Middle-East Journal of Scientific Research 25 (2): 380-386, 2017
ISSN 1990-9233

DOI: 10.5829/idosi.mejsr.2017.380.386

Regular Split Perfect Domination Number in Fuzzy Graph

[1]S. Revathi, [2]C.V.R. Hari Narayanan and [3]R. Muthuraj

[1]Assistant Professor, Department of Mathematics,
Saranathan College of Engineering, Trichy – 620 012, Tamilnadu, India
[2]Assistant Professor, Department of Mathematics,
Government Arts College, Paramakudi – 637 001, Tamilnadu, India
[3]Assistant Professor, PG & Research Department of Mathematics,
H.H. The Rajah's College, Pudukkottai – 622 001, Tamilnadu, India

Abstract: In this paper, the new kind of parameter split perfect domination number in a fuzzy graph is defined and established the parametric conditions. Another new kind of parameter regular split perfect domination number is defined and established the parametric conditions. The properties of split perfect domination number and regular split domination numbers are discussed.

Key words: Dominating set · Perfect dominating set · Split perfect dominating set · Regular split perfect dominating set · Regular split perfect domination number

INTRODUCTION

The study of dominating set in graphs was begun by Ore and Berge [1]. Kulli V.R. *et al.* [2] introduced the concept of split domination in graphs. In 1975, Rosenfeld introduced the notation of fuzzy graph theoretic concepts such as paths, cycles, connectedness and etc. Mahioub subatah *et al*. [3] investigated the split domination number of fuzzy graphs. Ponnappan C.Y. *et al*. [4] discussed the strong split domination number of fuzzy graphs. Nagoorgani, A. and K. Radha, [5] introduced the concept of regular fuzzy graphs. Ravi Narayanan, S., *et al*. [6] discussed the regular domination in fuzzy graphs. Revathi, S. *et al*. [7] introduced the concept of perfect domination in fuzzy graphs. In this paper we discussed the split perfect domination, regular perfect domination and regular split perfect domination in fuzzy graphs and establish the relationship with parameter which is also investigated.

Preliminaries

Definition: A fuzzy graph G = (σ, μ) is a pair of functions $s : V \to [0,1]$ is a fuzzy subset, $\mu : V \times V \to [0,1]$ is a fuzzy relation on the fuzzy sub set σ, such that $\mu(u,v) \leq s(u) \wedge s(v)$ for all u, v ∈ V.

Definition: A fuzzy graph G = (σ, μ) with the underlying set V, the order of G is defined and denoted by $O(G) = \sum_{u \in V} s(u)$ and size of G is define and denoted by $S(G) = \sum_{u,v \in V} \mu(u,v)$

Definition: Let G = (σ, μ) be a fuzzy graph. The degree of a node is defined as $d(u) = \sum_{v \neq u, v \in V} \mu(u,v)$.

Definition: Let $G = (s, \mu)$ be a fuzzy graph. Let u, v∈V. We say that u dominates v in G if (u, v) is a strong arc or strong edge. A subset D of V is called a dominating set of G if for each vertex v is not in D, there exists u∈D such that u dominates v.

Definition: A dominating set D of a fuzzy graph G is said to be a minimal dominating set, if for each vertex v in D, D-{v} is not a dominating set of G.

Definition: The minimum fuzzy cardinality of a minimal dominating set of G is called the domination number of a fuzzy graph G. It is denoted by $\gamma_f(G)$.

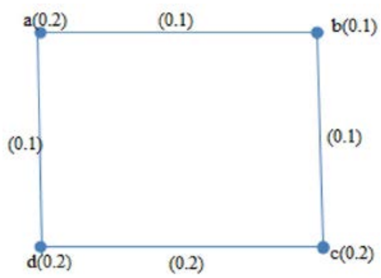
Corresponding Author: S. Revathi, Assistant Professor, Department of Mathematics, Saranathan College of Engineering, Trichy – 620 012, Tamilnadu, India. E- mail: revathi.soundar@gmail.com.

Definition: A dominating set D of a fuzzy graph G = (σ, μ) is a split dominating set if the induced sub graph <V-D> is disconnected.

Definition: A dominating set D of a fuzzy graph G is said to be a minimal split dominating set, if for each vertex v in D, D-{v} is not a split dominating set of G.

Definition: The minimum fuzzy cardinality taken over all minimal split dominating set of G is called the split domination number of a fuzzy graph G. It is denoted by $\gamma_s(G)$.

Example: Consider the fuzzy graph G,



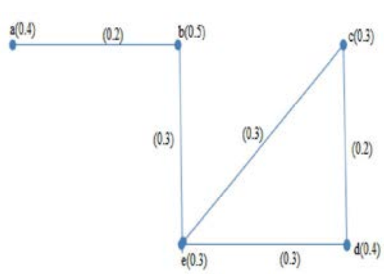
Split dominating set D = {b, d}
Split domination number $\gamma_s(G) = 0.3$
V-D = {a, c} is disconnected

Definition: The dominating set D of a fuzzy graph G = (σ, μ) is said to be regular dominating set if every vertex in D is of same degree.

Definition: A dominating set D of a fuzzy graph G is said to be a minimal regular dominating set, if for each vertex v in D, D-{v} is not a regular dominating set of G.

Definition: The minimum fuzzy cardinality taken over all minimal regular dominating set of G is called the regular domination number of a fuzzy graph G. It is denoted by $\gamma_r(G)$.

Example: Consider the fuzzy graph G



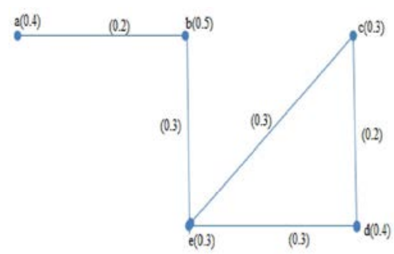
Regular dominating set D = {b, d}
Regular domination number = 0.9
deg(b) = 0.5
deg(d) = 0.5
Here, every vertex in D has same degree.

Definition: The dominating set D of a fuzzy graph G = (σ, μ) is said to be regular split dominating set if every vertex in D is of same degree and <V-D> is disconnected.

Definition: A dominating set D of a fuzzy graph G is said to be a minimal regular split dominating set, if for each vertex v in D, D-{v} is not a regular split dominating set of G.

Definition: The minimum fuzzy cardinality taken over all minimal regular split dominating set of G is called the regular split domination number of a fuzzy graph G. It is denoted by $\gamma_{rs}(G)$.

Example: Consider the fuzzy graph G



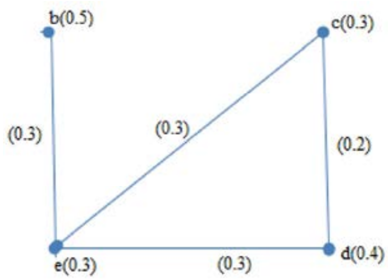
Regular split dominating set D = {b, d}
Regular split domination number $\gamma_{rs}(G) = 0.9$
V-D = {a, c, e} is disconnected.

Definition: Let $G=(s, \mu)$ be a fuzzy graph. Let u, v∈V. The vertex u dominates the vertex v in G if (u, v) is a strong arc or strong edge. A subset P of V is called a perfect dominating set of G if for each vertex v is not in P and v is dominated by exactly one vertex of P.

Definition: A perfect dominating set P of a fuzzy graph G is said to be a minimal perfect dominating set, if for each vertex v in P, P-{v} is not a perfect dominating set of G.

Definition: The minimum fuzzy cardinality of a minimal perfect dominating set of G is called the perfect domination number of a fuzzy graph G. It is denoted by $\gamma_{pf}(G)$.

Example: Consider the fuzzy graph G



Perfect dominating set = {e}
Perfect domination number = 0.3

Properties of Split Perfect Dominating Set in Fuzzy Graph Using Strong ARCS: In this section, we introduce the concept of split perfect dominating set in fuzzy graph using strong arcs and also discuss some parameters.

Definition: A perfect dominating set S_p of a fuzzy graph G is said to be a split perfect dominating set if the induced fuzzy sub graph <V- S_p > is disconnected.

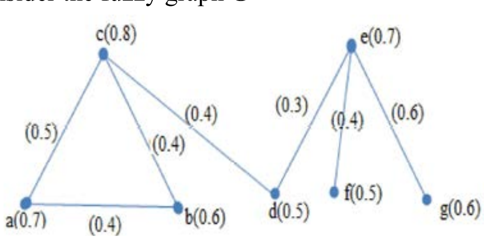
Definition: The minimum fuzzy cardinality of a split perfect dominating set is said to be a split perfect domination number and it is denoted by $?_{sp}(G)$.

Definition: A perfect dominating set S_{sp} of a fuzzy graph G is said to be a strong split perfect dominating set if the induced fuzzy sub graph <V- S_{sp} > is totally disconnected with at least two vertices.

Definition: The minimum fuzzy cardinality of a strong split perfect dominating set is said to be a strong split perfect domination number and it is denoted by $?_{ss_p}(G)$.

Theorem: A perfect dominating set S_{sp} of a fuzzy graph G is a strong split perfect dominating set if and only if there exists two fuzzy vertices u, v $\in$ V- S_{sp} such that every u-v path contains a fuzzy vertex of S_{sp}.

Proof: Let us verify this by considering the following example.
Consider the fuzzy graph G



Split perfect dominating set = {a, e};
<V- S_p > = {b, c, d, f, g} is disconnected
Split perfect domination number $?_{sp}(G)$ = 1.4
Strong split perfect dominating set S_{sp} = {a, e}
<V- S_{sp} > = {b, c, d, f, g} is totally disconnected with at least two vertices
Strong split perfect domination number = $?_{ss_p}(G)$ = 1.4.
Here, we see that there exists b, c$\in$V- S_{sp} such that b-c path contains a vertex 'a'.

Theorem: Every split perfect dominating set of a fuzzy graph is a perfect dominating set.

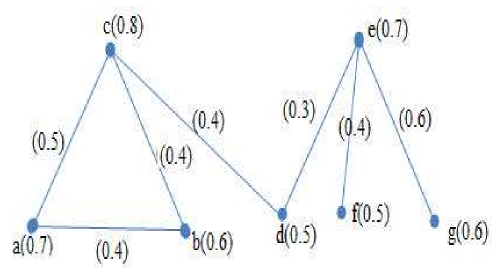
Proof: By using the definition of split perfect dominating set, if the induced fuzzy sub graph <V- S_p > is disconnected and every vertex v not in S_p, v is dominated by exactly one vertex in S_p which is a perfect dominating set of a fuzzy graph.

Theorem: Every strong split perfect dominating set of a fuzzy graph is a split perfect dominating set.

Proof: By using the strong split perfect dominating set and split perfect dominating set of a fuzzy graph, we have every strong split perfect dominating set of a fuzzy graph as a split perfect dominating set.

Theorem: Let G be a fuzzy graph, then $?_{ss_p}(G) \geq ?_p(G)$ where $?_{ss_p}(G)$ is the strong split perfect domination number and $?_p(G)$ is the perfect domination number of a fuzzy graph.

Proof: Let us verify this by considering the following example.
Consider the fuzzy graph G



Perfect dominating set = {b, e};
<V- S_p > = {a, c, d, f, g}
Perfect domination number = 1.3
Strong split perfect dominating set S_{sp} = {a, e}
<V- S_{sp} > = {b, c, d, f, g} is totally disconnected with at least two vertices
Strong split perfect domination number = $?_{ss_p}(G)$ = 1.4.
Here, we see that $?_{ss_p}(G) \geq ?_p(G)$

Properties of Regular Perfect Dominating Set in Fuzzy Graph Using Strong Arcs and Strong Fuzzy Graph: In this section, we introduce the concept of regular perfect dominating set of a fuzzy graph using strong arcs and discuss its properties.

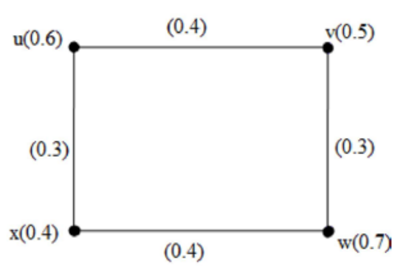
Definition: Let G be a fuzzy graph. A perfect dominating set P_r is said to be a regular perfect dominating set if all the vertices in P_r has the same degree of a fuzzy graph G.

Definition: A regular perfect dominating set P_r of a fuzzy graph G is said to be a minimal regular perfect dominating set if for each vertex v in P_r, P_r-{v} is not a dominating set of G.

Definition: The minimum fuzzy cardinality of a minimal regular perfect dominating set of G is called the regular perfect domination number of a fuzzy graph G. It is denoted by $\gamma_{rP}(G)$.

Definition: The maximum fuzzy cardinality of a minimal regular perfect dominating set of G is called the upper regular perfect domination number of a fuzzy graph G. It is denoted by $G_{rPf}(G)$.

Example: Consider the fuzzy graph G



Regular perfect dominating sets = {u, v}, {v, w}, {w, x}, {x, u}
Minimal perfect dominating set = {x, u}
Regular perfect domination number = 1
Upper regular perfect domination number = 1.2

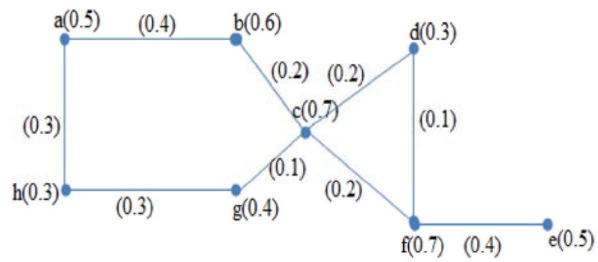
Theorem: Let G be any fuzzy graph. A perfect dominating set of a fuzzy graph G is minimal regular perfect dominating set if and only if for each vertex $u \in P_r$ at least one of the following conditions holds.

i. u is not a strong neighbor of any vertex in P_r
ii. For every vertex $u \in P_r$, there exists a vertex v in V-P_r such that N(v) n P_r = {u}

Proof: Suppose we assume that P_r be a minimal regular perfect dominating set of a fuzzy graph G. If for every vertex $u \in P_r$ such that u satisfy any conditions of (i) and (ii). Then P_r is not a minimal regular perfect dominating set. Hence proper subset P_r-{u} is a perfect dominating set of G.

Let Pr be a regular perfect dominating set and for each $u \in P_r$ at least one of two conditions holds. Suppose we assume P_r is not a minimal perfect dominating set of G, then for every vertex $u \in P_r$ such that P_r-{u} is a perfect dominating set. But u dominates to exactly one vertex v in P_r. That is when N(v)∩Pr = {u}, (ii) condition does not hold and this contradicts our assumption. Therefore, P is a minimal perfect dominating set of G.

Example: Consider the fuzzy graph G



Minimal regular perfect dominating set P_r = {c, f, h}
Regular perfect domination number = 1.7
N(c) = {b, d, f}; N(c)nP_r = {b, d, f}n{c, f, h} = {f}
N(f) = {c, e}; N(f)nP_r = {c, e}n{c, f, h} = {c}
N(h) = {a, g}; N(h)nP_r = {a, g}n{c, f, h} = ϕ
Here, (i) u is not a strong neighbor of any vertex in P_r is satisfied.
N(a) = {b, h}; N(a)nP_r = {b, h}n{c, f, h} = {h}
N(b) = {a, c}; N(b)nP_r = {a, c}n{c, f, h} = {c}
N(d) = {c}; N(d)nP_r = {c}n{c, f, h} = {c}
N(e)={f}; N(e)nP_r ={f}n{c, f, h}={f}
N(g)={h}; N(g)nP_r ={h}n{c, f, h}={h}
Here, (ii) For every vertex $u \in P_r$ there exists a vertex v in V-P_r such that N(v) n P_r = {u} is satisfied.

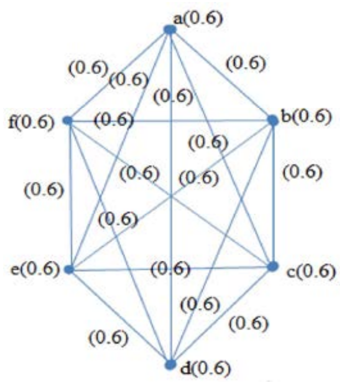
Theorem: A regular perfect dominating set exists for any regular strong fuzzy graph G.

Proof: Let G be a regular strong fuzzy graph. It is clear that $d(v_i) = k$, for every $v_i \in G$. Suppose a strong fuzzy graph G has a perfect dominating set, obviously, it contains the fuzzy vertices with $d(v_i) = k$ for every $v \in P_r$. Therefore, every regular strong fuzzy graph is a regular perfect dominating set and it exists for strong fuzzy graph.

Theorem: Let G be a complete fuzzy graph with same membership values for every vertex. Then $\gamma_{rP}(G) = \sigma(u)$ for all $u \in V$

Proof: Let G be a complete fuzzy graph with $\sigma(u_i)$'s equal. So, the degree of every vertex of G are equal. Also, we know that every vertex of G is a perfect dominating set. It is clear that, $\gamma_{rP}(G) = \sigma(u)$ for all $u \in V$.

Example: Consider the complete fuzzy graph G,



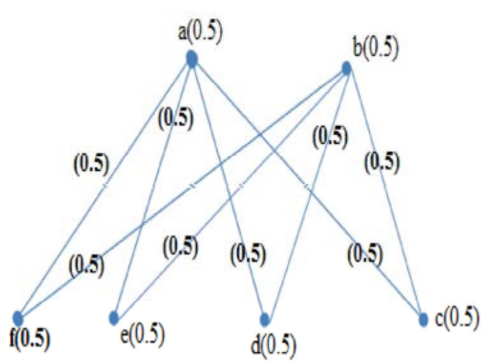
Regular perfect dominating set P_r = {a} or {b} or {c} or {d} or {e} or {f}

Regular perfect domination number $\gamma_{rP}(G) = \sigma(u) = (0.6)$ for all $u \in V$

Theorem: Let G be a complete bipartite fuzzy graph with $\sigma(u_i) = k$ and $\sigma(v_i) = k$ for every $u_i \in V_1$ and $v_i \in V_2$, then $\gamma_{rP}(G) = 2k$.

Proof: Let G be a complete fuzzy graph with $\sigma(u_i) = k$ and $\sigma(v_i) = k$ for every $u \in V_1$ and $v \in V_2$ By the definition of regular perfect dominating set = {min $\sigma(u_i)$, min $\sigma(v_i)$: $u_i \in V_1, v_i \in V_2$, clearly, d(u) = d(v) Therefore, regular perfect dominating set exists. That is, $\gamma_{rP}(G) = 2k$.

Example: Consider the complete bipartite fuzzy graph G



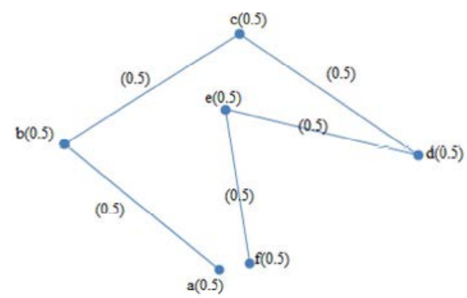
Regular perfect dominating set = {a, f} or {a, e} or {a, d} or {a, c} or {b, f} or {b, e} or {b, d} or {b,c}

Regular perfect domination number = k+k = 2k = 0.5+0.5 = 2(0.5) = 1

Theorem: For any strong fuzzy graph G, there is a fuzzy path with $\sigma(u_i) = k$ for every $u_i \in V$. Then a regular perfect dominating set exists.

Proof: For any strong fuzzy graph G there is a fuzzy path with $\sigma(u_i)= k$ for every $u_i \in V$. Regular perfect dominating set has the vertices of the same degree except the fuzzy pendent vertices. It is clear that $\gamma_{rP}(G) = \{u_i$: i is not equal to 1 or n$\}$. Therefore, regular perfect dominating set exists by using the definition of regular perfect dominating set.

Example: Consider the fuzzy graph G



Regular perfect dominating set = {b, e}

deg(b) = 1

deg(e) = 1

Properties of Regular Split Perfect Dominating Set in Fuzzy Graph Using Strong ARCS: In this section, we introduce the concept of regular split perfect dominating set in fuzzy graph using strong arcs, strong fuzzy graph and also discuss some parameters.

Definition: A split perfect dominating set S_{rp} of a fuzzy graph G is said to be a regular split perfect dominating set if all the vertices in S_{rp} has the same degree of a fuzzy graph G.

Definition: A regular split perfect dominating set S_{rp} of a fuzzy graph G is said to be a minimal regular split perfect dominating set if for each vertex v in S_{rp}, S_{rp} - {v} is not a dominating set of G.

Definition: The minimum fuzzy cardinality of a minimal regular split perfect dominating set of G is called the regular split perfect domination number of a fuzzy graph G. It is denoted by $?_{rsp}(G)$.

Definition: The maximum fuzzy cardinality of a minimal regular split perfect dominating set of G is called the upper regular split perfect domination number of a fuzzy graph G. It is denoted by $G_{rsPf}(G)$.

Theorem: For a regular fuzzy graph G, $\gamma_p(G) \leq \gamma_{rsp}(G)$

Proof: It is clear that every regular split perfect dominating set is a perfect dominating set. We get $\gamma_p(G) \leq \gamma_{rsp}(G)$

Theorem: For any fuzzy graph G, if S_{rp} is a regular split perfect dominating set, then V- S_{rp} is a dominating set of a fuzzy graph.

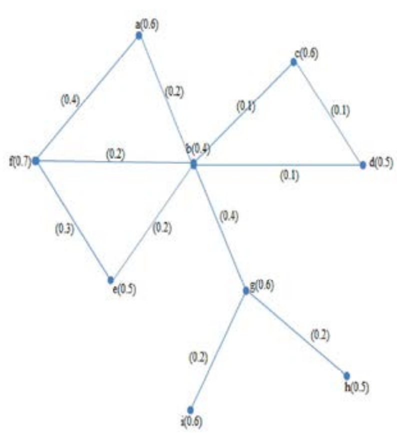
Proof: Let u be any vertex in S_{rp}. Since G has no isolated vertices, there is a vertex $v \in N(u)$. It is clear that every regular split perfect dominating set is a perfect dominating set such that $v \in V$- S_{rp}. Hence every vertex of S_{rp} dominates some vertices of V- S_{rp}. Hence, V- S_{rp} is a dominating set of fuzzy graph G.

Theorem: A regular split perfect dominating set of a fuzzy graph is minimal, if and only if for each vertex $v \in P_{rs}$ one of the following conditions holds.

(i) There exists a vertex $u \in V$-P_{rs} such that
$N(u) \cap P_{rs} = \{v\}$
(ii) v is an isolated vertex in <P_{rs}>
(iii) <V-D> is connected.

Proof: Suppose we assume that P_{rs} is minimal perfect dominating set and there exists a vertex $v \in P_{rs}$ such that v does not satisfy any one of the above conditions, then by conditions (i) and (ii), $P_{rs}' = P_{rs} - \{v\}$ is a perfect dominating set of a fuzzy graph G, also by (iii), <V- P_{rs}'> is disconnected which implies that P_{rs}' is a regular split perfect dominating set of a fuzzy graph G which is a contradiction to our assumption. Therefore, a regular split perfect dominating set is minimal satisfies one of the above conditions. Hence proved.

Example: Consider the fuzzy graph G



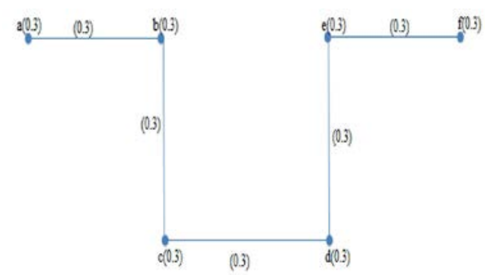
Regular split perfect dominating set $P_{rs} = \{b, g\}$ is also a perfect dominating set

Regular split perfect domination number = 1
V- $P_{rs} = \{ a, c, d, e, f, h, i\}$ is a dominating set
$N(a) \cap P_{rs} = \{b, f\} \cap \{b, g\} = \{b\}$
$N(c) \cap P_{rs} = \{b, d\} \cap \{b, g\} = \{b\}$
$N(d) \cap P_{rs} = \{b, c\} \cap \{b, g\} = \{b\}$
$N(e) \cap P_{rs} = \{b, f\} \cap \{b, g\} = \{b\}$
$N(f) \cap P_{rs} = \{a, e, b\} \cap \{b, g\} = \{b\}$
$N(h) \cap P_{rs} = \{g\} \cap \{b, g\} = \{g\}$
$N(i) \cap P_{rs} = \{g\} \cap \{b, g\} = \{g\}$

Theorem: Let G be a fuzzy path with $\sigma(u_i) = k$ for every $u_i \in V$ and having all effective edges (that is G be a strong fuzzy graph) then regular split perfect dominating set γ_{rsp} set exists.

Proof: Let G be a fuzzy path with $\sigma(u_i) = k$ for every $u_i \in V$ and having all effective edges, the regular split perfect dominating set = $\{ u_i : i \neq 1 \text{ or } n \}$ such that P_{rs} is a regular split perfect dominating set and <V-P_{rs}> is disconnected. Therefore, γ_{rsp} set exists.

Example: Consider the fuzzy graph G



Regular split perfect dominating set = {b, e} exists.
Regular split perfect domination number = 0.6

ACKNOWLEDGMENT

The authors are highly grateful to the referees for their valuable comments and suggestions for improving the paper.

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