -13, 13, -21, 21, 6, 5, -8, 12, -12, 18\}$

A $Q(6)$ -BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: $\{25, -25, 23, -23, -22, -26, 11, 7, 8, -7, -20, 24, 20, -24, 9, -6, 19, -19, -10, -14, -16, 14, 15, -15, -11, 17, 20, -17, -18, 16, -9, 22, -13, 13, -21, 21, 6, 26, -8, 12, -12, 18\}$

A $Q(7)$ -BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: $\{25, -25, 23, -23, -22, -26, 11, 7, 8, -7, -20, 24, 20, -24, 9, -27, 19, -19, -10, -14, -16, 14, 15, -15, -11, 17, 20, -17, -18, 16, -9, 22, -13, 13, -21, 21, 27, 26, -8, 12, -12, 18\}$

A $Q(8)$ -BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: $\{25, -25, 23, -23, -22, -26, 11, 28, 8, -28, -20, 24, 20, -24, 9, -27, 19, -19, -10, -14, -16, 14, 15, -15, -11, 17, 20, -17, -18, 16, -9, 22, -13, 13, -21, 21, 27, 26, -8, 12, -12, 18\}$

A $Q(9)$ -BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: $\{25, -25, 23, -23, -22, -26, 11, 28, 29, -28, -20, 24, 20, -24, 9, -27, 19, -19, -10, -14, -16, 14, 15, -15, -11, 17, 20, -17, -18, 16, -9, 22, -13, 13, -21, 21, 27, 26, -29, -12, 12, 18\}$

A $Q(10)$ -BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: $\{25, -25, 23, -23, -22, -26, 11, 28, 29, -28, -20, 24, 20, -24, -30, 27, 19, -19, 10, -10, -14, -16, 14, 15, -15, -11, 17, 20, -17, -18, 16, 22, -13, 13, -21, 21, 27, 26, -30, 12, -12, 18\}$

A $Q(11)$ -BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: $\{25, -25, 23, -23, -22, -26, 11, 28, 29, -28, -20, 24, 20, -24, -30, -27, 19, -19, 31, -31, -14, -16, 14, 15, -15, -11, 17, 20, -17, -18, 16, 22, -13, 13, -21, 21, 27, 26, -30, 12, -12, 18\}$

A $Q(12)$ -BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: $\{25, -25, 23, -23, -22, -26, 32, 28, 29, -28, -20, 24, 20, -24, -$

30,-27, 19,-19,31,-31,-14,-16,14,15,-15,-32,17,20,-17,-18,16,22,-13, 13, -21, 21,27,26,-30,12,18}

A Q(13)-BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: {25,-25,23,-23,-22,-26,32,28,29,-28, -20, 24, 20, -24, -30,-27, 19,-19,31,-31,-14,-16,14,15,-15,-32,17,20,-17,-18,16,22,-13, 13, -21,21,27,26,-30,33,18}

A Q(14)-BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: {25,-25,23,-23,-22,-26,32,28,29,-28, -20, 24, 20, -24, -30, -27,19,-19,31,-31,-14,-16,14,15,-15,-32,17,20,-17,-18,16,22,-34,34,-21,21,27,26,-30,33,18}

A Q(15)-BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: {25,-25,23,-23,-22,-26,32,28,29,-28, -20, 24, 20, -24, -30, -27, 19,-19,31,-31,-35,-16,35,15,-15,-32,17,20,-17,-18,16,22,-34, 34, -21,21,27,26,-30,18,33}

A Q(16)-BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: {25,-25,23,-23,-22,-26,32,28,29,-28, -20, 24, 20, -24, -30,-27,19,-19,31,-31,-35,-16,35,36,-36,-32,17,20,-17,-18,16,22,-34,34,-21,21,27,26,-30,18,33}

A Q(17)-BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: {25,-25,23,-23,-22,-26,32,28,29,-28, -20, 24, 20, -24, -30,-27, 18,19,-19,31,-31,-35,-37,35,36,-36,-32,17,20,-17,-18,37,22,-34, 34, -21,21,27,26,-30,33}

A Q(18)-BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: {25,-25,23,-23,-22,-26,32,28,29,-28, -20, 24, 20, -24, -30,-27, 19,-19,31,-31,-35,-37,35,36,-36,-32,37,20,-37,-18,37,22,-34, 34, -21,21,27,26,-30,18,33}

A Q(19)-BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: {38,25,-25,23,-23,-22,-26,32, 28,29,-28,-20, 24, 20, -24, -30, -27,19,-19,31,-31,-35,-37,35,36,-36,-32,37,20,-37,-38,37,22,-34, 34, -21,21,27,26,-30,33}

A Q(20)-BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: {39,-39,38,25,-25,23,-23,-22,-26, 32, 28, 29, -28,-20, 24, 20,-24,-30,-27,31,-31,-35,-37,35,36,-36,-32,37,20,-37,-38,37,22,-34,34, -21,21,27,26,-30,33}

A Q(21)-BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: {39,-39,38,25,-25,23,-23,-22,-26, 32, 28, 29, -28,-40, 24,40,-24,-30,-27,31,-31,-35,-37,35,36,-36,-32,37,20,-37,-38,37,22,-34,34, -21,21,27,26,-30,33}

A Q(22)-BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: {39,-39,38,25,-25,23,-23,-22,-26, 32, 28, 29, -28,-40, 24,40,-24,-30,-27,31,-31,-35,-37,35,36,-36,-32,37,20,-37,-38,37,22,-34,34, -41,41,27,26,-30,33}

A Q(23)-BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: {43,-43,39,-39,38,25,-25,23,-23,-26, 32, 28, 29,-28, -40,24,40,-24,-30,-27,31,-31,-35,-37,35,36,-36,-32,37,20,-37,-38,37,-34,34, -41,41,27,26,-30,33}

A Q(24)-BEM labelling for Hexagonal prism $Y_{(6,3)}$ graph: {43,-43,39,-39,38,25,-25,44,-44,-26, 32, 28, 29,-28, -40,24,40,-24,-30,-27,31,-31,-35,-37,35,36,-36,-32,37,20,-37,-38,37,-34,34, -41,41,27,26,-30,33}

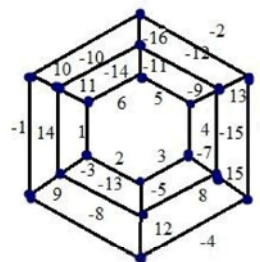


Fig. 8:

Example: Hexagonal prism $Y_{(6,2)}$ graph is weak Q(a)-BEM when $n = 18$

Let vertices $n = 18$,

Generalization of hexagonal prism graph edges is $2n - 6, q = 30$

Hexagonal prism $Y_{(6,2)}$ graph satisfies the Q(a)-balanced edge magic for $a = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18$

By the definition of Q(a) BEM, For $a = 1$ and q is even, fig 8, shows that Q(1)-balanced edge magic for $Y_{(6,2)}$.

A Q(2)-BEM labelling for Hexagonal prism $Y_{(6,2)}$ graph: {-2,7,13,-15,-4,-8,-16,-10,-6,-12,-15,-15,8, 12,-13,9, 14,10,-14,-11,5,-9,4,-7,3,-5,2,-3,16,11,6}

A Q(3)-BEM labelling for Hexagonal prism $Y_{(6,2)}$ graph: {3,-3,4,-4,5,-5,6,-6,7,-7,8,-8,9, 9,10,-10,11,-11, 12, -12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(4)-BEM labelling for Hexagonal prism $Y_{(6,2)}$ graph: {18,-18,4,-4,5,-5,6,-6,7,-7,8,-8,9,9,10,-10,11,-11,12,-12, 13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(5)-BEM labelling for Hexagonal prism $Y_{(6,2)}$ graph: {19,-19,18,-18,5,-5,6,-6,7,-7,8,-8,9,9,10,-10,11,-11,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(6)-BEM labelling for Hexagonal prism $Y_{(6,2)}$ graph: {19,-19,18,-18,20,-20,6,-6,7,-7,8,-8,9,9,10,-10,11,-11,12,-12, 13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(7)-BEM labelling for Hexagonal prism $Y_{(6,2)}$ graph: {21,-21,19,-19,18,-18,20,-20,7,-7, 8,-8,9,9,10,-10,11,-11, 12, -12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(8)-BEM labelling for Hexagonal prism $Y_{(6,2)}$ graph: {21,-21,19,-19,18,-18,20,-20,22,-22,8,-8,9,9,10,-10,11,-11, 12, -12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(9)-BEM labelling for Hexagonal prism $Y_{(6,2)}$ graph: {21,-21,19,-19,18,-18,20,-20,22,-22,23,-23,9,9,10,-10,11,-11, 12,-12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(10)-BEM labelling for Hexagonal prism $Y_{(6,2)}$ graph: {24,-24,21,-21,19,-19,18,-18,20,-20, 22,-22, 23,-23,10,-10,11, -11,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(11)-BEM labelling for Hexagonal prism $Y_{(6,2)}$
 graph: {24,-24,21,-21,19,-19,18,-18,20,-20,22,-22,23,-23,25,-25,11,-11,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(12)-BEM labelling for Hexagonal prism $Y_{(6,2)}$
 graph: {24,-24,21,-21,19,-19,18,-18,20,-20,22,-22,23,-23,25,-25,26,-26,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(13)-BEM labelling for Hexagonal prism $Y_{(6,2)}$
 graph: {24,-24,21,-21,19,-19,18,-18,20,-20,22,-22,23,-23,25,-25,26,-26,27,-27,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(14)-BEM labelling for Hexagonal prism $Y_{(6,2)}$
 graph: {24,-24,21,-21,19,-19,18,-18,20,-20,22,-22,23,-23,25,-25,26,-26,27,-27,28,-28,14,-14,15,-15,16,-16,17,-17}

A Q(15)-BEM labelling for Hexagonal prism $Y_{(6,2)}$
 graph: {29,-29,24,-24,21,-21,19,-19,18,-18,20,-20,22,-22,23,-23,25,-25,26,-26,27,-27,28,-28,14,15,-15,16,-16,17,-17}

A Q(16)-BEM labelling for Hexagonal prism $Y_{(6,2)}$
 graph: {30,-30,29,-29,24,-24,21,-21,19,-19,18,-18,20,-20,22,-22,23,-23,25,-25,26,-26,27,-27,28,-28,16,-16,17,-17}

A Q(17)-BEM labelling for Hexagonal prism $Y_{(6,2)}$
 graph: {31,-31,30,-30,29,-29,24,-24,21,-21,19,-19,18,-18,20,-20,22,-22,23,-23,25,-25,26,-26,27,-27,28,-28,17,-17}

A Q(18)-BEM labelling for Hexagonal prism $Y_{(6,2)}$
 graph: {31,-31,30,-30,29,-29,24,-24,21,-21,19,-19,18,-18,20,-20,22,-22,23,-23,25,-25,26,-26,27,-27,28,-28,32,-32}

Theorem 5: Prove that Heptagonal prism $Y_{(7,n)}$ graph is weak Q(a)-BEM for the vertices $n = 21$.

Proof:

Let $n = 21$,

Generalization of Heptagonal prism graph edges is $2n-7$. Therefore $q = 35$

Hexagonal prism $Y_{(7,2)}$ graph satisfies the Q(a)-balanced edge magic for $a = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21$

By the definition of Q(a) BEM, for $a = 1$ and q is odd, Fig 9, shows that Q(1)-balanced edge magic for $Y_{(7,2)}$

A Q(2)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {0,2,-2,3,-3,4,-4,-5,5,-6,6,-7,7,8,-8,9,-9,10,-10,11,-11,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(3)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {0,18,-18,3,-3,4,-4,-5,5,-6,6,-7,7,8,-8,9,-9,10,-10,11,-11,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(4)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {0,18,-18,19,-19,4,-4,-5,5,-6,6,-7,7,8,-8,9,-9,10,-10,11,-11,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(5)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {20,-20,0,18,-18,19,-19,-5,5,-6,6,-7,7,8,-8,9,-9,10,-10,11,-11,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17}

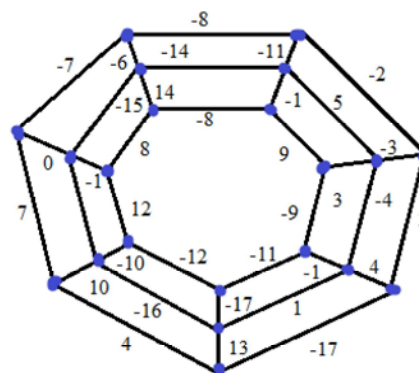


Fig. 9:

A Q(6)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {21,-21,20,-20,0,18,-18,19,-19,-6,6,-7,7,8,-8,9,-9,10,-10,11,-11,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(7)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {22,-22,21,-21,20,-20,0,18,-18,19,-19,-7,7,8,-8,9,-9,10,-10,11,-11,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(8)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {22,-22,21,-21,20,-20,0,18,-18,19,-19,-23,-23,8,-8,9,-9,10,-10,11,-11,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(9)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {24,-24,22,-22,21,-21,20,-20,0,18,-18,19,-19,23,-23,9,-9,10,-10,11,-11,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(10)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {25,24,-24,22,-22,21,-21,20,-20,0,18,-18,19,-19,23,-23,10,-10,11,-11,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17,25}

A Q(11)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {26,-26,0,25,24,-24,22,-22,21,-21,20,-20,18,-18,19,-19,23,-23,11,-11,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17,25}

A Q(12)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {27,-27,26,-26,0,25,24,-24,22,-22,21,-21,20,-20,18,-18,19,-19,23,23,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17,25}

A Q(13)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {27,-27,26,-26,0,25,24,-24,22,-22,21,-21,20,-20,18,-18,19,-19,23,23,29,-29,13,-13,14,-14,15,-15,16,-16,17,-17,25}

A Q(14)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {27,-27,26,-26,0,25,24,-24,22,-22,21,-21,20,-20,18,-18,19,-19,23,23,29,-29,30,-30,14,-14,15,-15,16,-16,17,-17,25}

A Q(15)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {27,-27,26,-26,0,25,24,-24,22,-22,21,-21,20,-20,18,-18,19,-19,23,23,29,-29,30,-30,31,-31,15,-15,16,-16,17,-17,25}

A Q(16)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {27,-27,26,-26,0,25,24,-24,22,-22,21,-21,20,-20,18,-18,19,-19,23,23,29,-29,30,-30,31,-31,32,-32,16,-16,17,-17,25}

A Q(17)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {27,-27,26,-26,0,25,24,-24,22,-22,21,-21,20,-20,18,-18,19,-19,23,-23,29,-29,30,-30,31,-31,32,-32,33,-33,17,-17,25}

A Q(18)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {27,-27,26,-26,0,25,24,-24,22,-22,21,-21,20,-20,18,-18,19,-19,23,-23,29,-29,30,-30,31,-31,32,-32,33,-33,34,-34,-25}

A Q(19)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {27,-27,26,-26,0,25,24,-24,22,-22,21,-21,20,-20,35,-35,19,-19,23,-23,29,-29,30,-30,31,-31,32,-32,33,-33,34,-34,-25}

A Q(20)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {27,-27,26,-26,0,25,24,-24,22,-22,21,-21,20,-20,35,-35,35,-36,36,-23,-23,29,-29,30,-30,31,-31,32,-32,33,-33,34,-34,-25}

A Q(21)-BEM labelling for Heptagonal prism $Y_{(7,2)}$
 graph: {27,-27,26,-26,0,25,24,-24,22,-22,21,-21,37,-37,35,-35,35,-36,36,-23,-23,29,-29,30,-30,31,-31,32,-32,33,-33,34,-34,-25}

Theorem 6: Prove that the Antiprism graph A_n is weak Q(a)-BEM when $n > 2$

Proof :

Given $n = 6, p = 12, q = 24$

A_6 is Q(a)-BEM for $a = 1, 2, 3, 4, 5, 6, 7, 8, 0, 10, 11, 12$

By the definition of Q(a) BEM.

Here q is even, Fig 10, shows that Q(1)-balanced edge magic for A_6 .

A Q(2)-BEM labelling for A_6 : {-2,2,3,-3,4,-4,5,-5,6,-6,7,-7,8,-8,9,-9,10,-10,11,-11,12,-12,13,-13}

A Q(3)-BEM labelling for A_6 : {3,-3,4,-4,5,-5,6,-6,7,-7,8,-8,9,-9,10,-10,11,-11,12,-12,13,-13,14,-14}

A Q(4)-BEM labelling for A_6 : {4,-4,5,-5,6,-6,7,-7,8,-8,9,-9,10,-10,11,-11,12,-12,13,-13,14,-14,-15,-15}

A Q(5)-BEM labelling for A_6 : {5,-5,6,-6,7,-7,8,-8,9,-9,10,-10,11,-11,12,-12,13,-13,14,-14,15,-15,16,-16}

AQ(6)-BEMlabelling for A_6 : {6,-6,7,-7,8,-8,9,-9,10,-10,11,12,-12,13,-13,14,-14,15,-15,16,-16,17,-17}

A Q(7)-BEM labelling for A_6 : {7,-7,8,-8,9,-9,10,-10,11,12,-12,13,-13,14,-14,15,-15,16,-16,17,18,-18}

A Q(8)-BEM labelling for A_6 : {-16,16,17,-17,18,-18,6,8,-8,9,-9,10,-10,11,12,-12,13,-13,14,14,15,-15,19,-19}

A Q(9)-BEM labelling for A_6 : {-16,16,17,-17,18,-18,6,19,-19,9,-9,10,-10,11,12,-12,13,-13,14,14,15,-15,20,-20}

A Q(10)-BEM labelling for A_6 : {-16,16,17,-17,18,-18,6,19,-19,20,-9,10,-10,11,12,-12,13,-13,-14,14,15,-15,21,-21}

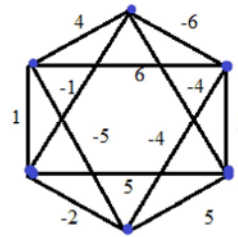


Fig. 10 :

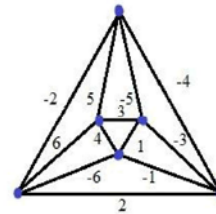


Fig. 11:

A Q(11)-BEM labelling for A_6 : {-16,16,17,-17,18,-18,6,19,-19,11,12,-12,13,-13,-14,14,15,-15,20,-20,21,-21,22,22}

A Q(12)-BEM labelling for A_6 : {-16,16,17,-17,18,-18,6,19,-19,11,12,-12,13,-13,-14,-23,23,14,15,-15,20,-20,21,21,22,-22}

Example: Cube Antiprism graph A_3 is weak Q(a) BEM

Proof:

Given $n = 3, p = 6, q = 12$

A_3 is Q(a) BEM for $a = 1, 2, 3, 4, 5, 6$

By the definition of Q(a), for $a = 1$ and q is even, Fig 11, shows that Q(1)-balanced edge magic for A_3

A Q(2)-BEM labelling for A_3 : {-2,2,3,-3,4,-4,5,-5,6,-6,7,-7}

A Q(3)-BEM labelling for A_3 : {3,-3,4,-4,5,-5,6,-6,7,-7,8,-8}

A Q(4)-BEM labelling for A_3 : {4,-4,5,-5,6,-6,7,-7,8,-8,9,-9}

A Q(5)-BEM labelling for A_3 : {5,-5,6,-6,7,-7,8,-8,9,-9,10,-10}

A Q(6)-BEM labelling for A_3 : {6,-6,7,-7,8,-8,9,-9,10,-10,11,-11}

Similarly square Antiprism graph A_4 , Hexa Antiprism A_5 are also Q(a)-BEM

CONCLUSION

In this above work defined weak Q(a)-Balance Edge Magic for the following graphs such as generalised Triangular prism $Y(3,n)$ graph for $n = 12$, generalised

cubical prism $Y(4,n)$ graph for $n = 12$, generalised Pentagonal prism $Y(5,n)$ graph is $n = 15$, generalised Hexagonal prism $Y(6,n)$ for $n > 23$, generalised Heptagonal prism $Y(7,n)$ for $n > 20$, Cube Antiprism graph, Square Antiprism graph, Antiprism graph. Author is working in other special types of graphs.

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REFERENCES

1. Stewart, B.M., 1966. Magic graphs, Canadian J. Math., 18: 1031-1059.
2. Stewart, B.M., 1967. Supermagic complete graphs, Canadian J. Math., 19: 427-438.
3. Jiri Sedláček, 1976. On magic graphs, Mathematica Slovaca, 26(4): 329-335.
4. Gallian, J.A., 2007. A dynamic survey of graph labeling, the Electronic J of Combin, # DS6, pp: 1-180.
5. Alison M. Marr and W.D. Wallis, 2013. Magic Graphs, 2nd edn., Springer New York Heidelberg Dordrecht London, 2013.
6. Enomoto, H., K. Masuda and T. Nakamigawa, 2000. Induced graph theorem on magic valuations, Ars Combinatoria, 56: 25-32.
7. Hegde, S.M. and S. Shetty, 2003. On magic graphs, Australasian Journal of Combinatorics, 27: 277-284.
8. Kotzig and A. Rosa, 1970. Magic valuations of finite graphs, Canada Math. Bull., 13: 451-461.
9. Kotzig and A. Rosa, 1972. Magic valuations of complete graphs, Publications du Centre de Recherches Mathematiques Universite de Montreal, pp: 175.
10. Ping-Tsai Chung and Sin-Min Lee, 2009. On Computing edge magic graphs and $Q(a)$ Balance Edge Magic Graphs, Congressus Numerantium, 199: 153-165.
11. Seah, E, S.M. Lee and S.K. Tan, 1992. On Edge-Magic Graphs, Congress Num., 86: 179-191.
12. Sin-Min Lee, Eric Seah and S.K. Tan, 1992. On edge-magic graphs, Congressus Numerantium, 86: 179-191.
13. Sin-Min Lee and Thomas Wong and Sheng - Ping Bill Lo, 2007. On the $Q(a)$ Balance Edge-magic Graphs and $Q(a)$ Balance Super Edge-magic Graphs, Congressus Numerantium, 188: 33-57.
14. Vimala, S., 2015. Some New Results On $Q(a)$ -Balance Edge-Magic Graphs, The Research Journal of Science & IT Management(RJSITM), 05(01): 31-38.
15. Vimala, S. and R. Prabavathi, XXXX. $Q(a)$ Balance Edge Magic of Sun family Graphs, International Journal of Engineering and Management Research, Volume-6, Issue-3 of May-June 2016, ISSN (ONLINE): 2250-0758, ISSN (PRINT): 2394-6962, pp: 143-149.