

## Standard Error of the Sample Autocorrelation as a Measure of ‘Non-Stationarity’

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**Abstract:** In this paper the feasibility of using Sample Autocorrelation Function (SACF) and the indices derived from SACF such as Standard Error (SE) of ACF as a measure of non-stationary of signals is investigated. Even though autocorrelation is a method widely used to analyze the non-stationary of signals, its association with non-stationarity is not objectively studied till now. Two sets of test signals are used for investigating the feasibility; in the first set of test signals different sinusoidal frequency components are concatenated at different scales to obtain different levels of non-stationarity. The second set of test signals are random sequences. The simulation of test signals and computations are performed in Matlab®. The value of SE is found to be robust to change in sampling rate and offset. Entropy is observed to be following the variation in non-stationarity of the signal. The SE and Absolute deviation of SE from the reciprocal of the square root of number of samples ( $\beta$ ) is not correlated with the entropy ( $r=-0.1200$ ). The dynamic variability of the SE and  $\beta$  is less when compared with the variability of entropy. This investigation emphasize on the need for reformulating the ACF based non-stationarity measures.

**Key words:** Non-stationarity • Autocorrelation • Standard error • Randomness • Variance

### INTRODUCTION

The term, ‘non-stationarity’ refers to the characteristic of a signal or a system, in which the statistical or spectral features change over time. In non-stationarity signals and systems statistical properties could be deterministic or non-deterministic functions of time. A stable system normally is stationary, provided it may turn to be non-stationary at any time. It is a well-known fact that ‘signals characterize the system’. Consequently, the study of non-stationarity of signals do have great important in system studies and signal processing.

In a random process  $X(t)$ , initiated at  $t = -8$ , given  $X(t_1), X(t_2) \dots X(t_k)$  denote the random variables obtained by observing the random process  $X(t)$  at the instants  $t_1, t_2, \dots, t_k$  respectively. The joint probability distribution function of this random variable is  $F_X(t_1), \dots, X(t_k)$ ,

$(x_1, \dots, x_k)$ . A shift of time ‘ $\tau$ ’ introduced in samples, yields a new set of random variables,  $X(t_{1+\tau}), X(t_{2+\tau}), \dots, X(t_{k+\tau})$ . So that, the joint probability distribution function of the latter set random variable is  $F_X(t_{1+\tau}, \dots, X(t_{k+\tau})) (x_1, \dots, x_k)$ . The random process  $X(t)$  is said to be stationary in the strict sense, strict stationary or strictly if the following condition in eq. (1) is met.

$$F_X(t_1, \dots, X(t_k)) (x_1, \dots, x_k) = F_X(t_1, \dots, X(t_k)) (x_1, \dots, x_k) \quad (1)$$

In other words, a random process  $X(t)$ , initiated at time  $t = -8$  is strictly stationary if the joint probability distribution function of any set of random variables obtained by observing the random process  $X(t)$  is invariant with respect to the origin at time instant,  $t=0$  [1]. This confirms that a stationary process is one whose statistical properties do not change over time. More clearly, all moments of all degrees, the expectations,

variances, third order and higher order moments of the process, at any part of the signal remain equal. A random process  $X(t)$  is said to be non-stationary if the following condition in eq. (2) is met.

$$F_X(t_1, \dots, t_n)(X_1, \dots, X_n) \neq F_X(t_1, \dots, t_n)(X_1, \dots, X_n) \quad (2)$$

Many researchers from system studies, process instrumentation, data analytics, time series analysis, biomedical instrumentation, vibro-acoustics and signal processing have already explored the scope of non-stationarity based features. C. Cao and S. Slobounov [2] used Shannon Entropy of peak frequency shifting to measure the non-stationarity of Electro Encephalograph (EEG) signals. To quantify the non-linear dynamics of the underlying attractors in healthy, inter-ictal and ictal EEG, N. P. Subramaniam and J. Hyttinen [3] recommended a method based on Recurrence Network (RN). Application of non-linear and wavelet based features for the automated identification of epileptic EEG signals were illustrated by Acharya *et al.*, [4]. S. K. Chuan [5] employed Hurst Exponent (HE), Fractal Dimension (FD), Approximate Entropy (ApEn), Largest Lyapunov Exponent (LLE) and Correlation Dimension (CD) to discriminate between normal, pre-ictal and ictal classes of EEG. Subha *et al.*, [6] used CD, HE, bi-spectrum features of the higher order spectra, LLE, different variations of entropy and phase space as well as recurrence plots to analyze the chaotic behavior of EEG signals. Melkonian *et al.*, [7] introduced fragmentary spectrum as a measure that brings the frequency contents, timing and duration of segments of Heart Rate Variability (HRV) signals.

Chu *et al.*, [8] utilized Rate-Transient Analysis (RTA) for characterizing and simulating the non-linear and non-stationary features caused by the changes in confined Pressure-Volume Temperature (PVT) properties of unconventional oil reservoirs. Su *et al.*, [9] recommended Re-Scaled range analysis (R/S), Brock Dechert Scheinkman (BDS) test, power spectra, recurrence plot, LLE, Kolmogorov entropy and CD to identify the nonlinearity of time series data. An Autoregressive-fit Residuals Kurtosis (ARK) method to detect nonlinearity and non-stationarity in time series were introduced by M. D. Domenico and V. Latora [10]. W. H. J. Toonen [11] observed that the non-stationarity had significant effect on the outcomes of flood frequency analysis in both short and long input data series. Szupiluk *et al.*, [12] developed a multi-stage structural analysis technique, capable to assess the level of randomness, to identify the cycles in

the data and to detect the long memory effects in financial time series. Ahn *et al.*, [13] observed that signal kurtosis is useful to characterize the repeating peaks of the vibration signal, generated due to faults in roller bearing systems. Guo *et al.*, [14] proposed the combination of spectral kurtosis and Ensemble Empirical Mode Decomposition (EEMD) to extract random signals produced by faulty bearing from noise. System properties of structures like buildings and bridges may vary with time due to the variation in factors like temperature, aging and loading. To identify such time-varying system properties, Y. Guo and A. Kareem [15] proposed a system identification framework based on instantaneous spectra derived from the Time-Frequency (T-F) representation obtained via Short Time Fourier Transform (STFT) and Wavelet Transform (WT). In the study of S. Chen and J. Lin [16], intra and inter wave frequency modulation were utilized to quantify the nonlinearity and non-stationarity of dynamic response of high speed vehicle-track coupling system. G. A. Salini and P. Perez [17] used LLE, the shape of decay of ACF and CD and Hurst exponents to investigate deterministic chaotic behavior in the time series data of fine particulate matter concentration in atmosphere.

Autocorrelation is a method widely used to measure periodicity or stationarity of a signal. SACF was used for order determination of mixed stationary and non-stationary ARMA models by R.S. Tsay and G. C. Tiao [18]. B. Ahmadi and R. Amirfatahi [19] used CD and Higuchi Fractal Dimension (HFD) to estimate the bispectral index of EEG. F. C. Blondeau [20] pointed out that auto information function and autocorrelation function remain complementary tools for investigating random signals. S. Degerine [21] illustrated that the Partial Auto Correlation Function (PACF) like Auto Correlation Function (ACF) can be used in order to parameterize non-stationary time series. Y. Takizawa and A. Fukasawa [22] proposed a time-dependent autocorrelation method to study the non-stationary characteristics of EEG signal in human sleep. In order to extract the frequency components produced by faults in bearing systems a combination of Hilbert-Huang Transform (HHT) and autocorrelation was put forth by Xue *et al.*, [23]. B. J. Shannon and K. K. Paliwal [24] computed the spectral features using higher-lag autocorrelation coefficients for speech recognition. L. Rabiner [25] used autocorrelation for the analysis of pitch in speech signals. The use of an autocorrelation function in the seasonality analysis for the fatigue strain data to identify the seasonal pattern had been presented by Nopiah *et al.*, [26].

Box *et al*, [27] suggested that SE of the ACF can be used to estimate the non-stationary behavior of a signal. It was stated that, as the signal become more non-stationary, the value of SE moves close to the reciprocal of the square root of the number of samples in the signal. The interdependency between the non-stationarity and the numerical value of SE and is not yet investigated. In this paper, the variation of SE and  $\beta$  are investigated on test signals of different levels of non-stationarity. Section 2 of this article describes the procedure for simulating the test signals and analytical formulation for computing the SE of SACF. The numerical values of sample entropy, statistical variance, SE and  $\beta$  computed from the standard test signals are analyzed in Section 3, qualitatively and objectively.

**Methodology:** To test the feasibility of using the SE of ACF two sets of test signals are used, in the first set of test signals different sinusoidal frequency components are concatenated. Three sinusoids, 5 Hz, 10 Hz and 15 Hz are generated with a sampling frequency of 2.5 KHz. These frequency components are randomly concatenated such that at different parts of the signal the frequency is different. The number of samples in all the three components is kept equal in the test signal. To generate test signals of different non-stationarity levels, the number of samples in the frequency components which form the test signals is varied. The first set of test signal comprises five signals. In the first test signal the number of samples in each frequency component or the length of each sinusoid is 125. Sinusoids of 5 Hz, 10 Hz and 15 Hz, each with a length of 125 samples are concatenated unevenly to form the first test signal. The total number of samples in the test signals is kept constant typically at 9000. In the second test signal the number of samples in each frequency component is 250. Sinusoids of 5 Hz, 10 Hz and 15 Hz, each with a length of 250 samples are concatenated unevenly to form the second test signal. Likewise, by varying the number of samples in the sinusoids which constitute the test signal is varied as a multiple of two, as 125, 250, 500, 1000 and 2000. The change in frequency with respect to time is faster when the number of samples in the sinusoids which constitute the test signal is 125 than when the number of samples in the sinusoids which constitute the test signal is 250 or more. Consequently, the non-stationarity of the test signals increases with respect to the increase in the number of samples in the sinusoids which constitute the test signal. The second set of test signals are random sequences of length 10000, with normalized values.

The consistency of the statistics at different instant of time is considered to be a good indicator of stationarity in signals. Similarly, entropy is an index extensively employed for estimating the relative degree of randomness of signals. In this article the pattern of variation of SE of SACF computed from the test signals of different non-stationarity levels are compared with the patterns of variation of global entropy and pattern of variation of SD of the variance of epochs of equal length in the test signals. The influence of the sampling frequency and offset on the ACF and SE are also observed. The simulation of test signals and computation of the ACF, SE and its statistics are performed in Matlab®.

$$SE = \sqrt{\frac{1}{N} \left( 1 + 2 \sum_{j=1}^q (r_j^2) \right)} \tag{3}$$

where ‘N’ is total number of samples, ‘q’ is lag beyond which the theoretical ACF is effectively 0 and ‘ $r_j$ ’ is the SACF function. If the series is fully uncertain, SE degrades to  $1/\sqrt{N}$  [30].

The SACF ‘ $r_k$ ’ at lag ‘k’ is

$$r_k = \frac{C_k}{C_0} \tag{4}$$

where ‘ $C_0$ ’ is the variance of the signal with zero shift and ‘ $C_k$ ’ is the co-variance of the signal.

$$C_0 = \frac{1}{N} \sum_{t=1}^N (Y_t - \mu(Y_t))^2 \tag{5}$$

where ‘ $\mu(Y_t)$ ’ is the mean value of the stochastic signal ‘ $Y_t$ ’ given by,

$$\mu(Y_t) = \frac{1}{N} \sum_{t=1}^N Y_t \tag{6}$$

$$C_k = \frac{1}{N-1} \sum_{t=1}^{N-k} (Y_t - \mu(Y_t))(Y_{(t+k)} - \mu(Y_t)) \tag{7}$$

where ‘ $Y_{(t+k)}$ ’ is the shifted version of the signal ‘ $Y_t$ ’ and given  $k=0,1,2 \dots \dots K$ .

### RESULTS AND DISCUSSIONS

The two test signals formed by concatenating sinusoids of frequency 5 Hz, 10 Hz and 15 Hz, each with a length of 1000 and 500 samples for a total 9000 samples,

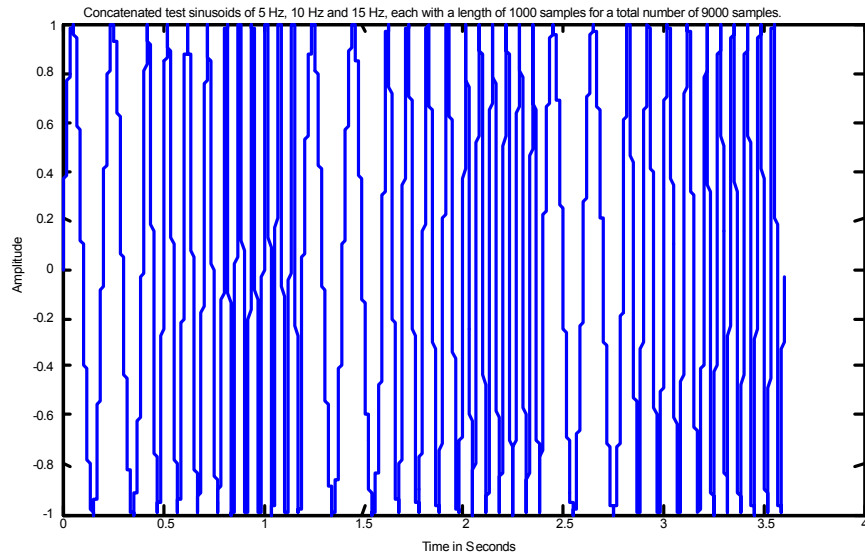


Fig. 1: Unevenly concatenated sinusoids of 5 Hz, 10 Hz and 15 Hz, each with a length of 1000 samples for a total number of 9000 samples.

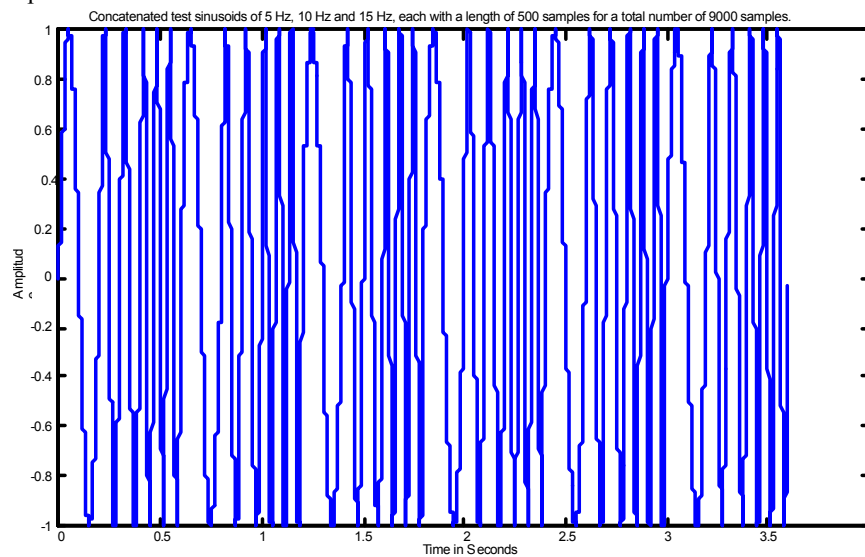


Fig. 2: Unevenly concatenated sinusoids of 5 Hz, 10 Hz and 15 Hz, each with a length of 500 samples for a total number of 9000 samples.

repeating unevenly, are shown in Fig. 1 and Fig. 2, respectively. In the first test signal (Fig. 1) the change in frequency with respect to time is less compared to the second test signal in Fig. 2. This is because, each sinusoidal frequency in the first test signal persists over 1000 samples. Whereas, in the test signal in Fig. 2, the frequency of the signal apparently changes in every 500 samples. Consequently, the second signal is more non-stationary than the first test signal.

The numerical values of entropy, variance, SE of the SACF and  $\beta$ , computed from the test signals formed by concatenating the sinusoids are furnished

in Table 1. In Table 1, the number of samples over which each sinusoid appears in the first test signal is 125. To control the non-stationarity of the test signals, the number of samples over which each sinusoid appears which constitute the test signals is increased as a multiple of two in the consecutive sinusoids. When the number of samples over which the frequency components persist increases the non-stationarity decreases. The decreasing pattern of entropy with respect to decreasing non-stationarity can be observed in Table 1. But the patterns SE of SACF and  $\beta$  are not monotonically decreasing.

Table 1: Numerical values of Entropy, variance, SE and  $\beta$  of the test signals formed by concatenating the sinusoids

Frequency of components (Hz)	Number of samples in sinusoids (Samples)	Sampling Frequency (Hz)	Total number of samples in the test signal (N)	Entropy	Variance	1/vN	SE	$\beta$
5,10,15	125	2500	9000	6.3564	0.2522	0.0105	0.2770	0.2665
5,10,15	250	2500	9000	5.6393	0.4200	0.0105	0.3140	0.3035
5,10,15	500	2500	9000	4.30427	0.5001	0.0105	0.1952	0.1847
5,10,15	1000	2500	9000	4.30424	0.5001	0.0105	0.1720	0.1615
5,10,15	2000	2500	18000	4.30421	0.5000	.0075	0.1643	0.1569

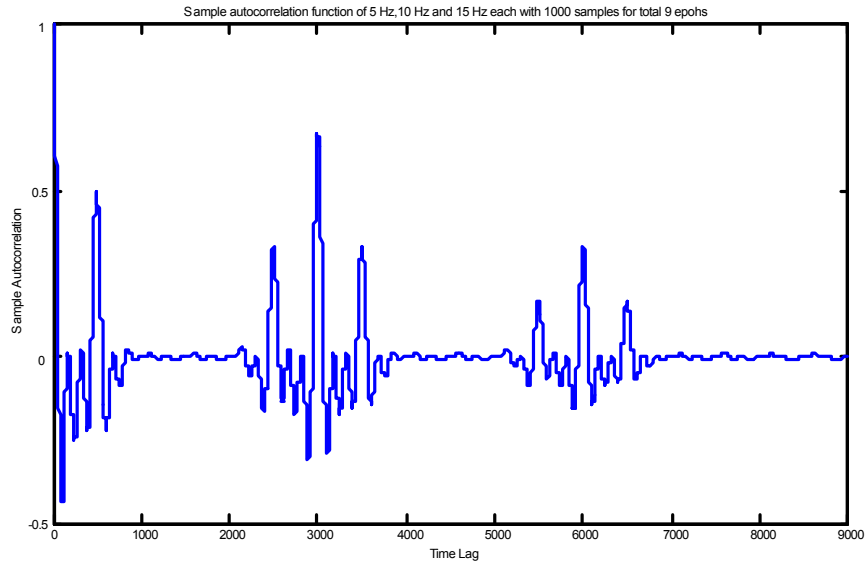


Fig. 3: SACF of test signal in Fig. 1

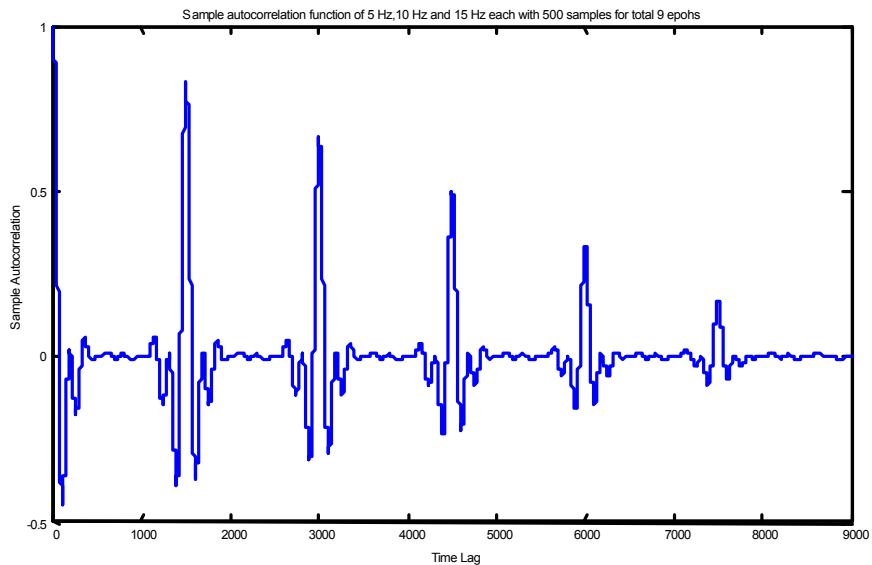


Fig. 4: SACF of test signal in fig. 2

The variation of SE of SACF and  $\beta$  shows randomness, irrespective of the falling pattern of non-stationarity. Entropy, to a certain extent is able to follow non-stationarity of the test signals, compared to SE and  $\beta$ . The plot of the normalized SACF of non-stationary test

signal in Fig. 1 and Fig. 2 are shown in Fig. 3 and Fig. 4, respectively. The wave pattern of the autocorrelation in Fig. 3 and Fig. 4 are abruptly distinct. This is a clear indication of the scope of ACF to be used to characterize the non-stationarity of signals.

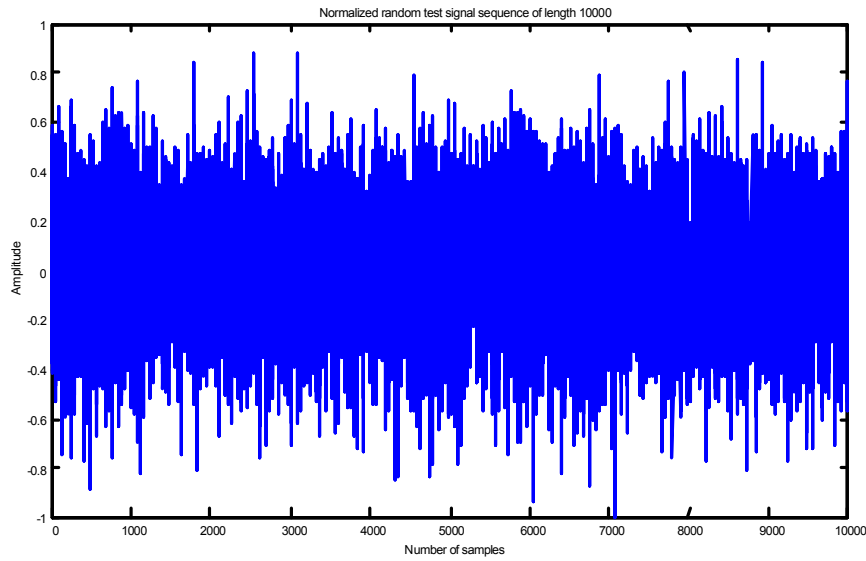


Fig. 5: Normalized random test signal sequence of length 10000.

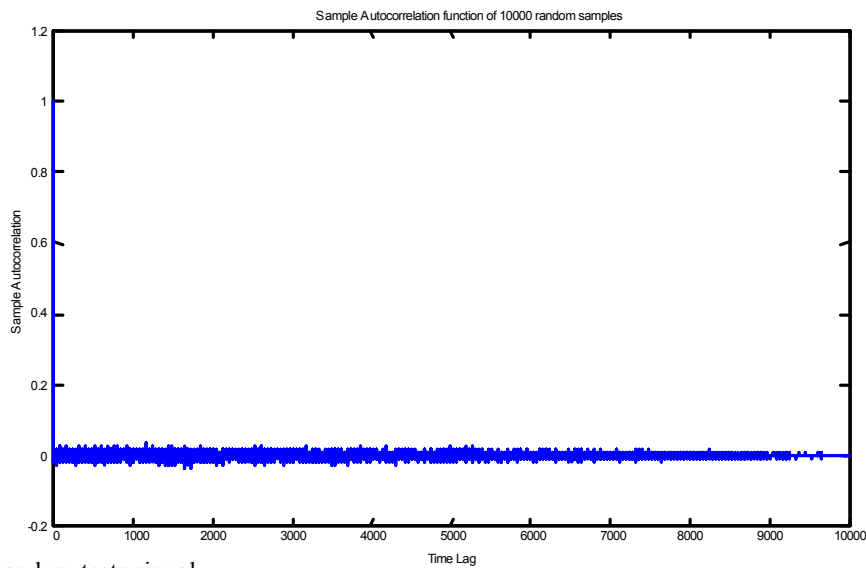


Fig. 6: SACF of random tests signal.

The plot of a simulated random test signal sequence is shown in Fig. 5. SACF of the random test signal is shown in Fig. 6. The numerical values of entropy, variance, SD of the variance of the epochs, SE of the SACF and  $\beta$ , computed from 50 random test signals are furnished in Table 2. To compute the SD of variance of the epochs the test signals are divided into epochs, each carrying 1000 samples. The SD of variance may exhibit good correspondence with non-stationarity if the epoch size is reduced further. Unlike the ACF of test signals in fig. 3 and Fig. 4, the ACF of random test signal in Fig. 6 does not follow any cyclic pattern.

$$SE = \frac{1}{N} \left( 1 + 2 \sum_{j=1}^q \left( \frac{\frac{1}{N-1} \sum_{i=1}^{N-k} (Y_i - \mu(Y_i))(Y_{(i+k)} - \mu(Y_i))}{\frac{1}{N} \sum_{i=1}^N (Y_i - \mu(Y_i))^2} \right)^2 \right) \quad (8)$$

Over the 50 random test signals the range of entropy is from 4.3226 to 4.6140. Whereas, the range of SD of variance of the epochs is 0.0016 to 0.0049 and the range of  $\beta$  is from 0.0099 to 0.0101. The SE varies between 0.0199 and 0.0201. Compared to the range of entropy, the range of SE and  $\beta$  is less. The poor dynamic variability of the SE and  $\beta$  is because of the influence of normalization factor N, which is the number of samples of signal, as evident in Eq. 8.

Table 2: Entropy, variance, SD, SE and  $\beta$  of random test signals

Test Signal	Entropy	Variance	SD of the			Test Signal	Entropy	Variance	SD of the		
			varianceof epochs	SE	$\beta$				variance of epochs	SE	$\beta$
1	4.5444	0.0702	0.0021	0.0200	0.0100	26	4.3854	0.0509	0.0023	0.0201	0.0101
2	4.4724	0.0586	0.0036	0.0200	0.0100	27	4.3762	0.0525	0.0019	0.0200	0.0100
3	4.4531	0.0614	0.0027	0.0201	0.0101	28	4.4614	0.0605	0.0025	0.0200	0.0100
4	4.4256	0.0529	0.0035	0.0200	0.0100	29	4.4044	0.0548	0.003	0.0200	0.0100
5	4.4121	0.0567	0.0016	0.0201	0.0101	30	4.4478	0.0635	0.0034	0.0199	0.0099
6	4.4464	0.0640	0.0031	0.0201	0.0101	31	4.3226	0.0500	0.0022	0.0201	0.0101
7	4.4479	0.0577	0.0021	0.0200	0.0100	32	4.4033	0.0588	0.0032	0.0200	0.0100
8	4.4971	0.0573	0.0028	0.0201	0.0101	33	4.4897	0.0638	0.0033	0.0199	0.0099
9	4.4715	0.0632	0.0043	0.0200	0.0100	34	4.4362	0.0564	0.0025	0.0200	0.0100
10	4.5279	0.0691	0.0025	0.0200	0.0100	35	4.4784	0.0667	0.002	0.0199	0.0099
11	4.4893	0.0573	0.0017	0.0200	0.0100	36	4.6140	0.0832	0.0024	0.0201	0.0101
12	4.4981	0.0668	0.0029	0.0201	0.0101	37	4.4681	0.0606	0.0023	0.0199	0.0099
13	4.5104	0.0739	0.0049	0.0200	0.0100	38	4.5454	0.0676	0.0016	0.0201	0.0101
14	4.5019	0.0663	0.0022	0.0201	0.0101	39	4.3823	0.0628	0.0023	0.0200	0.0100
15	4.4864	0.0607	0.0035	0.0200	0.0100	40	4.4207	0.0538	0.0028	0.0200	0.0100
16	4.5018	0.0594	0.0036	0.0201	0.0101	41	4.4422	0.0572	0.0026	0.0199	0.0099
17	4.4837	0.0617	0.0029	0.0200	0.0100	42	4.5044	0.0609	0.0027	0.0199	0.0099
18	4.4431	0.0565	0.0029	0.0201	0.0101	43	4.5540	0.0669	0.0033	0.0199	0.0099
19	4.4268	0.0464	0.0018	0.0200	0.0100	44	4.4880	0.0598	0.0021	0.0200	0.0100
20	4.5005	0.0692	0.0033	0.0200	0.0100	45	4.5693	0.0655	0.0025	0.0200	0.0100
21	4.4407	0.0624	0.0023	0.0200	0.0100	46	4.4342	0.0652	0.0025	0.0201	0.0101
22	4.5376	0.0667	0.0029	0.0199	0.0099	47	4.4666	0.0607	0.0032	0.0201	0.0101
23	4.5372	0.0644	0.003	0.0200	0.0100	48	4.5311	0.0747	0.0039	0.0201	0.0101
24	4.4066	0.0511	0.0031	0.0200	0.0100	49	4.4085	0.0537	0.0033	0.0201	0.0101
25	4.5464	0.0714	0.0022	0.0200	0.0100	50	4.5019	0.0601	0.0035	0.0200	0.0100

It also has been observed that the value of SE is robust to change in sampling rate and offset. The robustness to the offset is because of the demeaning in Eq. 5. The correlation between entropy and  $\beta$ , the correlation between entropy and SE is -0.1200. The correlation between SD of the variance of epochs and  $\beta$ , the correlation between SD of the variance of epochs and SE is -0.0655. It has been already made perspective that entropy to a certain extent is able to follow the variation in non-stationarity. But the SE and  $\beta$  is not correlated with the entropy. This reveals the inability of SE and  $\beta$  to account for the non-stationarity of the signal.

**CONCLUSION**

The feasibility of using SE of the SACF and absolute deviation of SE from the reciprocal of the square root of number of samples as a measure of non-stationary is evaluated on simulated test signals. Entropy to a certain extent is able to follow the variation in non-stationarity of the signals. SE and absolute deviation of SE from the reciprocal of the square root of number of samples are not able to follow the variation in non-stationarity. However, SE is robust to sampling rate and offset. Moreover, SE and  $\beta$  is not correlated with the entropy. The dynamic variability of the SE and the absolute

deviation of SE from the reciprocal of the square root of number of samples is less when compared with the variability of entropy. The auto correlation based indices which are robust to sampling rate, offset and ultimately the length of signals have to be developed.

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