Middle-East Journal of Scientific Research 24 (9): 2970-2972, 2016
ISSN 1990-9233

DOI: 10.5829/idosi.mejsr.2016.2970.2972

A Simple Note on EOQ Model for Buyer-Vendor with System Cost Using Algebraic Method

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Abstract: In this article, we analyze EOQ inventory model for buyer - vendor with supply chain. The model considered shortages for buyer alone and the total system cost is developed to benefit of both buyer and vendor. An algebraic derivation method is presented to solve the model. To widen the application of this approach, this proposed method finds the optimal lot size and backorder level for buyer-vendor. The goal is determined the optimal order quantity and backorder cost to minimize the total system cost. Finally, the results of the models are illustrated with some numerical examples.

Key words: Inventory · EOQ · Backorder · Order quantity · Algebraic Method

INTRODUCTION

The Economic Order Quantity (EOQ) inventory model was first initiated in February by 1913 by Harris [1]. Inventory models have been studied considerably since the economic order quantity (EOQ, also known as the Wilson lot size) is one of the most well known results of inventory theory. The various research efforts have been undertaken to expand the basic EOQ model so as to develop the models that conform real situation. Also, the EOQ models with/without backorder can be found in many literatures and textbooks. To derive the EOQ formula with/without shortages indicates to minimize the total relevant costs or total system cost.

Many researchers and academicians had been suggested different and changed methods of problem solving. Likewise, passing years, they have tried to derive the inventory models by making use of different approaches. The various mathematical methods are differential calculus, graphical representation, algebraic, arithmetic-geometric mean inequality, Cauchy Bunyakovsky-Schwartz (CBS) inequality etc. In most published papers that have derived using differential calculus to find the optimal solution. And this mathematical methodology is difficult to many younger students who lack the knowledge of calculus, derivatives and inequalities. So we choose some simple knowledge by using algebraic method.

A simple method to compute economic order quantities was studied by Cardenas-Barron [2]. Cardenas-Barron [3] has presented the derivation of EOQ/EPQ inventory models with two backorders costs by using analytic geometry and algebra. Samak-Kulkarni and Rajhans [4] has analyzed the determination of Optimum Inventory Model for Minimizing Total Inventory Cost. Teerapabolarn and Khamrod [5] had developed the inventory models with backorders and defective items derived algebraically and Arithmetic-Geometric Mean. And Hung – Chi Chang [6] analyzed a note on the EPQ model with shortage and variable lead time.

And then, Muniappan *et al*. [7] has developed a production inventory model for vendor–buyer coordination with quantity discount, backordering and rework for fixed life time products. Ravithammal *et al*. [8] studied a deterministic production inventory model for buyer- manufacturer with quantity discount and completely backlogged shortages for fixed life time product. Ravithammal *et al*. [9] developed an Integrated Production Inventory System for Perishable Items with Fixed and Linear Backorders. Muniappan *et al*. [10] concentrated an EOQ model for deteriorating items with inflation and time value of money considering time-dependent deteriorating rate and delay payments.

Therefore, in this paper, we provide easy-to-understand algebraic method approach without using derivatives to obtain the optimal solution.

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The detailed description of the paper is as follows: First we have given notations and then assumptions. Next the model formulated individually for buyer and vendor. Then analytically easily understandable solutions are obtained in this model. Next a numerical example is given in detail to illustrate the models. Finally conclusion and summary are presented.

Notations and Assumptions

Notations:

D = Demand rate per time unit,
r = Ordering cost per order,
B = Backorders level,
Q = Order quantity,
K = Setup cost per order,
h_b = Per unit holding cost per time unit,
h_v = Per unit setup cost per time unit,
c_1 = Per unit backorder cost per time unit
n = Vendors multiples of order

Assumptions:

(I) Demand rate is known and constant.
(ii) Lead time is zero.
(iii) Shortages are allowed for buyer only.
(iv) For the purpose of the equal benefits of buyer and vendor the system cost is developed.
The system cost can be written as $TC_{system} = TC_{buyer} + TC_{vendor}$

Model Formulation: The total annual cost for the buyer is formulated as ordering cost plus the annual average inventory holding cost plus backordering cost. Thus the total cost for buyer can be written as

$$TC_{buyer} = \frac{rD}{Q} + \frac{h_b(Q-B)^2}{2Q} + \frac{c_1B^2}{2Q} \quad (1)$$

The total annual cost for the vendor is formulated as setup cost plus the annual average inventory holding cost. Thus the total cost for vendor can be written as

$$TC_{vendor} = \frac{KD}{nQ} + \frac{h_v nQ}{2} \quad (2)$$

Now, the system cost can be written as

$$TC_{system} = TC_{buyer} + TC_{vendor}$$
$$= \frac{rD}{Q} + \frac{h_b(Q-B)^2}{2Q} + \frac{c_1B^2}{2Q} + \frac{KD}{nQ} + \frac{h_v nQ}{2} \quad (3)$$

In terms of B, equation (3) can be written as

$$TC_{system} = B^2\left[\frac{c_1}{2Q} + \frac{h_b}{2Q}\right] + B\left[\frac{-2Qh_b}{2Q}\right] + \frac{rD}{Q} + \frac{h_bQ^2}{2Q} + \frac{KD}{nQ} + \frac{nQh_v}{2} \quad (4)$$

Since, $B = \frac{a_2}{-2a_1}$ (by L. E. Cardenas Barron [3])

$$= \frac{-(-h_b)}{2\left(\frac{c_1}{2Q} + \frac{h_b}{2Q}\right)} = \frac{Qh_b}{h + c_1} \quad (5)$$

In terms of Q, equation (3) can be written as

$$TC_{system} = Q\left[\frac{h_b}{2} + \frac{nh_v}{2}\right] + \frac{1}{Q}\left[rD + \frac{h_bB^2}{2} + \frac{c_1B^2}{2} + \frac{KD}{n}\right] - h_bB \quad (6)$$

Since, $Q = \sqrt{\frac{a_2}{a_1}}$ (by L. E. Cardenas Barron [3])

$$= \sqrt{\frac{\frac{rD}{Q} + \frac{h_bB^2}{2} + \frac{c_1B^2}{2Q} + \frac{KD}{n}}{\frac{1}{2}(h_{b+nh_v})}}$$

$$= \sqrt{\frac{2D\left[r + \frac{K}{n}\right] + \frac{Q^2h_b^2}{c_1 + h_b}}{(h_{b+nh_v})}} \qquad \text{Since, } B = \frac{Qh_b}{c_1 + h_b}$$

$$= \sqrt{\frac{2D\left[r + \frac{K}{n}\right](c_1 + h_b)}{c_1h_b + (c_1 + h_b)nh_v}} \quad (7)$$

Numerical Example:

1. Given D = 500 units per year, K = 200$ per order, r = 100$ per order, h_v = 20$, h_b = 10$, n = 2, c_1 = 2.

The optimal solution is Q = 2343.2, B = 2702.7, $TC_{system} = 2.8429 \times 10^{10}$

2. Given D = 1000 units per year, K = 300$ per order, r = 100$ per order, h_v = 30$, h_b = 20$, n = 4, c_1 = 3 $.

The optimal solution is Q = 2343.2, B = 2702.7, $TC_{system} = 2.8429 \times 10^{10}$

CONCLUSION

This paper focused on EOQ model for buyer-vendor with total system cost. This model assumes with shortages for buyer alone and without shortages for vendor. There is numerous mathematical methods to derive the solution, here we preferred algebraic method to derive the optimal solution and backorder cost for buyer-vendor. The determined the total system cost by summing up the total cost of buyer and the total cost of vendor. In order to minimize the total system cost by applying the optimal order quantity and backorder cost for buyer-vendor. The developed paper reveals that to determine the total system cost provides equal benefits for both the buyer and the vendor. The numerical example is also given to illustrate the solution procedure. This approach could be used easily to introduce the basic inventory theories in EOQ model to younger students who lack the knowledge of calculus.

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