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New Kinds of Neighborhood Connected Domination Parameters in an Intuitionistic Fuzzy Graph

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Abstract: In this paper, neighborhood connected total domination in an intuitionistic fuzzy graph, neighborhood disconnected total domination in an intuitionistic fuzzy graph, neighborhood connected perfect domination in an intuitionistic fuzzy graph, neighborhood disconnected perfect domination in an intuitionistic fuzzy graphs are introduced and discuss its properties. Furthermore this new domination parameter is compare with other known domination parameters.

Key words: Neighborhood connected total domination number · Neighborhood disconnected total domination number · Neighborhood connected perfect domination number and neighborhood disconnected perfect domination number

INTRODUCTION

The study of domination set in graphs was begun by ore and Berge. The connected domination number was first introduced by E. Sampathkumar and H.B. Walikar [1] Rosenfield [2] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as path, cycles and connectedness. A Somasundram and S. Somasundaram [3] discussed domination in fuzzy graphs. K.T. Atanassov [4] initiated the concept of intuitionistic fuzzy relations and intuitionitic fuzzy graphs. R.Parvathi and M.G. Karunambigai [5] gave a definition of IFG as a special case of IFGS defined by K.T Atanassov and A.Shannon. R.Parvathi and G.Thamizhendhi [6] was introduced dominating set and domination number in IFGS. In this paper, we discuss some new kinds of neighborhood connected domination number of an intuitionstic fuzzy graph and obtain the relationship with other known parameters of an IFG G.

Preliminaries

Definition: 2.1: An Intuitionistic Fuzzy Graph (IFG) is of the form G = (V, E) where

- $V = \{V_1, V_2, \dots, V_n\}$ such that $\sigma_1 : V \to [0, 1]$ and$\sigma_2 : V \to [0, 1]$ denote the degree of membership and non-membership of the element $v_i \in V$ respectively and$0 \leq \sigma_1 (v_i) + \sigma_2 (v_i) \leq 1$, for every $v_i \in V$.
- $E \subset V \times V$ where

$\mu_1 : V\times V \to [0, 1]$ and $\mu_2 : V \times V \to [0, 1]$ are such that
$\mu_1 (v_i, v_j) \leq \min \{\sigma_1(v_i), \sigma_1(v_j)\}$
$\mu_2 (v_i, v_j) = \max \{\sigma_2(v_i), \sigma_2(v_j)\}$
and $0 \leq \mu_1(v_i, v_j) + \mu_2(v_i, v_j) \leq 1$
for every $(v_i, v_j) \in E$.

Note: when $\mu_{1ij} = \mu_{2ij} = 0$ for some i and j then there is no edge between v_i and v_j otherwise there exits an edge between v_i and v_j.

Definition: 2.2: An IFG H = (V', E') is said to be an IF subgraph (IFSG) of G = (V, E) if $V' \subseteq E$ and $E' \subseteq E$. That is $\sigma^1_{1i} \leq \sigma_{1i}$; $\sigma^1_{2i} \geq \sigma_{2i}$ and $\mu^1_{1i} \leq \mu_{1i}$; $\mu^1_{2j} \geq \mu_{2j}$ for every i = 1, 2….n.

Definition: 2.3: The intuitionistic fuzzy subgraph H = (V′, E′) is said to be a spanning fuzzy subgraph of an IFG G = (V, E) if $\sigma_1'(u) = \sigma_1(u)$ and $\sigma_2'(u) = \sigma_2(u)$ for all $u \in V'$ and $\mu_1'(u,v) \leq \mu_1(u,v)$ and $\mu_2'(u,v) \geq \mu_2(u,v)$ and for all $u, v \in V$.

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Definition: 2.4: Let G = (V, E) be an IFG. Then the vertex cardinality of G is defined by $P = |v| = \sum_{v_i} \frac{1}{2}(1+\sigma_1(v_1)-\sigma_2(v_2))$ for all $v_i \in V$.

Definition: 2.5: Let G = (V, E) be an IFG. Then the edge cardinality of E is defined by $q = |E| = \sum_{v_i v_j} \frac{1}{2}(1 + \mu_1 (v_i, v_j) - \mu_2 (v_i, v_j))$ for all $v_i v_j \in E$.

Definition: 2.6: Let G = (V, E) be an IFG. Then the cardinality of G is defined to be |G| = |V| + |E| = p +q.

Definition: 2.7: The number of vertices is called the order of an IFG and is denoted by O(G). The number of edge is called size of IFG and is denoted by S(G).

Definition: 2.8: The vertices v_i and v_j are said to the neighbors in IFG either one of the following conditions hold

- $\mu_1 (v_i, v_j) > 0$, $\mu_2 (v_i, v_j) > 0$
- $\mu_1 (v_i, v_j) = 0$, $\mu_2 (v_i, v_j) > 0$
- $\mu_1 (v_i, v_j) > 0$, $\mu_2 (v_i, v_j) = 0$, $v_i, v_j \in V$

Definition: 2.9: A path in an IFG is a sequence of distinct vertices $v_1, v_2, \ldots v_n$ such that either one of the following conditions is satisfied.

- $\mu_1 (v_i, v_j) > 0$, $\mu_2 (v_i, v_j) > 0$ for some i and j
- $\mu_1 (v_i, v_j) = 0$, $\mu_2 (v_i, v_j) > 0$ for some i and j
- $\mu_1 (v_i, v_j) > 0$, $\mu_2 (v_i, v_j) = 0$ for some i and j

Note: The length of the path $P = v_1, v_2, \ldots v_{n+1}$ (n > 0) is n

Definition: 2.10: Two vertices that are joined by a path is called connected.

Definition: 2.11: An edge e=(x, y) of an IFG G = (V, E) is called an effective edge if $\mu_1 (x, y) = \min \{\sigma_1(x), \sigma_1(y)\}$ and $\mu_2 (x, y) = \max \{\sigma_2(x), \sigma_2(y)\}$.

Definition: 2.12: An IFG G = (V, E) is said to be complete IFG if $\mu_{1ij} = \min \{\sigma_{1i}, \sigma_{1j}\}$ and $\mu_{2ij} = \max \{\sigma_{2i}, \sigma_{2j}\}$ for every $v_i, v_j \in V$.

Definition: 2.13: The complement of an IFG, G = (V, E) is an IFG, $\bar{G} = (\bar{V}, \bar{E})$, where,

- $\bar{V} = V$
- $\overline{\sigma_{1i}} = \sigma_{1i}$ and $\overline{\sigma_{2i}} = \sigma_{2i}$, for all i = 1, 2, … n
- $\overline{\mu_{1ij}} = \min\{\sigma_{1i}, \sigma_{1j}\} - \mu_{1ij}$
- $\overline{\mu_{2ij}} = \max\{\sigma_{2i}, \sigma_{2j}\} - \mu_{2ij}$ for all i = 1, 2, … n

Definition: 2.14: An IFG, G = (V, E) is said to bipartite the vertex set V can be partitioned into two non empty sets V_1 and V_2 such that

- $\mu_1 (v_i, v_j) = 0$ and $\mu_2 (v_i, v_j) = 0$ if $v_i, v_j \in V_1$(or) $v_i, v_j \in V_2$
- $\mu_1 (v_i, v_j) > 0$ and $\mu_2 (v_i, v_j) > 0$ if $v_i \in V_1$ and $v_j \in V_2$ for some i and jor

$\mu_1 (v_i, v_j) = 0$ and $\mu_2 (v_i, v_j) > 0$ if $v_i \in V_1$ and $v_j \in V_2$ for some i and jor
$\mu_1 (v_i, v_j) > 0$ and $\mu_2 (v_i, v_j) = 0$ if $v_i \in V_1$ and $v_j \in V_2$for some i and j

Definition: 2.15:

A bipartite IFG, G = (V, E) is said to be complete if
$\mu_1 (v_i, v_j) = \min \{\sigma_1(v_i), \sigma_1(v_j)\}$
$\mu_2 (v_i, v_j) = \max \{\sigma_2(v_i), \sigma_2(v_j)\}$
forall $v_i \in V_1$ and $v_j \in V_2$. It is denoted by $K_{(\sigma_1 \sigma_2,\ \mu_1 \mu_2)}$

Definition: 2.16: A vertex u ∈ V of an IFG G = (V, E) is said to be an isolated vertex if $\mu_1 (u, v) = 0$ and $\mu_2 (u, v) = 0$, for all v ∈ V. That is N (u) = ϕ. Thus an isolated vertex does not dominate any other vertex in G.

Definition: 2.17: Let G = (V, E) be an IFG on V. Let u, v ∈ V, we say that u dominate v in G if there exits an effective edge between them.

Definition: 2.18: Let G = (V, E) be an intuitionistic fuzzy graph G on the vertex set V. Let x , y∈V , we say that x dominates y in G if $\mu_1 (x, y) = \min \{\sigma_1(x), \sigma_1(y)\}$ and $\mu_2 (x, y) = \max \{\sigma_2(x), \sigma_2(y)\}$.A subset D of V is called a dominating set in IFG G if for every v ∈ V – D, there exists u ∈ D such that u dominates v. A dominating set D of an IFG is said to be minimal dominating set if no proper subset of D is a dominating set.

Intuitionistic Fuzzy Neighborhood Connected Total Domination Number: In this section, the concept of intuitionistic fuzzy neighborhood connected total domination number is introduced and its properties are investigated.

Definition 3.1: Let G = (V, E) be an intuitionistic fuzzy graph. The total dominating set D_t is called the neighborhood connected total dominating set if the induced intuitionistic fuzzy subgraph $\langle N(D_t) \rangle$ is connected. The intuitionistic fuzzy neighborhood connected total domination number $\gamma_{nct}(G)$ is the minimum intuitionistic fuzzy cardinality taken over all minimal neighborhood connected total dominating sets of G.

Example 3.2

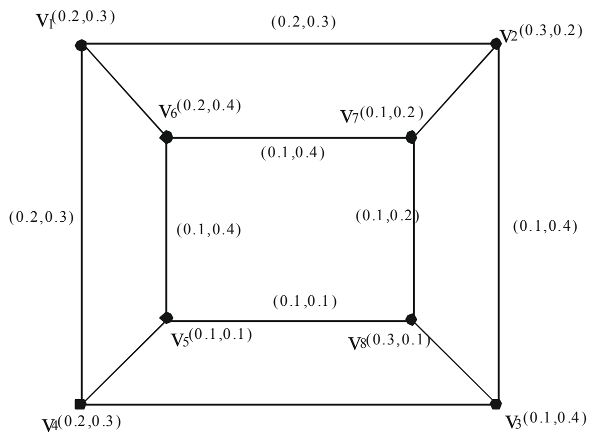

Fig. 3.1

Theorem 3.3: If G = (V, E) be an intuitionistic fuzzy k-regular graph with equal intuitionistic fuzzy vertex and edge cardinality then G has a neighborhood connected total dominating set.

Proof: Let G = (V, E) be an intuitionistic fuzzy k-regular graph with equal intuitionistic fuzzy vertex and edge cardinality. Then by definition of total domination, every vertex v∈V is adjacent to some vertex u∈ D. Every vertex is incident with equal number of edges since it is a k-regular intuitionistic fuzzy graph with equal vertex and edge cardinality. Therefore G has a total dominating set and the neighborhood of D is connected. Clearly G has a neighborhood connected total dominating set.

Theorem 3.4: If the k-regular intuitionistic fuzzy graph G = (V, E) is not a C_n with n intuitionistic fuzzy vertices then $\gamma_{nct}(G) \leq \frac{n}{2}\ |v|$.

Proof: Let G = (V, E) be an intuitionistic fuzzy k-regular graph with n intuitionistic fuzzy vertices and D_{nct} is the neighborhood connected total dominating set, $\gamma_{nct}(G)$ is the minimum neighborhood connected total domination number of G. The neighborhood connected total dominating set D_{nct} containing at most $\frac{n}{2}$ intuitionistic fuzzy vertices, since by the definition of total dominating set every v∈V is adjacent with atleast one vertex $u \in D_{nct}$. The induced subgraph < $N(D_t)$ > is connected. Clearly $\gamma_{nct}(G) \leq \frac{n}{2}\ |v|$.

Theorem 3.5: If the intuitionistic fuzzy graph G = (V, E) has a neighborhood connected total dominating set then G contains a cycle.

Proof: Let G = (V, E) be an intuitionistic fuzzy graph. Since G has a neighborhood connected total dominating set then by definition of neighborhood connected total dominating set, every vertex v∈V is adjacent with atleast one vertex $u \in D_{nct}$ and the neighborhood of D_{nct} is connected. This implies G has a cycle.

Theorem 3.6: If the intuitionistic fuzzy graph G = (V, E) is not a cycle, has a neighborhood connected total dominating set then G has a cycle non split dominating set.

Proof: Let the intuitionistic fuzzy graph G = (V, E) is not a cycle, has a neighborhood connected total dominating set D_{nct}. D_{nct} contains $\frac{n}{2}$ intuitionistic fuzzy vertices. Then the induced subgraph <$V-D_{nct}$> is a cycle. Therefore G has a cycle non split dominating set.

Theorem 3.7: If G = (V, E) is both a k-regular and r-totally regular intuitionistic fuzzy graph whose crisp graph is 3-regular then $\gamma_{ncp}(G) = \gamma_{nct}(G)$.

Proof:
Obvious.

Theorem 3.8: If the intuitionistic fuzzy graph G = (V, E) is K_2 then it has a neighborhood connected total dominating set.

Proof: Let G = (V, E) be an intuitionistic fuzzy graph K_2, obviously the dominating set is a singleton set, that is either of the intuitionistic fuzzy pendent vertices. Therefore the neighborhood is also a single intuitionistic fuzzy vertex. Clearly it is connected. Also the dominating set is total dominating set. Hence the K_2 has a neighborhood connected total dominating set.

Remark 3.9: If the intuitionistic fuzzy graph G = (V, E) is $C_n \circ K_1$ then G has a neighborhood connected total dominating set.

Remark 3.10: The intuitionistic fuzzy neighborhood connected total dominating set may be an intuitionistic fuzzy neighborhood connected perfect dominating set but the converse need not be true.

Intuitionistic Fuzzy Neighborhood Disconnected Total Domination Number: In this section, the concept of intuitionistic fuzzy neighborhood disconnected total domination number is introduced and its properties are discussed.

Definition 4.1: Let G = (V, E) be an intuitionistic fuzzy graph, The total dominating set $D_t$is called the neighborhood disconnected total dominating set if the induced intuitionistic fuzzy subgraph $<N(D_t)>$ is disconnected.The intuitionistic fuzzy neighborhood disconnected total domination number $\gamma_{ndct}(G)$ is the minimum intuitionistic fuzzy cardinality taken over all minimal neighborhood disconnected total dominating sets of G.

Example 4.2

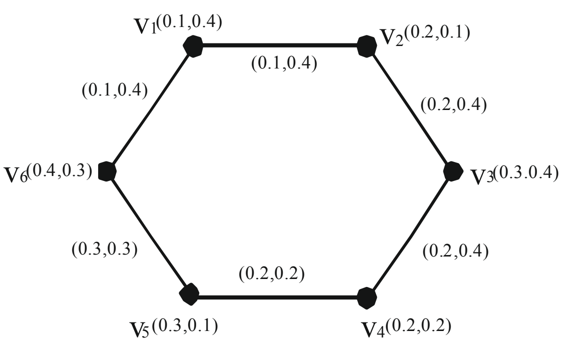

Fig. 4.1

$D_{ndct}(G) = \{ v_1, v_3, v_5\}$

Theorem 4.3: If G = (V, E) is a k-regular and r-totally regular intuitionistic fuzzy graph then $\gamma_{ndct}(G) = \leq \frac{n}{2} |v|$.

Proof: Let D be a γ_{ndct}-set . Since G has no isolated vertices, every V-D has at least one neighbor in V-D. This means that V-D is also a dominating set. If $|D| > \frac{n}{2}$, thenV-D is a smaller dominating set, contradicting the choice of D as a γ_{ndct}-set. Thus $\gamma_{ndct}(G) \leq \frac{n}{2} |v|$.

Theorem 4.4: If G = (V, E) is an intuitionistic fuzzy graph with equal intuitionistic fuzzy vertex and edge cardinality then $\gamma_{ndct}(G) = \leq \frac{n}{2} |v|$.

Proof:
Obvious.
Remark 4.5: If the intuitionistic fuzzy graph G = (V, E) is a cycle C_n or a path P_n with n =3 and equal intuitionistic fuzzy vertex and edge cardinality then,

- $\gamma_{ndct}(C_n) =\gamma_{ndct}(P_n) = \frac{n}{2} |v|$ if $n \equiv 0 \pmod 4$
- $\gamma_{ndct}(C_n) = \gamma_{ndct}(P_n) = \frac{n+2}{2} |v|$ if $n \equiv 2 \pmod 4$
- $\gamma_{ndct}(C_n) = \gamma_{ndct}(P_n) = \frac{n+1}{2} |v|$, otherwise.

Intuitionistic Fuzzy Neighborhood Connected Perfect Domination Number: This section, the concept of intuitionistic fuzzy neighborhood connected perfect domination number is introduced and related properties are discussed.

Definition 5.1: Let G = (V, E) be an intuitionistic fuzzy graph, The perfect dominating set D_p is called the neighborhood connected perfect dominating set if the induced intuitionistic fuzzy subgraph $< N(D_p) >$ is connected. The intuitionistic fuzzy neighborhood connected perfect domination number $\gamma_{ncp}(G)$ is the minimum intuitionistic fuzzy cardinality taken over all minimal neighborhood connected perfect dominating sets of G.

Example 5.2

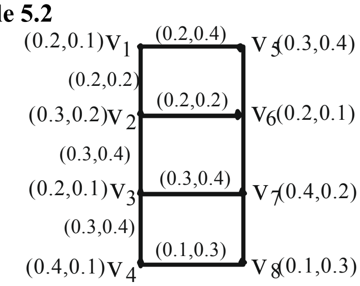

Fig. 5.1

Lemma 5.3: If D = {u, v} is a perfect dominating set of an intuitionistic fuzzy graph G = (V, E), then $N(u)\cap N(v) =\phi$ and $N[u]\cup N[v] =V(G)$.

Proof: Suppose $N(u)\cap N(v) \neq \phi$. Then there is a vertex $x\in V(G)$ such that $x\in N(u)\cap N(v)$. Then $x\in N(u)$ and $x\in N(v)$. This implies that $x \in V(G)-D$ is dominated by u and v contrary to the fact that D is a perfect dominating set of G. Thus $N(u)\cap N(v) =\phi$. Suppose that $N[u]\cup N[v] \neq V(G)$. Then there exists $w\in V(G)$ such that $w\notin N[u]\cup N[v]$, Thus $w\notin N[u]$ and $w\notin N[v]$. This implies that w is not dominated by any element of D contrary to the fact that D is a dominating set of G. Thus $N[u]\cup N[v] =V(G)$.

Theorem 5.4: If G = (V, E) is an intuitionistic fuzzy graph and D is the neighborhood connected perfect dominating set then V-D need not be a neighborhood connected perfect dominating set.

Theorem 5.5: If the intuitionistic fuzzy graph G = (V, E) has a neighborhood connected perfect dominating set then G has an even number of intuitionistic fuzzy vertices.

Proof: Let G = (V, E) be an intuitionistic fuzzy graph and D be the neighborhood connected dominating set. By the definition of perfect dominating set, every vertex u in D is adjacent with exactly one v in V-D. Clearly $|D| = |V - D|$. Therefore $|G| = |D| + |V - D|$, clearly the number of vertices in G is even.

Theorem 5.6: Let D_1 and D_2 be the two perfect dominating set of an intuitionistic fuzzy graph G = (V, E) then $|D_1| = |D_2|$ but the neighborhood connected perfect domination numbers are not equal, That is $\gamma_{ncp1}(G) \neq \gamma_{ncp2}(G)$.

Proof: Let $x \in D_1$.Then there is a unique vertex V(x) in D_2 which is adjacent to x, moreover for every y in D_2 there is a unique vertex U(y) in D_1 which is adjacent to y. It may be noted that these functions are inverse of each other. Therefore $|D_1| = |D_2|$ and hence D_1 and D_2 have the intuitionistic fuzzy cardinality $\sum_{x \in D1} |x|$ and $\sum_{y \in D2} |y|$. Therefore $\gamma_{ncp1}(G) \neq \gamma_{ncp2}(G)$.

Remark 5.7: Let D_1 and D_2 be the two perfect dominating set of an intuitionistic fuzzy graph G = (V, E) with equal intuitionistic fuzzy vertex and edge cardinality. Then the neighborhood connected perfect domination numbers are equal, that is $\gamma_{ncp1}(G) = \gamma_{ncp2}(G)$.

Corollary 5.8: If G = (V, E) is an intuitionistic fuzzy graph having two neighborhood connect perfect dominating sets D_1 and D_2 such that $|D_1| \neq |D_2|$ then $D_1 \cap D_2 \neq \phi$.

Corollary 5.9: Let G = (V, E)be an intuitionistic fuzzy graph with n intuitionistic fuzzy vertices. If there is a perfect dominating set D with the number of vertices in D is $|D| \neq \frac{n}{2}$ then V-D is not a perfect dominating set.

Intuitionistic Fuzzy Neighborhood Disconnected Perfect Domination Number: This section, introduces the concept of intuitionistic fuzzy neighborhood disconnected perfect domination number and carries out a discussion on its properties.

Definition 6.1: Let G = (V, E) be an intuitionistic fuzzy graph, The perfect dominating set D_p is called the neighborhood disconnected perfect dominating set if the induced intuitionistic fuzzy subgraph $\langle N(D_p) \rangle$ is disconnected. The intuitionistic fuzzy neighborhood disconnected perfect domination number $\gamma_{ndp}(G)$ is the minimum fuzzy cardinality taken over all minimal neighborhood disconnected perfect dominating sets of G.

Theorem 6.2: If G = (V, E) is an intuitionistic fuzzy cycle with 4n intuitionistic fuzzy vertices then $\gamma_{ndt}(C_{4n}) = 2n|v|$.

Proof:
Obvious.

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