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Efficient Dominating Energy

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Abstract: Let G = (n, m) be a simple graph. The ordinary energy of the graph is defined as the sum of the absolute values of the eigen values of the adjacency matrix. Recently, based on eigen values of a variety of graph matrices, various energies are found. Here we introduce efficient dominating energy of a graph. The efficient dominating energy of some families of graph is found and its properties are determined.

Key words: Energy · Efficient dominating set · Efficient dominating matrix · Efficient dominating energy

INTRODUCTION

In the year 1978, I. Gutman [1, 2] introduced the concept of energy of a graph. In Hukel Molecular Orbital theory, total energy of π electrons is equal to sum of the energies of all π electrons in the molecule.

Let G be a simple graph with m edges and n vertices. Let A = (a_{ij}) be the adjacency matrix of the graph. The adjacency matrix is defined as,

$$(a_{ij}) = \begin{cases} 1 \text{ if } v_i \text{ is adjacent to } v_j \\ 0 \text{ otherwise} \end{cases}$$

Let $\lambda_1, \lambda_2, \ldots \lambda_n$ be the eigen values of the adjacency matrix such that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$. The energy of the graph is defined as $E(G) = \Sigma^n_{i=1} |\lambda_1|$. The various types of energies are defined like Laplacian energy, distance energy, incidence energy, Harrary energy in [3-5]. The Efficient dominating energy for simple graph is defined in this paper. The efficient dominating energy for some graphs is found.

Efficient Dominating Energy: Let G be a simple graph of order n with vertex set $V = \{v_1, v_2, \ldots. v_n\}$ and the edge set E. A vertex set D in G is an Efficient Dominating Set(ED) for G if for every vertex $v \in V$, there is exactly one $d \in D$ dominating V which is also called as independent perfect dominating set.

The Efficient dominating number is the minimum cardinality taken over all the minimal efficient dominating sets of G. Let ED be the minimum efficient dominating set of graph G. The minimum efficient dominating matrix of G is n x n matrix $A_{ED}(G) = (a_{ij})$ where,

$$(a_{ij}) = \begin{cases} 1 \text{ if } v_i \text{ and } v_j \text{ are adjacent} \\ 1 \text{ if } i = j,\ v_i \in EDS \\ 0 \text{ otherwise} \end{cases}$$

The characteristic polynomial of $A_{ED}(G)$ is denoted by $P(G, \lambda) = \det(\lambda I - A_{ED}(G))$. The minimum efficient dominating eigen values of the graph G are the eigen values of $A_{ED}(G)$. Since $A_{ED}(G)$ is real and symmetric, the eigen values are $\lambda_1, \lambda_2, \ldots, \lambda_n$ in the decreasing order $\lambda_1 \geq \lambda_2 \geq, \ldots \geq \lambda_n$. The Efficient dominating energy of G is defined as $E_{ED}(G) = \Sigma^n_{i=1} |\lambda_1|$

Example 2.1: Let G be 3 vertex cycle C_3, with vertices v_1, v_2, v_3 and let the minimum efficient dominating set be ED = $\{v_1\}$. Then $A_{ED}(G) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

The Characteristic polynomial is $x^3 - x^2 - 3x - 1$
The eigen values are $\lambda_1 = 2.4142$, $\lambda_2 = -1$, λ_3 - 0.4142
Therefore, the Efficient dominating energy is $E_{ED}(G)$ = 3.8284

Properties of Efficient Dominating Energy

Theorem 3.1: Let G = (n, m) be a simple graph. Let $\lambda_1, \lambda_2, \ldots \lambda_n$ be eigen values of $A_{ED}(G)$, then $\Sigma^n_{i=1} \lambda_1^2 = 2m + |ED|$

Proof: The sum of square of the eigen value of $A_{ED}(G)$ is the trace of $A_{ED}(G)^2$.

$$\sum_{i=1}^{n} \lambda_i^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} a_{ji}$$

$$= 2\sum_{i<j} (a_{ij})^2 + \sum_{i=1}^{n} (a_{ii})^2 = 2m + |ED|$$

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Theorem 3.2:
Let G = (n, m) be a simple graph. Let ED be an efficient dominating set of G. Then $E_{ED}(G) \leq \sqrt{n(2m+|ED|}$

Proof: Let $\lambda_1 \geq \lambda_2 \geq, ... \geq \lambda_n$ be the eigen values of $A_{ED}(G)$. By Cauchy-Schwarz inequality,

$$\sum_{i=1}^{n}(a_i b_i)^2 \leq \sum_{i=1}^{n}(a_i)^2 \sum_{i=1}^{n}(b_i)^2$$

Let $a_i = 1$ and $b_i = |\lambda_i|$,

$$EDE\,(G)^2 = \left(\sum_{i=1}^{n}|\lambda_i|\right)^2$$

$$\leq n\left(\sum_{i=1}^{n}|\lambda_i|^2\right) = n \sum_{i=1}^{n}\lambda_i^2 = n(2m+|ED|)$$

Theorem 3.3: Let G = (n, m) be a simple graph with efficient dominating set ED. Let $\eta = |\det A_{ED}(G)|$, then $E_{ED}(G) \geq \sqrt{2m+|ED|+n(n-1)\eta^{\frac{2}{n}}}$

Proof:

$$(E_{ED}(G))^2 = \left(\sum_{i=1}^{n}|\lambda_i|\right)^2$$

$$= \sum_{i=1}^{n}|\lambda_i|^2 + \sum_{i \neq j}|\lambda_i||\lambda_j| =$$

By using the relation between arithmetic and geometric mean, we obtain.

$$\frac{1}{n(n-1)}\sum_{i \neq j}|\lambda_i||\lambda_j| \geq \left(\prod_{i \neq j}|\lambda_i||\lambda_j|\right)^{\frac{1}{n(n-1)}}$$

$$\sum_{i \neq j}|\lambda_i||\lambda_j| \geq n(n-1)\left(\prod_{i \neq j}|\lambda_i|^{2(n-1)}\right)^{\frac{1}{n(n-1)}}$$

$$\geq n(n-1)\left(\prod_{i \neq j}|\lambda_i|\right)^{\frac{2}{n}}$$

$$\geq n(n-1)|\det A_{ED}(G)|^{\frac{2}{n}}$$

$$\sum_{i \neq j}|\lambda_i||\lambda_j| \geq n(n-1)\eta^{\frac{2}{n}}$$

$$(E_{ED}(G))^2 \geq \sum_{i=1}^{n}|\lambda_i|^2 + n(n-1)\eta^{\frac{2}{n}}$$

$$\geq 2m + |ED| + n(n-1)\eta^{\frac{2}{n}}$$

$$E_{ED}(G) \geq \sqrt{2m + |ED| + n(n-1)\eta^{\frac{2}{n}}}$$

Efficient Dominating Energy of Some Families of Graphs

Definitions 4.1: Wheel graph is a graph formed by connecting a single vertex to all vertices of a cycle. The wheel graph W_n can be defined as the graph $K_1 + C_{n-1}$, where K_1 is the singleton graph and C_n is the cycle graph.

Theorem 4.1: For $n \geq 4$, The efficient dominating energy of wheel graph is equal to (n-2) $+\sqrt{n^2-2n+5}$

Proof: The wheel graph with vertex set $\{v_1, v_2, \ldots v_n\}$, the Efficient dominating set ED = $\{v_1\}$

$$A_{ED}(W_n) = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 0 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 \end{pmatrix}$$

The Characteristic polynomial $\det(\lambda I - A_{ED}(W_n))$ =

$$\begin{vmatrix} \lambda-1 & -1 & -1 & \cdots & -1 & -1 \\ -1 & \lambda & -1 & \cdots & -1 & -1 \\ -1 & -1 & \lambda & \cdots & -1 & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & -1 & -1 & \cdots & \lambda & -1 \\ -1 & -1 & -1 & \cdots & -1 & \lambda \end{vmatrix}$$

$R_2 \rightarrow R_i - R_2$, i = 3, …n

$$\begin{vmatrix} \lambda-1 & -1 & -1 & \cdots & -1 & -1 \\ -1 & \lambda & -1 & \cdots & -1 & -1 \\ 0 & -(\lambda+1) & \lambda+1 & \cdots & 0 & 0 \\ 0 & -(\lambda+1) & 0 & \cdots & \lambda & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & -(\lambda+1) & 0 & \cdots & \lambda+1 & 0 \\ 0 & -(\lambda+1) & 0 & \cdots & 0 & \lambda+1 \end{vmatrix}$$

$C_2 \rightarrow \Sigma_{i=3}^{n} C_i + C_2$

$$\begin{vmatrix} \lambda-1 & -(n-1) & -1 & \cdots & -1 & -1 \\ -1 & \lambda & -1 & \cdots & -1 & -1 \\ 0 & 0 & \lambda+1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & \lambda & -1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda+1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \lambda+1 \end{vmatrix}$$

The Characteristic equation is $(\lambda+1)^{n-2}(\lambda^2 - \lambda(n-1) - 1)$

$\lambda = -1$, (n-2 times)

$\lambda = (n-1) \pm \sqrt{\frac{n^2-2n+5}{2}}$ (one time)

Efficient dominating energy, $E_{ED}(W_n)$ = (n-2) + $\sqrt{n^2-2n+5}$

Theorem 4.2: The efficient dominating energy of the complete graph K_n is equal to (n-2) + $\sqrt{n^2-2n+5}$

Proof: The complete graph K_n with vertex set V = {v_1, v_2, ….v_n}, the efficient dominating set ED = {v_1}, Then,

$$A_{ED}(K_n) = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 1 & \cdots & 1 & 1 \\ 1 & 1 & 0 & \cdots & 1 & 1 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 1 & 1 & 1 & \cdots & 0 & 1 \\ 1 & 1 & 1 & \cdots & 1 & 0 \end{pmatrix}$$

The calculations are as in wheel graph.

The efficient dominating energy E_{ED} (K_n) = (n-2) + $\sqrt{n^2 - 2n + 5}$

Theorem 4.3: The efficient dominating energy of star graph S_n is $\sqrt{4n-3}$

Proof: The star graph S_n with vertex set {v_1, v_2, ….v_n }, the efficient dominating set ED = {v_1}.

$$A_{ED}(S_n) = \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 & 1 \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}$$

$$\det(\lambda I - A_{ED}(S_n)) = = \begin{vmatrix} \lambda - 1 & -1 & -1 & \cdots & -1 & -1 \\ -1 & \lambda & 0 & \cdots & 0 & 0 \\ -1 & -1 & \lambda & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -1 & 0 & 0 & \cdots & \lambda & 0 \\ -1 & 0 & 0 & \cdots & 0 & \lambda \end{vmatrix}$$

The characteristic polynomial is $\lambda^{n-2}(\lambda^2 - \lambda + (1 - n)$

$\lambda = 0$ (n-2 times)

$$\lambda = \frac{1 \pm \sqrt{4n-3}}{2}$$

The Efficient dominating energy, E_{ED} (S_n) = $\sqrt{4n-3}$

CONCLUSION

The Efficient dominating energy of various families of graph is computed. Also, various properties of efficient dominating energy are obtained. In future, Efficient dominating energy of various families of graph can be computed. Various properties can be determined.

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