Middle-East Journal of Scientific Research 25 (2): 348-351, 2017
ISSN 1990-9233

DOI: 10.5829/idosi.mejsr.2017.348.351

On Supra $\tilde{g}$ -Closed Sets

[1]G. Ramkumar, [2]S. Suriaprakash and [3]S. Tharmar

[1]Department of Mathematics, Arul Anandar College, Karumathur, Madurai District, Tamil Nadu, India
[2]Department of Mathematics, JeppiaarMaamallan Engineering College, Chennai, India
[3]Department of Mathematics, Mohamed Sathak Engineering College, Kilakarai, India

Abstract: In this paper, we offer the notation of supra $\tilde{g}$ -closed sets, supra *g- closed sets, supra gsp-closed sets in topological space and characterizations and properties of such new notions are studied. Also investigate the relationship with other supra closed sets like supra g-closed.2010 Mathematics Subject Classifications: 54A10, 54A20.

Key words: Supra $\tilde{g}$ closed set · Supra *g- closed set supra gsp-closed set and supra #gs-closed set supra topological space

INTRODUCTION

Quiet recently, the study of supra topological spaces is the order of the day. Many researchers are introducing many new notions and investigating the properties and characterizations of such new notions. In line with the research, in this paper we introduce the concepts of supra $\tilde{g}$ -closed set and study their basic properties and investigate several properties of the new notions.

Preliminaries: Throughout this paper (X,τ), (Y, σ) and (Z, ν) (or simply, X, Y and Z) denote topological spaces on which no separation axioms are assumed unless explicitly stated.

Definition 2.1 [1, 2]: Let X be a non-empty set. The subfamily $\mu \subseteq P(X)$ where P(X) is the power set of X is said to be a supra topology on X if $X \in \mu$ and μ is closed under arbitrary unions.

The pair (X, μ) is called a supra topological space.

The elements of μ are said to be supra open in (X, μ).

Complements of supra open sets are called supra closed sets.

Definition 2.2 [3]:
Let A be a subset of (X, μ). Then

The supra closure of a set A is, denoted by $cl^{\mu}(A)$, defined as
$cl^{\mu}(A) = \cap \{ B: B \text{ is a supra closed and } A \subseteq B\}$;

The supra interior of a set A is, denoted by $int^{\mu}(A)$, defined as
$int^{\mu}(A) = \cup\{G: G \text{ is a supra open and } A \supseteq G\}$.

Definition 2.3 [1]: Let (X, τ) be a topological space and μ be a supra topology on X. We call μ is a supra topology associated with τif$\tau\subseteq\mu$.

Definition 2.4: Let (X,μ) be a supra topological space. A subset A of X is called

Supra semi-open set [3] if $A \subseteq cl^{\mu}(int^{\mu}(A))$;
Supraα-open set [3, 4] if $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$;
supra pre-open set [4] if $A \subseteq int^{\mu}(cl^{\mu}(A))$.
supraβ-open set [5] if $A \subseteq cl^{\mu}(int^{\mu}(cl^{\mu}(A)))$;

The complements of the above mentioned open sets are called their respective closed sets.

Definition 2.5: Let (X, μ) be a supra topological space. A subset A of X is called

Supra g-closed [6] if $cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ).

Supra sg-closed [7] if $scl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra semi-open in (X, μ).

Supra gs-closed [7] if $scl^{\mu}(A) \subseteq U$ whenever $A\subseteq U$ and U is supra open in (X, μ).

supra gα-closed (resp. supra αg-closed) [8] if $\alpha cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra α-open (resp. supra open) in (X, μ).

Corresponding Author: G. Ramkumar, Department of Mathematics, Arul Anandar College, Karumathur, Madurai District, Tamil Nadu, India.

Supra ω-closed [9] if $cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra semi-open in (X, μ).

Supra *g-closed [10] if $cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra ω-open in (X, μ).

3. Supra $\tilde{g}$ -closed set

Definition 3.1: Let (X, μ) be a topological space. A subset A of X is called

(i) a supra $\tilde{g}$ s-closed set if $scl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra $^{\#}$gs -open in X.
(ii) a supra $\tilde{g}$ -closed set if $cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra $^{\#}$gs -open in X.

Proposition 3.2: Every supra closed is supra $\tilde{g}$ -closed.

Proof: Let $A \subseteq X$ be supra closed set and $A \subseteq U$, where U is supra $^{\#}$gs-open. Since A is supra closed $A=cl^{\mu}(A) \subseteq U$. Thus A is supra $\tilde{g}$ -closed.

Converse of the above proposition need not be true as seen from the following example.

Example 3.3: Let X= {a,b,c,d} with μ = {X, ϕ, {a},{a,c}, {b,c}, {a,b,c}}. It is evident that {a,b,d} is supra $\tilde{g}$ -closed set but not supra-closed.

Proposition 3.4: Every supra $\tilde{g}$ -closed set is supra ω-closed.

Proof: It follows from the fact that, every supra semi-open set is supra$^{\#}$gs-open

Proposition 3.5: Every supra ω-closed set is supra g-closed but not conversely.

Proof: It follows from the fact that, every supra-open set is supra semi-open.

Example 3.6: Consider Example 3,3. Here {a,c,d} is supra g-closed but not supra ω-closed.

Proposition 3.7: Every supra $\tilde{g}$ -closed set is suprags-closed, supra gsp-closed and hence supra rg-closed but not conversely.

Proof: Let A be supra $\tilde{g}$ -closed set. Then by proposition 3.4 and 3.5, A is supra g-closed. It is evident that every supra g-closed set is supra gs-closed, supra gsp-closed and supra rg-closed. By proposition 3.5. A is supra gs-closed, supra gsp-closed and supra rg-closed.

Example 3.8: Consider the Example 3.3. Here {c,d} is supra gs-closed supra gsp-closed and supra rg-closed set but not supra $\tilde{g}$ -closed set.

Proposition: 3.9: Every supra $\tilde{g}$ -closed set. is supra $\tilde{g}$ s-closed but not conversely.

Proof: It follows from the fact that $scl^{\mu}(A) \subseteq cl^{\mu}(A)$

Example 3.10: Consider the Example 3.3, Here {b} is supra $\tilde{g}$ s-closed set but not supra $\tilde{g}$ -closed.

Proposition 3.11: Every supra $\tilde{g}$ -closed set is supra $^{\#}$gs-closed but not conversely.

Proof: Every supra $\tilde{g}$ s-closed is supra $^{\#}$gs-closed and by Proposition 3.9, the proof follows.

Example 3.12: Consider Example 3.3, Here {a} is supra $^{\#}$gs-closed but not supra $\tilde{g}$ -closed.

Proposition 3.13: Every supra ω-closed set is supra sg-closed but not conversely.

Proof: It follows from the fact that every supra semi-open set is supra $^{\#}$gs-open and $scl^{\mu}(A) \subseteq cl^{\mu}(A)$.

Example 3.14: Let X= {a,b,c,d} with μ= {X, ϕ,{a,d},{b,d}, {a,b,d},{{a,c,d}. It is evident that {a} is supra sg-closed but not supra ω-closed.

Proposition 3.15: Every supra ω-closed set is supra gα-closed but not conversely.

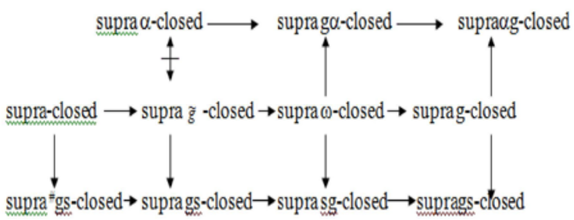
Proof: It follows from the fact, every supra open set is supra semi-open and $\alpha cl^{\mu}(A) \subseteq cl^{\mu}(A)$.

Example 3.16: Consider Example 3.14. Here {a} is supra gα-closed set but not supra ω-closed.

Remark 3.17: The following Example shows that supra $\tilde{g}$ -closed sets are independent of supraα-closed sets and supra semi-closed sets.

Example 3.18: Let X={a,b,c,d} with μ={X,ϕ,{a}, {b,c}, {a,c}, {a,b,c}}.Here {a,b,d} is supra $\tilde{g}$ -closed set but not supra α-closed and supra semi-closed. Also {b} is supra α-closed and supra semi closed but not supra $\tilde{g}$ -closedset.

Remark 3.19: The above discussions, we have the following diagram



Theorem 3.20: Let (X,μ) be supra topological space. Aset A is supra $\tilde{g}$ -closed set then $cl^{\mu}(A)\setminus A$ contains no non empty closed set in X.

Proof: Let (X,μ) be supra topological space. Suppose that A is supra $\tilde{g}$ -closed. Let F be a supra closed subset of $cl^{\mu}(A)\setminus A$. Then $A\subseteq F^c$. But A is supra $\tilde{g}$ -closed. Therefore, $cl^{\mu}(A)\subseteq F^c$.Consequently, $F \subseteq (cl^{\mu}(A))^c$. We have $F \subseteq cl^{\mu}(A)$. Thus $F \subseteq cl^{\mu}(A) \cap (cl^{\mu}(A))^c$and F is empty.

The converse of the above theorem is not true.

Example 3.21: Consider Example 3.3. If A={c,d} then $cl^{\mu}(A)\setminus A$={b} does not contain non empty closed set, But A is not supra $\tilde{g}$ -closed set.

Theorem 3.22: Let (X,μ) be supra topological space. A set A is supra $\tilde{g}$ -closed if and only if $cl^{\mu}(A)\setminus A$ contains no non empty supra #gs-closed set

Proof

Necessity: If A is supra $\tilde{g}$ -closed. Let F be a supra #gs –closed set such that $F \subseteq cl^{\mu}(A)\setminus A$. Since F^c is supra #gs-open and $A\subseteq F^c$. Since A is supra $\tilde{g}$ -closed. We have $cl^{\mu}(A) \subseteq F^c$.consequently, $F \subseteq (cl^{\mu}(A))^c$. This implies that $F \subseteq cl^{\mu}(A) \cap (cl^{\mu}(A))^c=\phi$

Sufficiency: Let $A \subseteq U$, where U is supra #gs-open. If $cl^{\mu}(A)$ is not contained in U. Then $cl^{\mu}(A) \cap U^c \neq \phi$. Now because $cl^{\mu}(A) \cap U^c \subseteq cl^{\mu}(A)\setminus A$ and $cl^{\mu}(A) \cap U^c$ is a non- empty supra #gs closed. We obtain contradiction. Therefore $cl^{\mu}(A) \subseteq U$. Hence A is supra $\tilde{g}$ -closed.

Theorem 3.23: Let (X, μ) be supra topological space. A set A is supra $\tilde{g}$ -closed and $A \subseteq B \subseteq cl^{\mu}(A)$ then B is supra $\tilde{g}$ -closed.

Proof: Let A be supra $\tilde{g}$ -closed. Since $B \subseteq cl^{\mu}(A)$, we have $cl^{\mu}(B) \subseteq cl^{\mu}(A)$. Then $cl^{\mu}(B)\setminus B \subseteq cl^{\mu}(A)\setminus A$. Since $cl^{\mu}(A) \setminus A$ has no non-emptysupra #gs-closed subset neither does $cl^{\mu}(B)\setminus B$. By theorem 3.22, B is supra $\tilde{g}$ -closed.

Proposition 3.24: Let (X, μ) be supra topological space. Let $A \subseteq Y \subseteq X$ and suppose that A is supra $\tilde{g}$ -closed then A is supra $\tilde{g}$ -closed relative to Y.

Proof: Let $A \subseteq Y\cap F$, where F is supra #gs –open. $A \subseteq F$ and hence $cl^{\mu}(A) \subseteq F$.This implies that $Y\cap cl^{\mu}(A) \subseteq Y\cap G$. Thus A is supra $\tilde{g}$ -closed relative to Y.

Proposition 3.25: Let (X,μ) be supra topological space. If A is a supra #gs-open and supra $\tilde{g}$ -closed set. Then A is supra closed in X.

Proof: Since A is supra #gs–openand supra $\tilde{g}$ -closed,$cl^{\mu}(A) \subseteq A$ and hence A is supra closed in X.

Definition 3.26: Let (X,μ) be supra topological space A subset A is said to be supra locally closed if $A= U\cap F$. Where U is supra open and F is supra closed in X.

Theorem 3.27: Let (X,μ) be supra topological space. The following properties are equivalent

A is supra closed.
A is supra locally closed and supra $\tilde{g}$ -closed.
A is supra locally closed and supra g-closed

Proof: It is obivious that (i) => (ii) =>(iii).

Now we have to show the implication (iii) => (i). Suppose that A is supra locally closed and supra g-closed then it follows that $A \cup (X\setminus cl^{\mu}(A))$ is an supra open set of X, since A is supra locally closed. Since A is supra g-closed and $A \subseteq A\cup(X \setminus cl^{\mu}(A))$,we obtain $cl^{\mu}(A) \subseteq A\cup(X\setminus cl^{\mu}(A))$. Thus we have $cl^{\mu}(A) \subseteq A$ and hence A is supra closed.

4. Supra $\tilde{g}$ -open set.

Definition: 4.1: Let (X,μ) be supra topological space. A set A is called supra $\tilde{g}$ -open in X if A^c is supra $\tilde{g}$ -closed in X.

Theorem 4.2: Let (X, μ) be supra topological space. A set A is supra $\tilde{g}$ -open in X if and only if $F \subseteq int^{\mu}(A)$ whenever F is supra #gs-closed and $F \subseteq A$.

Proof: Suppose $F \subseteq int^{\mu}(A)$ where F is supra #gs-closed and $F \subseteq A$. Let $A^c\subseteq G$ where G is supra #gs-open. Then $G^c\subseteq A$ and $G^c\subseteq int^{\mu}(A)$. Thus A^c is supra $\tilde{g}$ -closed set in X. Hence A is supra $\tilde{g}$ -open in X.

Conversely suppose that A is supra $\tilde{g}$ -open $F \subseteq A$ and F is supra $^{\#}$gs-closed in X. Then F^c is supra$^{\#}$gs-open and $A^c \subseteq F^c$. There fore $cl^{\mu}(A^c) \subseteq F^c$. But $cl^{\mu}(A^c) = (int^{\mu}(A))^c$. Hence $F \subseteq int^{\mu}(A)$.

Theorem 4.3: Let (X, μ) be supra topological space. A set A is supra $\tilde{g}$ -open in X if and only if G=X whenever G is supra $^{\#}$gs-open and $int^{\mu}(A) \cup A^c \subseteq G$.

Proof: Let A be supra $\tilde{g}$ -open, G be supra #gs-open and $int^{\mu}(A) \cup A^c \subseteq G$. This given $G^c \subseteq (int^{\mu}(A))^c \cap (A^c)^c = (int^{\mu}(A))^c \backslash A^c = cl^{\mu}(A^c) \backslash A^c$. Since $A^c$is supra $\tilde{g}$ -closed and $G^c$is supra #gs-closed by Theorem 3.22, it follows that $G^c = \phi$ therefore X=G.

Conversely suppose that F is supra $^{\#}$gs-closed and $F \subseteq A$. Then $int^{\mu}(A) \cup A^c \subseteq int^{\mu}(A) \cup F^c$. It follows that $int^{\mu}(A) \cup F^c = X$ and hence $F \subseteq int^{\mu}(A)$.Therefore A is supra $\tilde{g}$ -open.

Proposition 4.4: Let (X, μ) be supra topological space if $int^{\mu}(A) \subseteq B \subseteq A$ and A is supra $\tilde{g}$ -open in X, then B is supra $\tilde{g}$ open.

Proof: Suppose $int^{\mu}(A) \subseteq B \subseteq A$ and supra $\tilde{g}$ -open in X. Then $A^c \subseteq B^c \subseteq cl^{\mu}(A^c)$ and since A^c is supra $\tilde{g}$ -closed by Theorem 3.23, B is supra $\tilde{g}$ -open in X

Theorem 4.5: Let (X,μ) be supra topological space. A set A is supra $\tilde{g}$ -closed set. If and if only if $cl^{\mu}(A) \backslash A$ is supra $\tilde{g}$ -open.

Proof

Necessity: Suppose that A is supra $\tilde{g}$ -closed in X. Let $F \subseteq cl^{\mu}(A) \backslash A$ where F is supra $^{\#}$gs-closed. By Theorem 3.22, $F \neq \phi$. Therefore $F \subseteq int^{\mu}(cl^{\mu}(A) \backslash A)$ and by Theorem 4.2$cl^{\mu}(A) \backslash A$ is supra $\tilde{g}$ -open.

Sufficiency: Let $A \subseteq U$ and U be supra $^{\#}$gs-open set then $cl^{\mu}(A) \cap U^c \subseteq cl^{\mu}(A) \cap A^c = cl^{\mu}(A) \backslash A$. Since $cl^{\mu}(A) \cap U^c$is supra $^{\#}$gs-closed set and $cl^{\mu}(A) \backslash A$ is supra $\tilde{g}$ -open, by Theorem 4.2. We have $cl^{\mu}(A) \cap U^c \subseteq int^{\mu}(cl^{\mu}(A) \backslash A) = \phi$, This shows that $cl^{\mu}(A) \subseteq U$. Hence A is supra $\tilde{g}$ -closed set.

Theorem 4.6: Let (X, μ) be supra topological space. For a subset A of X thefollowing are equivalent

A is supra $\tilde{g}$ -closed

$cl^{\mu}(A) \backslash A$ contains no non empty supra #gs-closed set

$cl^{\mu}(A) \backslash A$ is supra $\tilde{g}$ -open

Proof: The proof follows from theorem 3.23 and 4.5.

REFERENCES

1. Mashhour, A.S., A.A. Allam, F.S. Mahmoud and F.H. Khedr, 1983. On supra topological spaces, Indian J. Pure and Appl. Math., 14(4): 502-510.
2. Sayed, O.R. and T. Noiri, 2010. On supra b-open sets and supra b-continuity on topological spaces, European J. Pure and Applied Math., 3(2): 295-302
3. Devi, R., S. Sampathkumar and M. Caldas, 2008. On supra α-open sets and sα- continuous maps, General Mathematics, 16(2): 77-84.
4. Sayed, O.R., 2010. Supra pre-open sets and supra pre-continuity on topological spaces, Vasile Alecsandri, University of Bacau, Faculty of Sciences, Scientific Studies and Research Series Mathematics and Informatics., 20(2): 79-88.
5. Ravi, O., G. Ramkumar and M. Kamaraj, 2011. On supra β-open sets and supra β-continuity on topological spaces, Proceed. National Seminar held at Sivakasi, India, pp: 22-31.
6. Ravi, O., G. Ramkumar and M. Kamaraj, 2011. On supra g-closed sets, International Journal of Advances in Pure and Applied Mathematics, 1(2): 52-66.
7. Kamaraj, M., G. Ramkumar and O. Ravi, 2011. Suprasg-closed sets and Supra gs-closed sets, International Journal of Mathematical Archive, 2(11): 52-66.
8. Kamaraj, M., G. Ramkumar and O. Ravi, 2012. Two classes of supra generalized closed sets, Global Journal of Advances in Pure and Applied Mathematics, 1(1): 24-33.
9. Ramkumar, G. and O. Ravi, 2015. Decomposition of supra continuity,Proceed. National Seminar held at Kovilpatti India, pp: 32-39.
10. Ramkumar, G., O. Ravi and M. Joseph Israel, 2016. supra g#-closed set and its related maps Mathematical Sciences International Research Journal, 5: 71-75.