Middle-East Journal of Scientific Research 25 (2): 352-357, 2017
ISSN 1990-9233

DOI: 10.5829/idosi.mejsr.2017.352.357

Pseudo Fuzzy Coset of a Fuzzy HX Ideal of a HX Ring

[1]R. Muthuraj and [2]N. Ramila Gandhi

[1]Department of Mathematics, H.H.The Rajah's College,
Pudukkottai- 622001,Tamilnadu, India
[2]Department of Mathematics, PSNA College of Engineering
and Technology, Dindigul-624622, Tamilnadu, India

Abstract: The purpose of this paper is to develop a new approach to the concept of pseudo fuzzy coset of a fuzzy HX ideal of a HX ring. The aim is to study the properties of pseudo fuzzy coset of a fuzzy HX left ideal as well as fuzzy HX right ideal of a HX ring.

Key words: HX ring · Fuzzy HX right ideal · Fuzzy HX left ideal · Pseudo fuzzy coset of a fuzzy HX ideal

INTRODUCTION

In 1965, Lotfi. A. Zadeh [1] introduced the concept of fuzzy set. Fuzzy sets attracted many mathematicians and grew enormously by finding applications in many areas. In 1982 Wang-jin Liu [2] introduced the concept of fuzzy subring and fuzzy ideal. In 1988, Professor Li Hong Xing [3] proposed the concept of HX ring and derived some of its properties, then Professor Zhong [4, 5] gave the structures of HX ring on a class of ring. W.B.Vasantha kandasamy [6] introduced the concept of fuzzy cosets.

Pseudo Fuzzy Coset of a Fuzzy HX Ideal of a HX Ring: In this section, we introduce the notion of pseudo fuzzy cosets of a fuzzy HX ideal and discuss its properties.

Definition 2.1: Let μ be a fuzzy set defined on R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. Let λ^μ be a fuzzy HX right ideal of a HX ring $\Re$ and $A \in \Re$. Then the pseudo fuzzy coset of a fuzzy HX right ideal λ^μ of a HX ring $\Re$ determined by the element $A \in \Re$ is denoted as $(A + \lambda^\mu)^P$ is defined by

$(A + \lambda^\mu)^P(X) = p(A)\,\lambda^\mu(X)$ for every $X \in \Re$ and for some $p \in P$, where $P = \{\, p(X) / p(X) \in [0,1]$ for all $X \in \Re\}$.

Definition: 2.2 Let μ be a fuzzy set defined on R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. Let λ^μ be a fuzzy HX left ideal of a HX ring $\Re$ and $A \in \Re$. Then the pseudo fuzzy coset of a fuzzy HX left ideal λ^μ of a HX ring $\Re$ determined by the element $A \in \Re$ is denoted as $(A + \lambda^\mu)^P$ and is defined by

$(A + \lambda^\mu)^P(X) = p(A)\,\lambda^\mu(X)$ for every $X \in \Re$ and for some $p \in P$, where $P = \{\, p(X) / p(X) \in [0,1]$ for all $X \in \Re\}$.

Definition 2.3: Let μ be a fuzzy set defined on R. Let $\Re \subset 2^R - \{\phi\}$ be a HX ring. Let λ^μ be a fuzzy HX ideal of a HX ring $\Re$ and $A \in \Re$. Then the pseudo fuzzy coset of a fuzzy HX ideal λ^μ of a HX ring $\Re$ determined by the element $A \in \Re$ is denoted as $(A + \lambda^\mu)^P$ and is defined as $(A + \lambda^\mu)^P$ is both the pseudo fuzzy coset of a fuzzy HX right ideal λ^μ of a HX ring $\Re$ determined by the element $A \in \Re$ and the pseudo fuzzy coset of a fuzzy HX left ideal λ^μ of a HX ring $\Re$ determined by the element $A \in \Re$.

Theorem 2.4: Let λ^μ be a fuzzy HX right ideal of a HX ring $\Re$. Let $(A + \lambda^\mu)^P$ be a pseudo fuzzy coset of a fuzzy HX right ideal λ^μ of a HX ring $\Re$, then $(A + \lambda^\mu)^P$ is a fuzzy HX right ideal of a HX ring $\Re$.

Proof: Let λ^μ be a fuzzy HX right ideal of a HX ring $\Re$.
Let $(A + \lambda^\mu)^P$ be a pseudo fuzzy coset of a fuzzy HX right ideal λ^μ of a HX ring $\Re$.

For every X, Y, $A \in \Re$ we have

i. $(A + \lambda^\mu)^P(X-Y) = p(A)\,\lambda^\mu(X-Y)$
$\geq p(A) \min\{\lambda^\mu(X), \lambda^\mu(Y)\}$
$= \min\{p(A)\lambda^\mu(X), p(A)\,\lambda^\mu(Y)\}$
$= \min\{(A+\lambda^\mu)^P(X), (A+\lambda^\mu)^P(Y)\}$

Therefore, $(A+\lambda^\mu)^P(X-Y) \geq \min\{(A+\lambda^\mu)^P(X), (A+\lambda^\mu)^P(Y)\}$

ii. $(A + \lambda^\mu)^P(XY) = p(A)\,\lambda^\mu(XY)$

Corresponding Author: R. Muthuraj, PG & Research Department of Mathematics, H.H.The Rajah's College, Pudukkottai- 622001,Tamilnadu, India.

$\geq p(A)\ \lambda^{\mu}(X)$
$= (A + \lambda^{\mu})^{P}(X).$

Therefore, $(A + \lambda^{\mu})^{P}(XY) \geq (A + \lambda^{\mu})^{P}(X)$.
Hence, $(A + \lambda^{\mu})^{P}$ is a fuzzy HX right ideal of a HX ring $\Re$.

Theorem 2.5: Let λ^{μ} be a fuzzy HX left ideal of a HX ring $\Re$. Let $(A + \lambda^{\mu})^{P}$ be a pseudo fuzzy coset of a fuzzy HX left ideal λ^{μ} of a HX ring $\Re$, then $(A + \lambda^{\mu})^{P}$ is a fuzzy HX left ideal of a HX ring $\Re$.

Proof: Let λ^{μ} be a fuzzy HX left ideal of a HX ring $\Re$.

Let $(A + \lambda^{\mu})^{P}$ be a pseudo fuzzy coset of a fuzzy HX left ideal λ^{μ} of a HX ring $\Re$.

For every $X, Y, A \in \Re$ we have

i. $(A + \lambda^{\mu})^{P}(X-Y) = p(A)\ \lambda^{\mu}(X-Y)$
$\geq p(A) \min\{\lambda^{\mu}(X), \lambda^{\mu}(Y)\}$
$= \min\{p(A)\lambda^{\mu}(X), p(A)\ \lambda^{\mu}(Y)\}$
$= \min\{(A+\lambda^{\mu})^{P}(X), (A+\lambda^{\mu})^{P}(Y)\}$

Therefore, $(A + \lambda^{\mu})^{P}(X-Y) \geq \min\{(A+\lambda^{\mu})^{P}(X), (A+\lambda^{\mu})^{P}(Y)\}$

ii. $(A + \lambda^{\mu})^{P}(XY) = p(A)\ \lambda^{\mu}(XY)$
$\geq p(A)\ \lambda^{\mu}(Y)$
$= (A + \lambda^{\mu})^{P}(Y).$

Therefore, $(A + \lambda^{\mu})^{P}(XY) \geq (A + \lambda^{\mu})^{P}(Y)$.
Hence, $(A + \lambda^{\mu})^{P}$ is a fuzzy HX left ideal of a HX ring $\Re$.

Theorem 2.6: Let λ^{μ} be a fuzzy HX ideal of a HX ring $\Re$.

Let $(A+ \lambda^{\mu})^{P}$ be a pseudo fuzzy coset of a fuzzy HX ideal λ^{μ} of a HX ring $\Re$, then $(A + \lambda^{\mu})^{P}$ is a fuzzy HX ideal of a HX ring $\Re$.

Proof: It is clear.

Theorem 2.7: Let μ and η be any two fuzzy sets defined on R. Let $(A + \lambda^{\mu})^{P}$ and $(A + \gamma^{\eta})^{P}$ be any two pseudo fuzzy coset of a fuzzy HX right ideals λ^{μ} and γ^{η} of a HX ring $\Re$ determined by the element $A \in \Re$. Then $(A + \lambda^{\mu})^{P} \cap (A + \gamma^{\eta})^{P}$ is a fuzzy HX right ideal of $\Re$.

Proof: Let $(A + \lambda^{\mu})^{P}$ and $(A + \gamma^{\eta})^{P}$ be any two pseudo fuzzy coset of a fuzzy HX right ideals of a HX ring $\Re$ determined by the element $A \in \Re$.

By Theorem 2.4, $(A + \lambda^{\mu})^{P}$ and $(A + \gamma^{\eta})^{P}$ are fuzzy HX right ideals of a HX ring $\Re$ determined by the element $A \in \Re$.

By Theorem 3.3 [10], $(A + \lambda^{\mu})^{P} \cap (A + \gamma^{\eta})^{P}$ is a fuzzy HX right ideals of $\Re$ determined by the element $A \in \Re$.

Theorem 2.8: Let μ and η be any two fuzzy sets defined on R. Let $(A + \lambda^{\mu})^{P}$ and $(A + \gamma^{\eta})^{P}$ be any two pseudo fuzzy coset of fuzzy HX left ideals λ^{μ} and γ^{η} of a HX ring $\Re$ determined by the element $A \in \Re$. Then $(A + \lambda^{\mu})^{P} \cap (A + \gamma^{\eta})^{P}$ is a fuzzy HX left ideal of $\Re$.

Proof: Let $(A + \lambda^{\mu})^{P}$ and $(A + \gamma^{\eta})^{P}$ be any two pseudo fuzzy coset of a fuzzy HX left ideals of a HX ring $\Re$ determined by the element $A \in \Re$.

By Theorem 2.5, $(A + \lambda^{\mu})^{P}$ and $(A + \gamma^{\eta})^{P}$ are fuzzy HX left ideals of a HX ring $\Re$ determined by the element $A \in \Re$.

By Theorem 4.3 [10], $(A + \lambda^{\mu})^{P} \cap (A + \gamma^{\eta})^{P}$ is a fuzzy HX left ideals of $\Re$ determined by the element $A \in \Re$.

Theorem 2.9: Let μ and η be any two fuzzy sets defined on R. Let $(A+ \lambda^{\mu})^{P}$ and $(A + \gamma^{\eta})^{P}$ be any two pseudo fuzzy coset of a fuzzy HX ideals λ^{μ} and γ^{η} of a HX ring $\Re$ determined by the element $A \in \Re$. Then $(A+ \lambda^{\mu})^{P} \cap (A+ \gamma^{\eta})^{P}$ is a fuzzy HX ideal of $\Re$.

Proof: It is clear.

Theorem 2.10: Let μ and η be any two fuzzy sets defined on R. Let $(A + \lambda^{\mu})^{P}$ and $(A + \gamma^{\eta})^{P}$ be any two pseudo fuzzy coset of a fuzzy HX right ideals λ^{μ} and γ^{η} of a HX ring $\Re$ determined by the element $A \in \Re$. Then $(A + \lambda^{\mu})^{P} \cup (A + \gamma^{\eta})^{P}$ is a fuzzy HX right ideal of $\Re$.

Proof: Let $(A + \lambda^{\mu})^{P}$ and $(A + \gamma^{\eta})^{P}$ be any two pseudo fuzzy coset of a fuzzy HX right ideals of a HX ring $\Re$ determined by the element $A \in \Re$.

By Theorem 2.4, $(A + \lambda^{\mu})^{P}$ and $(A + \gamma^{\eta})^{P}$ are fuzzy HX right ideals of a HX ring $\Re$ determined by the element $A \in \Re$.

By Theorem 3.4 [10], $(A + \lambda^{\mu})^{P} \cup (A + \gamma^{\eta})^{P}$ is a fuzzy HX right ideals of $\Re$ determined by the element $A \in \Re$.

Theorem 2.11: Let μ and η be any two fuzzy sets defined on R. Let $(A + \lambda^{\mu})^{P}$ and $(A+ \gamma^{\eta})^{P}$ be any two pseudo fuzzy coset of a fuzzy HX left ideals λ^{μ} and γ^{η} of a HX ring $\Re$ determined by the element $A \in \Re$. Then $(A + \lambda^{\mu})^{P} \cup (A + \gamma^{\eta})^{P}$ is a fuzzy HX left ideal of $\Re$.

Proof: Let $(A + \lambda^{\mu})^{P}$ and $(A + \gamma^{\eta})^{P}$ be any two pseudo fuzzy coset of fuzzy HX left ideals of a HX ring $\Re$ determined by the element $A \in \Re$.

By Theorem 2.5, $(A + \lambda^{\mu})^{P}$ and $(A + \gamma^{\eta})^{P}$ are fuzzy HX left ideals of a HX ring $\Re$ determined by the element $A \in \Re$.

By Theorem 4.4 [10], $(A + \lambda^{\mu})^{P} \cup (A + \gamma^{\eta})^{P}$ is a fuzzy HX left ideal of $\Re$ determined by the element $A \in \Re$.

Theorem 2.12: Let μ and η be any two fuzzy sets defined on R. Let $(A + \lambda^{\mu})^p$ and $(A + \gamma^{\eta})^p$ be any two pseudo fuzzy coset of fuzzy HX ideals λ^{μ} and γ^{η} of a HX ring $\Re$ determined by the element $A \in \Re$.Then $(A+ \lambda^{\mu})^p \cup (A + \gamma^{\eta})^p$ is a fuzzy HX ideal of $\Re$.

Proof: It is clear.

Theorem 2.13: Let μ and η be any two fuzzy sets defined on R. Let $(A + \lambda^{\mu})^p$ and $(A + \gamma^{\eta})^p$ be any two pseudo fuzzy coset of a fuzzy HX right ideals λ^{μ} and γ^{η} of a HX ring $\Re$ determined by the element $A \in \Re$.Then $(A + \lambda^{\mu})^p \times (A + \gamma^{\eta})^p$ is a fuzzy HX right ideal of $\Re \times \Re$.

Proof: Let $(A + \lambda^{\mu})^p$ and $(A + \gamma^{\eta})^p$ be any two pseudo fuzzy coset of a fuzzy HX right ideals of a HX ring $\Re$ determined by the element $A \in \Re$.

By Theorem 2.4, $(A + \lambda^{\mu})^p$ and $(A + \gamma^{\eta})^p$ are fuzzy HX right ideals of a HX ring $\Re$ determined by the element $A \in \Re$.

By Theorem 3.5 [10], $(A + \lambda^{\mu})^p \times (A + \gamma^{\eta})^p$ is a fuzzy HX right ideal of $\Re \times \Re$.

Theorem 2.14: Let μ and η be any two fuzzy sets defined on R. Let $(A + \lambda^{\mu})^p$ and $(A + \gamma^{\eta})^p$ be any two pseudo fuzzy coset of a fuzzy HX left ideals λ^{μ} and γ^{η} of a HX ring $\Re$ determined by the element $A \in \Re$.Then $(A + \lambda^{\mu})^p \times (A + \gamma^{\eta})^p$ is a fuzzy HX left ideal of $\Re \times \Re$.

Proof: Let $(A + \lambda^{\mu})^p$ and $(A + \gamma^{\eta})^p$ be any two pseudo fuzzy coset of a fuzzy HX left ideals of a HX ring $\Re$ determined by the element $A \in \Re$.

By Theorem 3.5, $(A + \lambda^{\mu})^p$ and $(A + \gamma^{\eta})^p$ are fuzzy HX left ideals of a HX ring $\Re$ determined by the element $A \in \Re$.

By Theorem 4.5 [10], $(A + \lambda^{\mu})^p \times (A + \gamma^{\eta})^p$ is a fuzzy HX left ideal of $\Re \times \Re$.

Theorem 2.15: Let μ and η be any two fuzzy sets defined on R. Let $(A + \lambda^{\mu})^p$ and $(A + \gamma^{\eta})^p$ be any two pseudo fuzzy coset of a fuzzy HX ideals λ^{μ} and γ^{η} of a HX ring $\Re$ determined by the element $A \in \Re$. Then $(A + \lambda^{\mu})^p \times (A + \gamma^{\eta})^p$ is a fuzzy HX ideal of $\Re \times \Re$.

Proof: It is clear.

Homomorphism and Anti Homomorphism of a Pseudo Fuzzy Coset of a Fuzzy Hx Ideal of a Hx Ring: In this section, we introduce the concept of an image, pre-image of pseudo fuzzy coset of a HX ring and discuss the properties of homomorphic and anti homomorphic images and pre images of pseudo fuzzy coset of a fuzzy HX ideal λ^{μ} of a HX ring $\Re$ determined by the element $A \in \Re$.

Theorem 3.1: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be a homomorphism onto HX rings. Let $(A+ \lambda^{\mu})^p$ be the pseudo fuzzy coset of a fuzzy HX right ideal λ^{μ} of a HX ring $\Re_1$ determined by the element $A \in \Re_1$.Then $f((A+ \lambda^{\mu})^p)$ is the pseudo fuzzy coset of a fuzzy HX right ideal $f(\lambda^{\mu})$ of a HX ring $\Re_2$ determined by the element $f(A) \in \Re_2$ and $f((A + \lambda^{\mu})^p) = (f(A) + f(\lambda^{\mu}))^p$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof: Let $f : \Re_1 \to \Re_2$ be a homomorphism onto HX rings.

Let $(A+\lambda^{\mu})^p$ be the pseudo fuzzy coset of a fuzzy HX right ideal λ^{μ} of a HX ring $\Re_1$ determined by the element $A \in \Re_1$.By Theorem, if λ^{μ} is a fuzzy HX right ideal of $\Re_1$ then $f(\lambda^{\mu})$ is a fuzzy HX right ideal of $\Re_2$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

$f((A + \lambda^{\mu})^p)$ is the pseudo fuzzy coset of a fuzzy HX right ideal $f(\lambda^{\mu})$ of a HX ring $\Re_2$ determined by the element $f(A) \in \Re_2$.

Let $A, X \in \Re_1$, then $f(A), f(X) \in \Re_2$.

$$
\begin{aligned}
\text{Now, } (f(A) + f(\lambda^{\mu}))^p f(X) &= p(f(A))(f(\lambda^{\mu})(f(X)) \\
&= p(A)\, \lambda^{\mu}(X) \\
&= (A + \lambda^{\mu})^p(X) \\
&= f((A + \lambda^{\mu})^p)f(X). \\
(f(A) + f(\lambda^{\mu}))^p f(X) &= f((A + \lambda^{\mu})^p)f(X), \\
&\qquad \text{for any } f(X) \in \Re_2. \\
\text{Hence, } f((A + \lambda^{\mu})^p) &= (f(A) + f(\lambda^{\mu}))^p.
\end{aligned}
$$

Theorem 3.2: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let $f : \Re_1 \to \Re_2$ be a homomorphism onto HX rings. Let $(A + \lambda^{\mu})^p$ be the pseudo fuzzy coset of a fuzzy HX left ideal λ^{μ} of a HX ring $\Re_1$ determined by the element $A \in \Re_1$.Then $f((A + \lambda^{\mu})^p)$ is the pseudo fuzzy coset of a fuzzy HX left ideal $f(\lambda^{\mu})$ of a HX ring $\Re_2$ determined by the element $f(A) \in \Re_2$ and $f((A + \lambda^{\mu})^p) = (f(A) + f(\lambda^{\mu}))^p$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof: Let $f : \Re_1 \to \Re_2$ be a homomorphism onto HX rings.

Let $(A + \lambda^{\mu})^p$ be the pseudo fuzzy coset of a fuzzy HX left ideal λ^{μ} of a HX ring $\Re_1$ determined by the element $A \in \Re_1$.By theorem, if λ^{μ} is a fuzzy HX left ideal of $\Re_1$ then $f(\lambda^{\mu})$ is a fuzzy HX left ideal of $\Re_2$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

$f((A + \lambda^{\mu})^p)$ is the pseudo fuzzy coset of a fuzzy HX left ideal $f(\lambda^{\mu})$ of a HX ring $\Re_2$ determined by the element $f(A) \in \Re_2$.

Let A, X $\in \Re_1$, then f(A), f(X)$\in \Re_2$.

Now, $(f(A) + f(\lambda^{\mu}))^{p} f(X) = p(f(A))(f(\lambda^{\mu})(f(X))$
$= p(A)\, \lambda^{\mu}(X)$
$= (A + \lambda^{\mu})^{P}(X)$
$= f((A + \lambda^{\mu})^{P})f(X).$

$(f(A) + f(\lambda^{\mu}))^{p} f(X)= f((A + \lambda^{\mu})^{P})f(X),$ for any f(X)$\in \Re_2$.

Hence, $f((A + \lambda^{\mu})^{P}) = (f(A) + f(\lambda^{\mu}))^{p}$.

Theorem 3.3: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f : $\Re_1 \to \Re_2$ be a homomorphism onto HX rings. Let $(A + \lambda^{\mu})^{P}$ be the pseudo fuzzy coset of a fuzzy HX ideal λ^{μ} of a HX ring $\Re_1$ determined by the element A$\in \Re_1$. Then $f((A + \lambda^{\mu})^{P})$ is the pseudo fuzzy coset of a fuzzy HX ideal $f(\lambda^{\mu})$ of a HX ring $\Re_2$ determined by the element f(A)$\in \Re_2$ and $f((A + \lambda^{\mu})^{P}) = (f(A) + f(\lambda^{\mu}))^{p}$,
if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof: It is clear.

Theorem 3.4: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f : $\Re_1 \to \Re_2$ be a homomorphism on HX rings. Let $(B + \eta^{\alpha})^{P}$ be the pseudo fuzzy coset of a fuzzy HX right ideal η^{α} of a HX ring $\Re_2$ determined by the element B$\in \Re_2$.Then $f^{-1}((B + \eta^{\alpha})^{P})$ is the pseudo fuzzy coset of a fuzzy HX right ideal $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B)\in \Re_1$ and

$f^{-1}((B + \eta^{\alpha})^{P}) = (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^{p}$.

Proof: Let f: $\Re_1 \to \Re_2$ be a homomorphism on HX rings.

Let $(B + \eta^{\alpha})^{P}$ be the pseudo fuzzy coset of a fuzzy HX right ideal η^{α} of a HX ring $\Re_2$ determined by the element B$\in \Re_2$. By Theorem, if η^{α} is a fuzzy HX right ideal of $\Re_2$ then $f^{-1}(\eta^{\alpha})$ is a fuzzy HX right ideal of $\Re_1$. $f^{-1}((B + \eta^{\alpha})^{P})$ is the pseudo fuzzy coset of a fuzzy HX right ideal $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B)\in \Re_1$.

Let X $\in \Re_1$ and B$\in \Re_2$.

Now, $(f^{-1}(B) + f^{-1}(\eta^{\alpha}))^{p} (X)= p(f^{-1}(B))(f^{-1}(\eta^{\alpha}))(X)$
$= p(B)(\eta^{\alpha}(f(X))$
$= (B + \eta^{\alpha})^{P}f(X)$
$= f^{-1}((B + \eta^{\alpha})^{P})(X)$

$(f^{-1}(B) + f^{-1}(\eta^{\alpha}))^{p} (X) = f^{-1}((B + \eta^{\alpha})^{P})(X)$

Hence, $f^{-1}((B + \eta^{\alpha})^{P}) = (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^{p}$.

Theorem 3.5: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f : $\Re_1 \to \Re_2$ be a homomorphism on HX rings. Let $(B + \eta^{\alpha})^{P}$ be the pseudo fuzzy coset of a fuzzy HX left ideal η^{α} of a HX ring $\Re_2$ determined by the element B$\in \Re_2$.Then $f^{-1}((B + \eta^{\alpha})^{P})$ is the pseudo fuzzy coset of a fuzzy HX left ideal $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B) \in \Re_1$ and $f^{-1}((B + \eta^{\alpha})^{P}) = (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^{p}$.

Proof: Let f : $\Re_1 \to \Re_2$ be a homomorphism on HX rings.

Let $(B + \eta^{\alpha})^{P}$ be the pseudo fuzzy coset of a fuzzy HX left ideal η^{α} of a HX ring $\Re_2$ determined by the element B$\in \Re_2$.By Theorem if η^{α} is a fuzzy HX left ideal of $\Re_2$ then

$f^{-1}(\eta^{\alpha})$ is a fuzzy HX left ideal of $\Re_1$. $f^{-1}((B + \eta^{\alpha})^{P})$ is the pseudo fuzzy coset of a fuzzy HX left ideal $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B)\in \Re_1$.

Let X $\in \Re_1$ and B$\in \Re_2$.

Now, $(f^{-1}(B) + f^{-1}(\eta^{\alpha}))^{p} (X)= p(f^{-1}(B))(f^{-1}(\eta^{\alpha}))(X)$
$= p(B)(\eta^{\alpha}(f(X))$
$= (B + \eta^{\alpha})^{P}f(X)$
$= f^{-1}((B + \eta^{\alpha})^{P})(X)$

$(f^{-1}(B) + f^{-1}(\eta^{\alpha}))^{p} (X) = f^{-1}((B + \eta^{\alpha})^{P})(X)$

Hence, $f^{-1}((B + \eta^{\alpha})^{P}) = (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^{p}$.

Theorem 3.6: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f: $\Re_1 \to \Re_2$ be a homomorphism on HX rings. Let $(B + \eta^{\alpha})^{P}$ be the pseudo fuzzy coset of a fuzzy HX ideal η^{α} of a HX ring $\Re_2$ determined by the element

B$\in \Re_2$.Then $f^{-1}((B + \eta^{\alpha})^{P})$ is the pseudo fuzzy coset of a fuzzy HX ideal $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B)\in \Re_1$ and $f^{-1}((B + \eta^{\alpha})^{P}) = (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^{p}$.

Proof: It is clear.

Theorem 3.7: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f : $\Re_1 \to \Re_2$ be an anti homomorphism onto HX rings. Let $(A + \lambda^{\mu})^{P}$ be the pseudo fuzzy coset of a fuzzy HX right ideal λ^{μ} of a HX ring $\Re_1$ determined by the element A$\in \Re_1$.Then $f((A + \lambda^{\mu})^{P})$ is the pseudo fuzzy coset of a fuzzy HX left ideal $f(\lambda^{\mu})$ of a HX ring $\Re_2$ determined by the element f(A)$\in \Re_2$ and $f((A + \lambda^{\mu})^{P}) = (f(A) + f(\lambda^{\mu}))^{p}$, if λ^{μ} has a supremum property and λ^{μ} is f-invariant.

Proof: Let f : $\Re_1 \to \Re_2$ be an anti homomorphism onto HX rings. Let $(A + \lambda^{\mu})^{P}$ be the pseudo fuzzy coset of a fuzzy HX right ideal λ^{μ} of a HX ring $\Re_1$ determined by the element

A∈ $\Re_1$. By Theorem, if λ^μ is a fuzzy HX right ideal of $\Re_1$, then f(λ^μ) is a fuzzy HX left ideal of $\Re_2$, if λ^μ has a supremum property and λ^μ is f-invariant.

f($(A + \lambda^\mu)^P$) is the pseudo fuzzy coset of a fuzzy HX left ideal f(λ^μ) of a HX ring $\Re_2$ determined by the element f(A)∈ $\Re_2$.

Let A, X ∈ $\Re_1$, then f(A), f(X)∈ $\Re_2$.

Now, $(f(A) + f(\lambda^\mu))^p f(X) = p(f(A))(f(\lambda^\mu)(f(X)))$
$= p(A)\, \lambda^\mu(X)$
$= (A + \lambda^\mu)^P(X)$
$= f((A + \lambda^\mu)^P)f(X).$

$(f(A) + f(\lambda^\mu))^p f(X) = f((A + \lambda^\mu)^P)f(X),$ for any f(X)∈ $\Re_2$.

Hence, $f((A + \lambda^\mu)^P) = (f(A) + f(\lambda^\mu))^p$.

Theorem 3.8: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f : $\Re_1 \to \Re_2$ be an anti homomorphism onto HX rings. Let $(A + \lambda^\mu)^P$ be the pseudo fuzzy coset of a fuzzy HX left ideal λ^μ of a HX ring $\Re_1$ determined by the element A∈ $\Re_1$.Then f($(A + \lambda^\mu)^P$) is the pseudo fuzzy coset of a fuzzy HX right ideal f(λ^μ) of a HX ring $\Re_2$ determined by the element f(A)∈ $\Re_2$ and $f((A + \lambda^\mu)^P) = (f(A) + f(\lambda^\mu))^p$, if λ^μ has a supremum property and λ^μ is f-invariant.

Proof: Let f : $\Re_1 \to \Re_2$ be an anti homomorphism onto HX rings. Let $(A + \lambda^\mu)^P$ be the pseudo fuzzy coset of a fuzzy HX left ideal λ^μ of a HX ring $\Re_1$ determined by the element A∈ $\Re_1$.By Theorem if λ^μ is a fuzzy HX left ideal of $\Re_1$, then f(λ^μ) is a fuzzy HX right ideal of $\Re_2$, if λ^μ has a supremum property and λ^μ is f-invariant. f($(A + \lambda^\mu)^P$) is the pseudo fuzzy coset of a fuzzy HX right ideal f(λ^μ) of a HX ring $\Re_2$ determined by the element f(A)∈ $\Re_2$.

Let A, X ∈ $\Re_1$, then f(A), f(X)∈ $\Re_2$.

Now,$(f(A) + f(\lambda^\mu))^p f(X) = p(f(A))(f(\lambda^\mu)(f(X)))$
$= p(A)\, \lambda^\mu(X)$
$= (A + \lambda^\mu)^P(X)$
$= f((A + \lambda^\mu)^P)f(X)$

$(f(A) + f(\lambda^\mu))^p f(X) = f((A + \lambda^\mu)^P)f(X),$ for any f(X)∈ $\Re_2$.

Hence, $f((A + \lambda^\mu)^P) = (f(A) + f(\lambda^\mu))^p$.

Theorem 3.9: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f : $\Re_1 \to \Re_2$ be an anti homomorphism onto HX rings. Let $(A + \lambda^\mu)^P$ be the pseudo fuzzy coset of a fuzzy HX ideal λ^μ of a HX ring $\Re_1$ determined by the element A∈ $\Re_1$.Then f($(A + \lambda^\mu)^P$) is the pseudo fuzzy coset of a fuzzy HX ideal f(λ^μ) of a HX ring $\Re_2$ determined by the element f(A)∈ $\Re_2$ and $f((A+ \lambda^\mu)^P) = (f(A) + f(\lambda^\mu))^p$, if λ^μ has a supremum property and λ^μ is f-invariant.

Proof: It is clear.

Theorem 3.10: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f : $\Re_1 \to \Re_2$ be an anti homomorphism on HX rings. Let $(B + \eta^\alpha)^P$ be the pseudo fuzzy coset of a fuzzy HX right ideal η^α of a HX ring $\Re_2$ determined by the element B∈ $\Re_2$.Then $f^{-1}((B + \eta^\alpha)^P)$ is the pseudo fuzzy coset of a fuzzy HX left ideal $f^{-1}(\eta^\alpha)$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B)$∈ $\Re_1$ and $f^{-1}((B + \eta^\alpha)^P) = (f^{-1}(B) + f^{-1}(\eta^\alpha))^p$.

Proof: Let f: $\Re_1 \to \Re_2$ be an anti homomorphism on HX rings.Let $(B + \eta^\alpha)^P$ be the pseudo fuzzy coset of a fuzzy HX right ideal η^α of a HX ring $\Re_2$ determined by the element

B∈ $\Re_2$.By Theorem η^α be a fuzzy HX right ideal of $\Re_2$ then

$f^{-1}(\eta^\alpha)$ is a fuzzy HX left ideal of $\Re_1$. $f^{-1}((B + \eta^\alpha)^P)$ is the pseudo fuzzy coset of a fuzzy HX left ideal $f^{-1}(\eta^\alpha)$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B)$∈ $\Re_1$.

Let X ∈ $\Re_1$ and B∈ $\Re_2$.

Now, $(f^{-1}(B) + f^{-1}(\eta^\alpha))^p (X) = p(f^{-1}(B))(f^{-1}(\eta^\alpha))(X)$
$= p(B)(\eta^\alpha(f(X))$
$= (B + \eta^\alpha)^P f(X)$
$= f^{-1}((B + \eta^\alpha)^P)(X)$

$(f^{-1}(B) + f^{-1}(\eta^\alpha))^p (X) = f^{-1}((B + \eta^\alpha)^P)(X).$

Hence,$f^{-1}((B + \eta^\alpha)^P) = (f^{-1}(B) + f^{-1}(\eta^\alpha))^p$.

Theorem 3.11: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively. Let f : $\Re_1 \to \Re_2$ be an anti homomorphism on HX rings. Let $(B + \eta^\alpha)^P$ be the pseudo fuzzy coset of a fuzzy HX left ideal η^α of a HX ring $\Re_2$ determined by the element B∈ $\Re_2$.Then $f^{-1}((B + \eta^\alpha)^P)$ is the pseudo fuzzy coset of a fuzzy HX right ideal $f^{-1}(\eta^\alpha)$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B)$∈ $\Re_1$ and $f^{-1}((B + \eta^\alpha)^P) = (f^{-1}(B) + f^{-1}(\eta^\alpha))^p$.

Proof: Let f: $\Re_1 \to \Re_2$ be an anti homomorphism on HX rings. Let $(B + \eta^\alpha)^P$ be the pseudo fuzzy coset of a fuzzy HX left ideal η^α of a HX ring $\Re_2$ determined by the element

B∈ $\Re_2$.By Theorem, if η^α be a fuzzy HX left ideal of $\Re_2$ then

$f^{-1}(\eta^\alpha)$ is a fuzzy HX right ideal of $\Re_1$. $f^{-1}((B + \eta^\alpha)^P)$ is the pseudo fuzzy coset of a fuzzy HX right ideal $f^{-1}(\eta^\alpha)$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B)$∈ $\Re_1$.

Let $X \in \Re_1$ and $B \in \Re_2$.

Now, $(f^{-1}(B) + f^{-1}(\eta^{\alpha}))^{p}(X) = p(f^{-1}(B))(f^{-1}(\eta^{\alpha}))(X)$

$= p(B)(\eta^{\alpha}(f(X))$

$= (B + \eta^{\alpha})^{P} f(X)$

$= f^{-1}((B + \eta^{\alpha})^{P})(X)$

$(f^{-1}(B) + f^{-1}(\eta^{\alpha}))^{p}(X) = f^{-1}((B + \eta^{\alpha})^{P})(X)$

Hence, $f^{-1}((B + \eta^{\alpha})^{P}) = (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^{p}$.

Theorem 3.12: Let $\Re_1$ and $\Re_2$ be any two HX rings on the rings R_1 and R_2 respectively.Let f: $\Re \to \Re$ be an anti homomorphism on HX rings. Let $(B+ \eta^{\alpha})^{P}$ be the pseudo fuzzy coset of a fuzzy HX ideal η^{α} of a HX ring $\Re_2$ determined by the element $B \in \Re_2$.Then $f^{-1}((B + \eta^{\alpha})^{P})$ is the pseudo fuzzy coset of a fuzzy HX ideal $f^{-1}(\eta^{\alpha})$ of a HX ring $\Re_1$ determined by the element $f^{-1}(B) \in \Re_1$ and $f^{-1}((B + \eta^{\alpha})^{P}) = (f^{-1}(B) + f^{-1}(\eta^{\alpha}))^{p}$.

Proof: It is clear.

REFERENCES

1. Zadeh, L.A., 1965. Fuzzy sets, Information and Control, 8: 338-353.
2. Liu, W.J., 1982. Fuzzy invariant subgroups and fuzzy ideals, Fuzzy Sets and Systems, 8: 133-139.
3. Li Hong Xing and HX ring, 1988. BUSEFAL, 34(1): 3-8, January 1988.
4. Bing-xueYao and Yubin-Zhong, 2007. The construction of power ring, Fuzzy information and Engineering (ICFIE), ASC, 40: 181-187.
5. Bing-xueYao and Yubin-Zhong, 2009. Upgrade of algebraic structure of ring, Fuzzy information and Engineering, 2: 219-228.
6. Vasantha Kandasamy, W.B., 2003. Smarandahe Fuzzy Algebra, American ResearchPress.
7. Manikandan, K.H., 2013. Muthuraj.R, Pseudo Fuzzy Cosets of a HX Group,Applied Mathematical Sciences, 7(86): 4259-4271.
8. Mashinchi, M. and M.M. Zahedi, 1990. On fuzzy ideals of a ring, J. Sci. I.R. Iran, 1(3): 208-210.
9. Mukherjee, T.K and M.K. Sen, 1987. On fuzzy ideals in rings, Fuzzy sets and Systems, 21: 99-104.
10. Muthuraj, R, N. Ramila Gandhi, 2014. Homomorphism and anti homomorphism of fuzzy HX ideals of a HX ring, Discovery, 21(65):,July 1, 2014, pp: 20-24.
11. Muthuraj, R., 2015. Ramila Gandhi.N, Pseudo fuzzy cosets of a HX ring, SS international Journal of Pure and Applied Mathematics, 1(1): 22-30.
12. Muthuraj, R. and N. Ramila Gandhi, 2016. Properties of fuzzy HX ideal of a HX ring , International Journal of Mathematics and its Applications, 4(2-B): 41-50.
13. Rajesh Kumar, 1993. Fuzzy Algebra, Publication division, University of Delhi, (1993).
14. Rosenfeld. A., 1971. Fuzzy groups, J. Math. Anal., 35(1971): 512-517.
15. Young Bae Jun, Kyoung Ja Lee, 2012. Generalised fuzzy cosets and radicals, Applied Mathematical Sciences, 6(126): 6273-6280.