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A Study on Intuitionistic Multi-Fuzzy Ideals in BG-Algebra

[1]R. Muthuraj and [2]S. Devi

[1]PG and Research Department of Mathematics,
H.H. The Rajah's College, Pudhukottai-622001, Tamilnadu, India
[2]Department of Mathematics, PSNA College of Engineering and Technology,
Dindigul-624622, Tamilnadu, India

Abstract: The purpose of this paper is to implement the concept of intuitionistic multi-fuzzy sets to ideals in BG-algebra. In this paper, we introduce the notion of intuitionistic multi-fuzzy ideals, intuitionistic multi-fuzzy closed ideals in BG-algebra and investigate some of their related properties. Also we discuss the relation between intuitionistic multi-fuzzy ideals and intuitionistic multi-fuzzy closed ideals of BG-algebra. Finally we define the upper level subset of intuitionistic multi-fuzzy ideals of BG-algebra and study some of its properties based on (α, β)-cut.

Key words: BG-algebra · BG-ideal · Multi-fuzzy set · Intuitionistic multi-fuzzy set · Intuitionistic multi-fuzzy BG-ideal · Intuitionistic multi-fuzzy closed ideal · Level subset · Homomorphism

INTRODUCTION

The notion of a fuzzy subset was initially introduced by Zadeh [1] in 1965, for representing uncertainity. The idea of intuitionistic fuzzy set was first published by Atanassov [2], as a generalization of the notion of the fuzzy set. In 2000, S.Sabu and T.V.Ramakrishnan [3, 4] proposed the theory of multi-fuzzy sets in terms of multi-dimensional membership functions and investigated some properties of multi-level fuzziness. Theory of multi-fuzzy set is an extension of theory of fuzzy sets. Complete characterization of many real life problems can be done by multi-fuzzy membership functions of the objects involved in the problem.

Y. Imai and K. Iseki introduced two classes of abstract algebras: BCK algebras and BCI-algebras [5-7]. It is shown that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. J.Neggers and H.S.Kim [8] introduced a new notion, called a B-algebra. In 2005, C.B.Kim and H.S.Kim [9] introduced the notion of a BG-algebra which is a generalization of B-algebras. With these ideas, fuzzy subalgebras of BG-algebra were developed by S.S.Ahn and H.D.Lee [10]. R.Muthuraj and S.Devi [11, 12] introduced the concept of multi- fuzzy subalgebras and intuitionistic multi-fuzzy subalgebras in BG-algebra in 2016. In this paper, we define a new algebraic structure of intuitionistic multi-fuzzy ideals and intuitionistic multi-fuzzy closed ideals of BG-algebra and discuss some of their related properties based on level subsets. Also we investigate the properties of intuitionistic multi-fuzzy ideals of BG-algebra under homomorphism.

Preliminaries: In this section, the basic definitions of a BG-algebra, BG-ideal, multi-fuzzy sets are recalled. We start with.

Definition 2.1 [9, 10]: Let X be a non-empty set. A multi-fuzzy set A in X is defined as a set of ordered sequences: $A = \{(x, \mu_1(x), \mu_2(x), ..., \mu_k(x)): x \in X\}$, where $\mu_i: X \rightarrow [0, 1]$ for all i. Here k is called the dimension of A.

Definition 2.2 [8]: Let X be a non-empty set. An intuitionistic fuzzy set(IFS) A in X is a set of the form $A = \{ (x, \mu(x), \nu(x)): x \in X \}$, where $\mu: X \rightarrow [0, 1]$ and $\nu: X \rightarrow [0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively, with $0 \leq \mu(x) + \nu(x) \leq 1$.

Corresponding Author: R. Muthuraj, PG and Research Department of Mathematics, H.H. The Rajah's College, Pudhukottai-622001, Tamilnadu, India.

Definition 2.3 [11]: Let X be a non-empty set . An intuitionistic multi-fuzzy set A in X is a set of the form $A = \{ (x, \mu_A(x), \nu_A(x)): x \in X \}$, where $\mu_A(x) = (\mu_1(x), \mu_2(x), ..., \mu_k(x))$, $\nu_A(x) = (\nu_1(x), \nu_2(x), ..., \nu_k(x))$ and each $\mu_i: X \to [0, 1]$, $\nu_i: X \to [0, 1]$ with $0 \le \mu_i(x) + \nu_i(x) \le 1$, $x \in X$ for all i = 1, 2, ……..k. Here $\mu_1(x) \ge \mu_2(x) \ge ...... \ge \mu_k(x)$, $x \in X$.

Remark: Note that although we arrange the membership sequence in decreasing order, the corresponding non-membership sequence need not be in decreasing or increasing order.

Definition 2.4 [5]: A non-empty set X with a constant 0 and a binary operation "*" is called a BG-algebra if it satisfies the following axioms:

- $x * x = 0$
- $x * 0 = x$
- $(x * y) * (0 * y) = x, \forall\ x, y \in X.$

Example 2.5: Let X = {0, 1, 2} be a set with the following table:

Table 2.1

*	0	1	2
0	0	1	2
1	1	0	1
2	2	2	0

Then (X; $\in$, 0) is a BG-algebra.

Definition 2.6 [5]: Let S be a non-empty subset of a BG-algebra X, then S is called a subalgebra of X if $x \in y \in S$ for all $x, y \in S$.

Definition 2.7: Let X be a BG-algebra and I be a subset of X. Then I is called a BG-ideal of X if it satisfies the following conditions:

- $0 \in I$
- $x * y \in I$ and $y \in I \Rightarrow x \in I$
- $x \in I$ and $y \in X \Rightarrow x * y \in I$

Definition 2.8 [6]: Let μ be a fuzzy set in a BG-algebra X. Then μ is called a fuzzy subalgebra of X if $\mu(x * y) \ge \min \{\mu(x), \mu(y)\}$, $x, y \in X$.

Definition 2.9 [13]: An intuitionistic multi-fuzzy subset $A = \{ (x, \mu_A(x), \nu_A(x)): x \in X \}$ in X is called an intuitionistic multi-fuzzy subalgebra of X if it satisfies:

- $\mu_A (x * y) \ge \min \{ \mu_A(x), \mu_A(y) \}$
- $\nu_A (x * y) \le \max \{ \nu_A (x), \nu_A(y) \}$, $x, y \in X$

Definition 2.10: A mapping $f: X \to Y$ of a BG-algebra is called a homomorphism if $f(x * y) = f(x) * f(y)$, $x, y \in X$.

Remark: If $f: X \to Y$ is a homomorphism of BG-algebra then $f(0) = 0$.

Definition 2.11: Let X and Y be any two non-empty sets and $f: X \to Y$ be a mapping. Let A and B be any two IMF subsets of X and Y respectively having the same dimension k. Then the pre-image of March 18, 2017B($\subseteq$Y) under the map f is denoted by $f^{-1}(B)$ and it is defined as: $f^{-1}(B) = (\mu_B(f(x)), \nu_B(f(x)))$, $x \in X$.

Definition 2.12 [11]: Let $A = \{ (x, \mu_A(x), \nu_A(x)): x \in X \}$ and $B = \{(x, \mu_B(x), \nu_B(x)): x \in X \}$ be any two IMFS's having the same dimension k of X. Then.

- $A \subseteq B$ if and only if, $\mu_A(x) \le \mu_B(x)$, $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$.
- $A = B$ if and only if $\mu_A(x) = \mu_B(x)$, $\nu_A(x) = \nu_B(x)$ for all $x \in X$.
- $A \cap B = \{ (x, (\mu_{A\cap B})(x), (\nu_{A\cap B})(x)): x \in X\}$, where $(\mu_{A\cap B})(x) = \min \{ \mu_A(x), \mu_B(x) \} = (\min\{ \mu_{iA}(x), \mu_{iB}(x)\})^k_{i=1}$ and $(\nu_{A\cap B})(x) = \max \{\nu_A(x), \nu_B(x) \} = (\max \{\nu_{iA}(x), \nu_{iB}(x)\})^k_{i=1}$
- $A \cup B = \{ (x, (\mu_{AUB})(x), (\nu_{AUB})(x)): x \in X \}$, where $(\mu_{AUB})(x) = \max\{ \mu_A(x), \mu_B(x) \} = (\max\{\mu_{iA}(x), \mu_{iB}(x) \})^k_{i=1}$ and $(\nu_{AUB})(x) = \min \{ \nu_A(x), \nu_B(x) \} = (\min \{ \nu_{iA}(x), \nu_{iB}(x) \})^k_{i=1}$

Intuitionistic Multi-Fuzzy Ideals of BG-Algebra: In this section, we define the new notion of intuitionistic multi-fuzzy ideals and intuitionistic multi-fuzzy closed ideals of BG-algebra and discuss some of its properties.

Definition 3.1: An intuitionistic multi-fuzzy subset $A = \{(x, \mu_A(x), \nu_A(x)): x \in X \}$ in X is called an intuitionistic multi-fuzzy ideal of X if it satisfies:

- $\mu_A(0) \ge \mu_A(x)$ and $\nu_A(0) \le \nu_A(x)$
- $\mu_A (x) \ge \min \{\mu_A(x * y), \mu_A(y)\}$
- $\nu_A (x) \le \max \{\nu_A (x * y), \nu_A(y)\} \forall x, y \in X$

Example 3.2: Consider a BG-algebra X = { 0, 1, 2, 3 }with the following table.

Table 3.1

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Let $A = (\mu_A, \nu_A)$ be an intuitionistic multi-fuzzy subset in X defined as follows:

$\mu_A(0) = \mu_A(1) = (1, 1)$, $\mu_A(2) = \mu_A(3) = (0.7, 0.5)$
and $\nu_A(0)= \nu_A(1) = (0, 0)$, $\nu_A(2) = \nu_A(3) = (0.2, 0.3)$

Then A is an intuitionistic multi-fuzzy ideal of X.

Theorem 3.3: Let $A = (\mu_A, \nu_A)$ be an intuitionistic multi-fuzzy ideal of a BG-algebra X. If $x * y \leq z$ then $\mu_A(x) \geq \min\{\mu_A(y), \mu_A(z)\}$, $\nu_A(x) \leq \max\{\nu_A(y), \nu_A(z)\}$ for all $x, y, z \in X$.

Proof:
Let $x, y, z \in X$ such that $x * y = z$.
Then $(x * y) * z = 0$

$$\begin{aligned}\mu_A(x) &\geq \min\{\mu_A(x * y), \mu_A(y)\}\\ &\geq \min\{\min\{\mu_A((x * y)*z), \mu_A(z)\}, \mu_A(y)\}\\ &= \min\{\min\{\mu_A(0), \mu_A(z)\}, \mu_A(y)\}\\ &= \min\{\mu_A(y)\}, \mu_A(z)\}\end{aligned}$$

$$\begin{aligned}\nu_A(x) &\leq \max\{\nu_A(x * y), \nu_A(y)\}\\ &= \max\{\max\{((\nu_A(x * y) * z), \nu_A(z)\}, \nu_A(y)\}\\ &= \max\{\max\{\nu_A(0), \nu_A(z)\}, \nu_A(y)\}\\ &= \max\{\nu_A(y), \nu_A(z)\}\end{aligned}$$

Theorem 3.4: Let $A = (\mu_A, \nu_A)$ be an intuitionistic multi-fuzzy ideal of a BG-algebra X. If $x \leq y$ then $\mu_A(x) \geq \mu_A(y)$, $\nu_A(x) \leq \nu_A(y)$ for all $x, y \in X$.

Proof:
Let $x, y \in X$ such that $x \leq y$. Then $x * y = 0$.

$$\begin{aligned}\mu_A(x) &\geq \min\{\mu_A(x * y), \mu_A(y)\}\\ &\geq \min\{\mu_A(0), \mu_A(y)\}\\ &= \mu_A(y)\end{aligned}$$

And

$$\begin{aligned}\nu_A(x) &\leq \max\{\nu_A(x * y), \nu_A(y)\}\\ &\leq \max\{\nu_A(0), \nu_A(y)\}\\ &= \nu_A(y)\end{aligned}$$

Theorem 3.5: Let $A = (\mu_A, \nu_A)$ and $B = (\mu_B, \nu_B)$ be two intuitionistic multi-fuzzy ideals of a BG-algebra X. Then the intersection $A \cap B$ is also an intuitionistic multi-fuzzy ideal of X.

Proof:
Let $x, y \in A\cap B$
Then $x, y \in A$ and $x, y \in B$

$$\begin{aligned}\mu_{A\cap B}(0) &= \mu_{A\cap B}(x * x)\\ &= \min\{\mu_A(x * x), \mu_B(x * x)\}\\ &\geq \min\{\min\{\mu_A(x), \mu_A(x)\}, \min\{\mu_B(x), \mu_B(x)\}\}\\ &= \min\{\mu_A(x), \mu_B(x)\}\\ &= \mu_{AnB}(x)\end{aligned}$$

$$\begin{aligned}\nu_{A\cap B}(0) &= \nu_{A\cap B}(x * x)\\ &= \max\{\nu_A(x * x), \nu_B(x * x)\}\\ &\leq \max\{\max\{\nu_A(x), \nu_A(x)\}, \max\{\nu_B(x), \nu_B(x)\}\}\\ &= \max\{\nu_A(x), \nu_B(x)\}\\ &= \nu_{A\cap B}(x)\end{aligned}$$

$$\begin{aligned}\mu_{A\cap B}(x) &= \min\{\mu_A(x), \mu_B(x)\}\\ &= \min\{\min\{\mu_A(x * y), \mu_A(y)\}, \min\{\mu_B(x * y), \mu_B(y)\}\}\\ &\geq \min\{\min\{\mu_A(x * y), \mu_B(x * y)\}, \min\{\mu_A(y), \mu_B(y)\}\}\\ &= \min\{\mu_{A\cap B}(x * y), \mu_{A\cap B}(y)\}\end{aligned}$$

$$\begin{aligned}\nu_{A\cap B}(x) &= \max\{\nu_A(x), \nu_B(x)\}\\ &= \max\{\max\{\nu_A(x * y), \nu_A(y)\}, \max\{\nu_B(x * y), \nu_B(y)\}\}\\ &= \max\{\max\{\nu_A(x * y), \nu_B(x /* y)\}, \max\{\nu_A(y), \nu_B(y)\}\}\\ &\leq \max\{\nu_{A\cap B}(x * y), \nu_{A\cap B}(y)\}\end{aligned}$$

Theorem 3.6: Let X be a BG-algebra. Then an intuitionistic multi-fuzzy set A is an intuitionistic multi-fuzzy ideal of X if and only if A is an intuitionistic multi-fuzzy subalgebra of X.

Proof: Every intuitionistic multi-fuzzy ideal of X is an intuitionistic multi-fuzzy subalgebra of X.

Conversely, let A be an intuitionistic multi-fuzzy subalgebra of X.

Let $x, y \in X$

$$\begin{aligned}\mu_A(0) &= \mu_A(x * x)\\ &\geq \min\{\mu_A(x), \mu_A(x)\} = \mu_A(x)\end{aligned}$$

$$\begin{aligned}\nu_A(0) &= \nu_A(x * x)\\ &\leq \max\{\nu_A(x), \nu_A(x)\}\\ &= \nu_A(x)\end{aligned}$$

$$\begin{aligned}\mu_A(x) &= \mu_A((x * y) * (0 * y))\\ &\geq \min\{\mu_A(x * y), \mu_A(0 * y)\}\\ &\geq \min\{\mu_A(x * y), \min\{\mu_A(0), \mu_A(y)\}\}\\ &= \min\{\mu_A(x * y), \mu_A(y)\}\end{aligned}$$

$$\begin{aligned}\nu_A(x) &= \nu_A((x * y) * (0 * y))\\ &\leq \max\{\nu_A(x * y), \nu_A(0 * y)\}\\ &\leq \max\{\nu_A(x * y), \max\{\nu_A(0), \nu_A(y)\}\}\\ &= \max\{\nu_A(x * y), \nu_A(y)\}\end{aligned}$$

Definition 3.7: An intuitionistic multi-fuzzy subset $A = \{(x, \mu_A(x), \nu_A(x)): x \in X\}$ in X is called an intuitionistic multi-fuzzy closed ideal of X if it satisfies:

- $\mu_A(0* x) \geq \mu_A(x)$ and $\nu_A(0*x) \leq \nu_A(x)$
- $\mu_A(x) \geq \min \{ \mu_A(x * y), \mu_A(y) \}$
- $\nu_A(x) \leq \max \{ \nu_A(x * y), \nu_A(y) \} \forall x, y \in X$

Example 3.8: Consider a BG-algebra X = {0, 1, 2, 3} with the following table:

Table 3.2

*	0	1	2	3
0	0	1	2	3
1	1	0	1	1
2	2	2	0	2
3	3	3	3	0

Let A = (μ_A, ν_A) be an intuitionistic multi-fuzzy subset in X defined as follows:

$\mu_A(0) = \mu_A(1) = (0.8, 0.5)$, $\mu_A(2) = \mu_A(3) = (0.4, 0.3)$
and $\nu_A(0) = \nu_A(1) = (0.2, 0.3)$, $\nu_A(2) = \nu_A(3) = (0.5, 0.4)$

Then A is an intuitionistic multi-fuzzy closed ideal of X.

Theorem 3.9: Every intuitionistic multi-fuzzy closed ideal of a BG-algebra X is an intuitionistic multi-fuzzy ideal of X.

Proof:
Let A = (μ_A, ν_A) be an intuitionistic multi-fuzzy closed ideal of X.
It is enough to prove that $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$
Now, $\mu_A(0) \geq \min \{ \mu_A(0*x), \mu_A(x) \}$
$\geq \min \{ \mu_A(x), \mu_A(x) \} = \mu_A(x)$
$\nu_A(0) \leq \max \{ \nu_A(0*x), \nu_A(x) \}$
$\leq \max \{ \nu_A(x), \nu_A(x) \}$
$= \nu_A(x)$

Remark: The converse of the above thorem is not true.
Let us prove this by the following example.

Example 3.10: Let Consider a BG-algebra X= { 0, 1, 2, 3 }with the following table:

Table 3.3

*	0	1	2	3
0	0	3	2	1
1	1	0	3	2
2	2	1	0	3
3	3	2	1	0

Let A = (μ_A, ν_A) be an intuitionitic multi-fuzzy subset in X defined as follows:
$\mu_A(0) = (0.8, 0.6)$, $\mu_A(1) = (0.6, 0.4)$,
$\mu_A(2) = \mu_A(3) = (0.5, 0.3)$
and $\nu_A(0) = (0.1, 0.2)$, $\nu_A(1) = (0.2, 0.4)$,
$\nu_A(2) = \nu_A(3) = (0.4, 0.6)$

Then A is an intuitionistic multi-fuzzy ideal of X but it is not an intuitionistic multi-fuzzy closed ideal of X.

Corollary 3.11: Every intuitionistic multi-fuzzy subalgebra satisfying the conditions $\mu_A(x) \geq \min \{ \mu_A(x * y), \mu_A(y) \}$, $\nu_A(x) \leq \max \{\nu_A(x * y), \nu_A(y) \}$, is an intuitionistic multi-fuzzy closed ideal.

Theorem 3.12: Every intuitionistic multi-fuzzy closed ideal of a BG-algebra X is an intuitionistic multi-fuzzy subalgebra of X.

Proof:
Let A = (μ_A, ν_A) be an intuitionistic multi-fuzzy closed ideal of X.
Then $\mu_A(x*y) \geq \min \{ \mu_A((x*y)*(0*y), \mu_A(0 *y)\}$
$= \min \{ \mu_A(x), \mu_A(0 *y) \}$
$\geq \min \{ \mu_A(x), \mu_A(y) \}$
$\nu_A(x*y) \leq \max \{ \nu_A ((x*y)*(0*y)), \nu_A (0*y)\}$
$= \max \{ \nu_A (x), \nu_A (0 *y) \}$
$\leq \max \{ \nu_A (x), \nu_A (y) \}$

Preoposition 3.13: If an intuitionistic multi-fuzzy set A = (μ_A, ν_A) in X is an intuitionistic multi-fuzzy closed ideal, then for all $x \in X$, $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$

Proof: Straight forward.
Definition 3.14: Let A = { (x, $\mu_A(x)$, $\nu_A(x)$): $x \in X$ } be an intuitionistic multi-fuzzy subset in X. Then the (α, β)-cut of A is denoted by $[A]_{(\alpha, \beta)}$ and is defined by $[A]_{(\alpha, \beta)} = \{ x \in X: \mu_A(x) \geq \alpha$ and $\nu_A(x) \leq \beta \}$ where $\alpha = (\alpha_1, \alpha_2, \ldots\alpha_k)$ and $\beta = (\beta_1, \beta_2, \ldots\beta_k)$ where each $\alpha_i, \beta_i \in [0, 1]$ with $0 \leq \alpha_i + \beta_i \leq 1$ for all i = 1, 2, ….k such that $\mu_i(x) \geq \alpha_i$ with the corresponding $\nu_i(x) \leq \beta_i$ for all i = 1, 2, …..k.

Theorem 3.15: If A is an intuitionistic multi-fuzzy ideal of X, then the subset $[A]_{(\alpha, \beta)}$ is an BG-ideal in X.

Proof:

- Since A = (μ_A, ν_A) is an intuitionistic multi-fuzzy ideal in X, $\mu_A(0) \geq \mu_A(x) \geq \alpha$

and $\nu_A(0) \leq \nu_A(x) \leq \beta$.
Then $0 \in [A]_{(\alpha, \beta)}$

- Let $x * y \in [A]_{(\alpha, \beta)}$ and $y \in [A]_{(\alpha, \beta)}$

Then $\mu_A(x * y) \geq \alpha$, $\nu_A(x * y) \leq \beta$ and
$\mu_A(y), \geq \alpha$, $\nu_A(y) \leq \beta$
$\mu_A(x) \geq \min \{ \mu_A(x * y), \mu_A(y) \}$
$\geq \min \{ \alpha, \alpha \} = \alpha$
and $\nu_A(x) \leq \max \{ \nu_A(x * y), \nu_A(y) \}$
$\leq \min \{ \beta, \beta \} = \beta$
This implies that $x \in [A]_{(\alpha, \beta)}$

- Let $x \in [A]_{(\alpha, \beta)}$ and $y \in X$

Choose y in X such that $\mu_A(y) \geq \alpha$, $\nu_A(y) \leq \beta$
$\mu_A(x*y) \geq \min \{ \mu_A(x), \mu_A(y) \}$
$\geq \min \{ \alpha, \alpha \} = \alpha$
and $\nu_A(x*y) \leq \max \{ \nu_A(x), \nu_A(y) \}$
$\geq \max \{ \beta, \beta \} = \beta$
This implies that $x * y \in [A]_{(\alpha, \beta)}$
Hence the subset $[A]_{(\alpha, \beta)}$ is a BG-ideal in X.

Theorem 3.16: Let X be a BG-algebra . If the set $[A]_{(\alpha, \beta)}$ is a BG-ideal in X then an intuitionistic multi-fuzzy set A = (μ_A, ν_A) is an intuitionistic multi-fuzzy ideal in X .

Proof: Let $[A]_{(\alpha, \beta)}$ be a BG-ideal in X.

Assume that A = (μ_A, ν_A) is not an intuitionistic multi-fuzzy ideal in X.

Then there exists $a, b \in X$ such that $\mu_A(a) < \min \{ \mu_A(a*b), \mu_A(b) \}$ and $\nu_A(a) > \max \{ \nu_A(a*b), \nu_A(b) \}$ hold.
Let $\alpha = [\mu_A(a) + \min \{\mu_A(a*b), \mu_A(b)\}] / 2$, $\beta = [\nu_A(a) + \max\{\nu_A(a*b), \nu_A(b)\}] / 2$
Then $\mu_A(a) < \alpha < \min \{ \mu_A(a*b), \mu_A(b) \}$ and
$\nu_A(a) > \beta > \max\{\nu_A(a*b), \nu_A(b) \}$
This implies that $a*b, b \in [A]_{(\alpha, \beta)}$ but $\mu_A(a) < \alpha$ and $\nu_A(a) > \beta$
That is, $a \notin [A]_{(\alpha, \beta)}$ which is a contradiction that $A]_{(\alpha, \beta)}$ is a BG-ideal of X.
Therefore $\mu_A(x) \geq \min \{ \mu_A(x * y), \mu_A(y) \}$ and $\nu_A(x) \leq \max \{ \nu_A(x * y), \nu_A(y) \}$ Hence A = (μ_A, ν_A) is an intuitionistic multi-fuzzy ideal in X.

Properties of Intuitionistic Multi-fuzzy Ideals under Homomorphism: In this section, we study the properties of intuitionistic multi-fuzzy ideals under homomorphism.

Theorem 4.1: Let f: X→Y is a BG-homomorphism of BG-algebras. If B = (μ_B, ν_B) is an intuitionistic multi-fuzzy ideal of Y then the pre-image $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B))$ of B under f is an intuitionistic multi-fuzzy ideal in X.

Proof:
For any $x \in X$,
i) $f^{-1}(\mu_B)(x) = \mu_B(f(x)) \leq \mu_B(0) = \mu_B(f(0)) = f^{-1}(\mu_B)(0)$
And $f^{-1}(\nu_B)(x) = \nu_B(f(x)) \geq \nu_B(0) = \nu_B(f(0)) = f^{-1}(\nu_B)(0)$
ii) $f^{-1}(\mu_B)(x) = \mu_B(f(x))$
$\geq \min \{ \mu_B(f(x) * f(y)), \mu_B(f(y)) \}$
$= \min \{ \mu_B(f(x*y)), \mu_B(f(y)) \}$
$= \min \{ f^{-1}(\mu_B)(x*y), f^{-1}(\mu_B)(y) \}$
$f^{-1}(\nu_B)(x) = \nu_B(f(x))$
$\leq \max \{ \nu_B(f(x) * f(y)), \nu_B(f(y)) \}$
$= \max \{ \nu_B(f(x*y)), \nu_B(f(y)) \}$
$= \max \{ f^{-1}(\nu_B)(x*y), f^{-1}(\nu_B)(y) \}$

Theorem 4.2: Let f: X → Y be an epimorphism of BG-algebra. Then B = (μ_B, ν_B) is an intuitionistic multi-fuzzy ideal of Y, if $f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\nu_B))$ of B under f is an intuitionistic multi-fuzzy ideal in X.

Proof:
For any $x \in Y$, there exists $a \in X$ such that f(a) = x. Then $\mu_B(x) = \mu_B(f(a)) = f^{-1}(\mu_B)(a) \leq f^{-1}(\mu_B)(0) = \mu_B(f(0)) = \mu_B(0)$ $\nu_B(x) = \nu_B(f(a)) = f^{-1}(\nu_B)(a) \leq f^{-1}(\nu_B)(0) = \nu_B(f(0)) = \nu_B(0)$.
ii) Let $x, y \in Y$
Then f(a) = x and f(b) = y for some $a, b \in X$
$\mu_B(x) = \mu_B(f(a))$
$= f^{-1}(\mu_B)(a)$
$\geq \min \{ f^{-1}(\mu_B(a*b)), f^{-1}(\mu_B(b)) \}$
$= \min \{ \mu_B(f(a*b)), \mu_B(f(b)) \}$
$= \min \{ \mu_B(f(a)*f(b)), \mu_B(f(b)) \}$
$= \min \{ \mu_B(x*y), \mu_B(y) \}$
$\nu_B(x) = \nu_B(f(a))$
$= f^{-1}(\nu_B)(a)$
$\leq \max \{ f^{-1}(\nu_B(a*b)), f^{-1}(\nu_B(b)) \}$
$= \max \{ \nu_B(f(a*b)), \nu_B(f(b)) \}$
$= \max \{ \nu_B(f(a)*f(b)), \nu_B(f(b)) \}$
$= \max \{ \nu_B(x*y), \nu_B(y) \}$

Hence the intuitionistic multi-fuzzy set B = (μ_B, ν_B) is an intuitionistic multi-fuzzy ideal of Y.

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