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An Approach for Solving Network Problem with Pentagonal Intuitionistic Fuzzy Numbers Using Ranking Technique

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Abstract: Critical path method is a network based method premeditated for scheduling and organization of complex project in real world application. A novel approach has been made to find the critical path in a directed acyclic graph, whose activity time is uncertain. The indistinguishable parameters in the network are represented by Intuitionistic pentagonal fuzzy numbers and include basic arithmetic operations like addition, subtraction, for Intuitionistic pentagonal fuzzy number. The problem is solved using a Ranking technique called Accuracy function for Pentagonal Intuitionistic fuzzy numbers.

Key words: Fuzzy sets · Fuzzy numbers · Intuitionistic fuzzy numbers · Pentagonal Intuitionistic fuzzy number · Fuzzy project network · Critical path

INTRODUCTION

Network optimization is a very popular and frequently applied field among the well studied areas of operations research. Many practical problems arising in the real life situations can be formulated as a network models. In reality, due to uncertainty of information as well as the variation of management scenario, it is often difficult to obtain the exact activity time estimates. Hence the fuzzy set theory proposed by Zadeh plays a vital role in this kind of decision-making environment. There were several methods reported to solve the fuzzy critical path (FCP) problem in the open literature [1-5]. In this paper we define Intuitionistic Pentagonal Fuzzy Number and include basic arithmetic operations like addition, subtraction for Intuitionistic Pentagonal Fuzzy Number. We present examples for the above defined operations between Intuitionistic Pentagonal Fuzzy Numbers and also Score and Accuracy Function of an Intuitionistic Pentagonal Fuzzy Numbers. Finally we give examples for Intuitionistic Pentagonal Fuzzy Number.

Preliminaries

Fuzzy Set: If X is a universal set, a fuzzy subset μ on a set X is a map $\mu: X \rightarrow [0, 1]$ and A(x) is interpreted as the degree of membership of element x in fuzzy A for each $x \in X$. The Value zero is used to represent complete non-membership; the value one is used to represent complete membership and values in between are used to represent intermediate degrees of membership. The mapping A is also called the membership function of fuzzy set A.

Intuitionistic Fuzzy Set: Let X is a universal set and then an Intuitionistic Fuzzy Set (IFS) A in X is given by $A = \{(x, \mu_A(x), \gamma_A(x)) / x \in X\}$, where the functions $\mu_A(x): X \rightarrow [0, 1]$ and $\gamma A(x): X \rightarrow [0, 1]$ determine the degree of membership and non-membership of the element $x \in X$,
$0 = \mu_A(x) + \gamma_A(x) = 1$.

Fuzzy Number: A fuzzy number A is a fuzzy set on the real line R, must satisfy the following conditions

- $\mu_A(x_0)$ is piecewise continuous
- There exist atleast one $x_0 \in R$ with $\mu_A(x_0) = 1$.
- A must be normal and convex.

Triangular Fuzzy Number: Triangular Fuzzy Number is defined as A= {a, b, c} where all a, b, c are real numbers and its membership function is given by,

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$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)} & \text{for } a \le x \le b \\ \frac{(c-x)}{(c-b)} & \text{for } b \le x \le c \\ 0 & \text{Otherwise.} \end{cases}$$

Trapezoidal Fuzzy Number [6]: A fuzzy set A= (a, b, c, d) is said to trapezoidal fuzzy number if its membership function is given by where a = b = c = d.

$$\mu_A(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{(x-a)}{(b-a)} & \text{for } a \le x \le b \\ 1 & \text{for } b \le x \le c \\ \frac{(d-x)}{(d-c)} & \text{for } c \le x \le d \\ 0 & \text{for } x > d \end{cases}$$

Pentagonal Fuzzy Number [7]: A Pentagonal Fuzzy Number of a fuzzy set A is defined as A_p= {a, b, c, d, e} and its membership function is given by

$$\mu_{A_p}(x) = \begin{cases} 0 & \text{for } x < a \\ \frac{(x-a)}{(b-a)} & \text{for } a \le x \le b \\ \frac{(x-b)}{(c-b)} & \text{for } b \le x \le c \\ 1 & x = c \\ \frac{(d-x)}{(d-c)} & \text{for } c \le x \le d \\ \frac{(e-x)}{(e-d)} & \text{for } d \le x \le e \\ 0 & \text{for } x > e \end{cases}$$

Intuitionistic Fuzzy Number: An Intuitionistic Fuzzy Set A_I is called an Intuitionistic Fuzzy Number if it satisfies the following conditions,

- A_I is normal i.e., there exists atleast two points x_0, $x_1 \in R$ such that $\mu_A(x_0) = 1$ and $\gamma_A(x_1) = 1$.
- A_I is convex i.e., its membership function is fuzzy convex and its non-membership function is concave.
- Its membership function is upper semi continuous and its non-membership function is lower semi continuous and the set A_I is bounded.

Pentagonal Intuitionistic Fuzzy Number: A Pentagonal Intuitionistic Fuzzy Number A^I of an Intuitionistic Fuzzy Set is defined as [8]

A^I={ (a_1,b_1, c_1, d_1, e_1) (a_2, b_2, c_2, d_2, e_2) } where all a_1, b_1, c_1, d_1, e_1, a_2, b_2, c_2, d_2,e_2 are real numbers and its membership function $\mu_A I$ (x), non-membership function $\gamma_A I$ (x) are given by,

$$\mu_{AI}(x) = \begin{cases} 0 & \text{for } x < a_1 \\ \frac{(x-a_1)}{(b_1-a_1)} & \text{for } a_1 \le x \le b_1 \\ \frac{(x-b_1)}{(c_1-b_1)} & \text{for } b_1 \le x \le c_1 \\ 1 & x = c_1 \\ \frac{(d_1-x)}{(d_1-c_1)} & \text{for } c_1 \le x \le d_1 \\ \frac{(e_1-x)}{(e_1-d_1)} & \text{for } d_1 \le x \le e_1 \\ 0 & \text{for } x > e_1 \end{cases}$$

$$\gamma_{AI}(x) = \begin{cases} 1 & \text{for } x < a_1 \\ \frac{(b_2-x)}{(b_2-a_2)} & \text{for } a_2 \le x \le b_2 \\ \frac{(c_2-x)}{(c_2-b_2)} & \text{for } b_2 \le x \le c_2 \\ 0 & x = c_1 \\ \frac{(x-c_2)}{(d_2-c_2)} & \text{for } c_2 \le x \le d_2 \\ \frac{(x-d_2)}{(e_2-d_2)} & \text{for } d_2 \le x \le e_2 \end{cases}$$

Arithmetic Operations of PIFN: Let A^I ={ (a_1,b_1, c_1, d_1, e_1) (a_2, b_2, c_2, d_2, e_2) } and B^I = {(a_3, b_3,c_3,d_3,e_3) (a_4, b_4,c_4,d_4,e_4)} be two Pentagonal Intuitionistic Fuzzy Numbers, then the arithmetic operations are as follows:

Addition of PIFN:

$A^I + B^I$ = {(a1 +a_3, $b_1 + b_3$, $c_1 + c_3$, $d_1 + d_3$, $e_1 + e_3$) ($a_2 + a_4$, $b_2 + b_4$, $c_2 + c_4$, $d_2 + d_4$, $e_2 + e_4$)}

Subtraction of PIFN:

$A^I - B^I$ = {(a1 - e_3, $b_1 - d_3$, $c_1 - c_3$, $d_1 - b_3$, $e_1 - a_3$) ($a_2 - e_4$, $b_2 - d_4$, $c_2 - c_4$, $d_2 - b_4$, $e_2 - a_4$)}

Ranking of Pifn Based on Accuracy Function: Accuracy function of a Pentagonal Intuitionistic Fuzzy Number

$A^I = \{ (a_1, b_1, c_1, d_1, e_1)\ (a_2, b_2, c_2, d_2, e_2) \}$ is defined as,

$$H(A^I) = \frac{(a_1+a_2 + b_1+b_2 + c_1+c_2 + d_1+d_2 + e_1+e_2)}{5}$$

Intuitionistic Fuzzy Critical Path Method: The following is the procedure for finding Intuitionistic Fuzzy Critical Path [9].

NOTATIONS

N: The set of all nodes in a project network.
EST: Earliest Starting Time.
EFT: Earliest Finishing Time.
LST: Latest Starting Time.
LFT: Latest Finishing Time.
TF: Total Float.

Forward Pass Calculation: Forward pass calculations are employed to calculate the Earliest Starting Time (EST) in the project network.

EFT = [EST + Intuitionistic fuzzy activity time]

Backward Pass Calculation: Backward pass calculations are employed to calculate the Latest Finishing Time (LFT) in the project network.

LST = [LFT + Intuitionistic fuzzy activity time]

Total Float:

TF = LFT - EFT (OR)
TF = LST – EST

Procedure to Find Intuitionistic Fuzzy Critical Path:

STEP 1: Construct a network G (V, E) where V is the set of vertices and E is the set of edges. Here G is an acyclic diagraph and arc length or edge weights are taken as Intuitionistic Pentagonal Fuzzy Numbers.

STEP 2: Expected time in terms of Intuitionistic Pentagonal Fuzzy Numbers are defuzzified in the network diagram.

STEP 3: Calculate Earliest Starting Time according to Forward pass calculation.

STEP 4: Calculate Earliest Finishing Time.

STEP 5: Calculate Latest Finishing Time according to Backward pass calculation.

STEP 6: Calculate Latest Starting Time.

STEP 7: Calculate Total Float.

STEP 8: In each activity whenever one get 0; such activities are called as Intuitionistic fuzzy critical activities [10] and the corresponding paths Intuitionistic critical paths [11].

Illustrative Example: Consider a small network with 6 vertices and 7 edges shown in the figure 1, where each arc length is represented as a Pentagonal Intuitionistic Fuzzy Number.

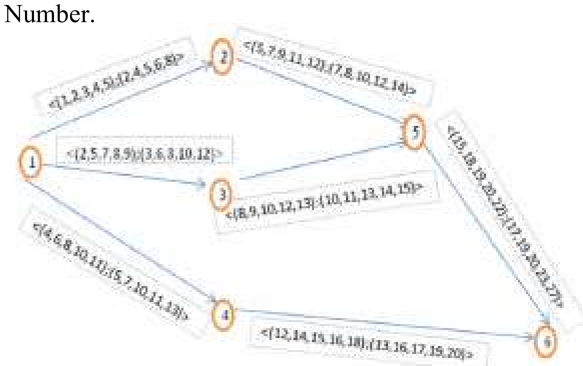

Fig. 1:

Table 1:

Activity	Intuitionistic Fuzzy Activity Time	Defuzzified Activity Time	Total Float
1-> 2	< (1,2,3,4,5) (2,4,5,6,8)>	8	10
1-> 3	< (2,5,7,8,9) (3,6,8,10,12)>	14	0
1->4	< (4,6,8,10,11) (5,7,10,11,13)>	17	28
2->5	< (5,7,9,11,12) (7,8,10,12,14)>	19	10
3-> 5	< (8,9,10,12,13) (10,11,13,14,15)>	23	0
4->6	< (12,14,15,16,18) (13,16,17, 19, 20)>	32	28
5->6	< (15,18,19,20,22) (17,19,20,23,27)>	40	0

Table 2:

sPATHS	IFCPM	RANK VALUE	RANK
1-2-5-6	< (21,27,31,35,39) (36,31,35,41,49)>	69	2
1-3-5-6	<(25,32,36,40,44) (30,36,41,47,54)>	77	1
1-4-6	<(16,20,23,26,30) (18,23,27,30,33)>	49.2	3

CONCLUSION

A method for finding critical path in an Intuitionistic fuzzy environment has been proposed. In this paper, we proposed Pentagonal Intuitionistic fuzzy number is defuzzified using Ranking method based on Accuracy function. Now the Intuitionistic fuzzy number is converted to crisp number. The critical paths for Intuitionistic fuzzy project network are 1-2-5-6, 1-3-5-6 and 1-4-6. Hence we conclude based on Accuracy function ranking method, the critical path is 1-3-5-6 and the project duration is 77.

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