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Slip Effects on Mhd Flow of Jeffrey Fluid over an Unsteady Shrinking Sheet with Wall Mass Transfer

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Abstract: An analysis is made to study the slip effects on MHD flow of Jeffrey fluid over an unsteady shrinking sheet with wall mss transfer. The governing partial differential equations are transformed into self similar ordinary differential equations using similarity transformations and solved it numerically by Runge-Kutta fourth order method with shooting technique. The effects of unsteady parameter, Hartmann number, Jeffrey parameter, Suction parameter and velocity slip parameter are represented graphically on velocity profiles while the local skin friction and Nusselt number are represented numerically through tables. It is found that the present results have been excellent agreement with the existed studies under some special cases.

Key words: Slip parameter · Jeffrey parameter · MHD · Unsteady shrinking sheet

INTRODUCTION

The boundary layer flow over a shrinking surface is encountered in several technological processes. It has been extensively used in many engineering fields and industries for expanding and contracting of surfaces such as shrinking wrapping, bundle wrapping, hot rolling, extrusion of sheet material, wire rolling and glass fiber. Such situations occur in polymer processing, manufacturing of glass sheets, paper production in textile industries and many others.

Sudipta Ghosh *et al*. [1] studied dual solutions of slip flow past a nonlinearly shrinking permeable sheet. Sandeep Naramgari *et al*. [2] investigated MHD flow over a permeable stretching/shrinking sheet of a nanofluid with suction/injection. Krishnendu Bhattacharyya [3] discussed MHD stagnation point flow of Casson fluid and heat transfer over a stretching sheet with thermal radiation. Stagnation point flow and heat transfer over an exponentially shrinking sheet was studied by Krishnendu Bhattacharyya *et al*. [4]. Effects of thermal radiation on micropolar fluid flow and heat transfer over a porous shrinking sheet was discussed by Krishnendu Bhattacharyya *et al*. [5]. Naddem *et al*. [6] analyzed MHD flow of a Casson fluid over an exponentially shrinking sheet. Slip effects on MHD mixed convection stagnation point flow of a micropolar fluid towards a shrinking vertical sheet was reported by Das [7]. Alin *et al*. [8] discussed flow and heat transfer over a vertical permeable stretching/shrinking sheet with a second order slip. Hamad *et al*. [9] studied thermal jump effects on boundary layer flow of a Jeffrey fluid near the stagnation point on a stretching/shrinking sheet with variable thermal conductivity. Turkyilmazoglu *et al*. [10] discussed exact analytical solution for the flow and heat transfer near the stagnation point on a stretching/shrinking sheet in a Jeffrey fluid.

In all these above studies the flow and temperature fields is considered to be steady state. However, in some cases, the flow field, heat and mass transfer can be unsteady due to a sudden shrinking sheet. A few papers have been published on the boundary layer flow and heat transfer problems where the shrinking force and surface temperature are varying with time. Nadeem *et al*. [11] obtained MHD boundary layer flow over an unsteady shrinking sheet: analytical and numerical approach. Azizah Mohad Rohni *et al*. [12] discussed flow and heat transfer over an unsteady shrinking sheet with suction in nanofluids. Ali *et al*. [13] studied an unsteady shrinking sheet with mass transfer in a rotating fluid. Unsteady stagnation point flow towards a shrinking sheet with radiation effect was investigated by Ali *et al*. [14]. Eshetu Haile *et al*. [15] presented heat and mass transfer in the boundary layer of unsteady viscous nanofluid along a vertical stretching sheet. Azizah Mohd Rohni *et al*. [16] investigated flow and heat transfer at a stagnation point

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over an exponentially shrinking vertical sheet with suction. Turkyilmazoglu [17] studied three dimensional MHD flow and heat transfer over a stretching/shrinking surface in a viscoelastic fluid with various physical effects. Hayat *et al*. [18] obtained effects of thermal radiation on unsteady magnetohydrodynamic flow of a micropolar fluid with heat and mass transfer. Hayat *et al*. [19] studied radiation and magnetic field effects on the unsteady mixed convection flow of a second grade fluid over a vertical stretching sheet.

The objective of the present study to investigate slip effects on MHD flow of Jeffrey fluid over an unsteady shrinking sheet with wall mss transfer. Numerical solution of the coupled non linear momentum equation is obtained using fourth order Runge-Kutta method with shooting technique. The velocity flow is computed and the results obtained are depicted graphically, for different controlling parameters [20].

Mathematical Formulation of the Problem: Consider the boundary layer flow of incompressible Jeffrey fluid over an unsteady shrinking sheet with wall mass transfer. The fluid electrically conducting under the influence of time dependent magnetic field $B(t)$ applied in the direction to the shrinking sheet. The induced magnetic field is neglected under the assumption of small magnetic Reynolds number. The x-axis parallel to the porous surface and y-axis normal to it. The governing equations of motion for steady two dimensional flow in the presence of uniform transverse magnetic field are written in usual notation as:

$$\frac{\partial u}{\partial x}+\frac{\partial \upsilon}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}=\frac{\nu}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2(t)}{\rho}u \tag{2}$$

The appropriate boundary conditions for the velocity components are

$$\left.\begin{aligned}&u(x,t)=U_w(x,t)+L\frac{\partial u}{\partial y},\ \upsilon(x,t)=\upsilon_w(x,t)\ at\ y=0\\ &u(x,t)\to 0\, as\, y\to\infty\end{aligned}\right\} \tag{3}$$

where u and υ are velocity components in x and y directions respectively. ν Is the kinematic viscosity of the fluid, ρ is the fluid density, σ is the electrical conductivity of the fluid and L is the slip length and λ_1 is Jeffrey parameter.

Introducing the stream function ψ, the velocity components u and v can be written as

$$u=\frac{\partial\psi}{\partial y}\ \text{and}\ \upsilon=\frac{-\partial\psi}{\partial x} \tag{4}$$

From relations of (4), the mass conservation equation (1) is satisfied automatically and the momentum equation (2) take the following forms

$$\frac{\partial^2\psi}{\partial t\partial y}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}=\frac{\nu}{1+\lambda_1}\frac{\partial^3\psi}{\partial y^3}-\frac{\sigma B^2(t)}{\rho}\frac{\partial\psi}{\partial y} \tag{5}$$

also the boundary conditions in (3) reduce to

$$\left.\begin{aligned}&\frac{\partial\psi}{\partial y}=U_w+L\frac{\partial^2\psi}{\partial y^2},\frac{\partial\psi}{\partial x}=-\upsilon_w\ at\ y=0\\ &\frac{\partial\psi}{\partial y}\to 0\ as\ y\to\infty\end{aligned}\right\} \tag{6}$$

We further assume the shrinking velocity as

$$U_w(x,t)=\frac{-U_0 x}{1-\gamma t} \tag{7}$$

where U_0 is a constant having a dimension of 1/time, also the time dependent magnetic field is chosen as

$$B(t)=B_0(1-\gamma t)^{-1} \tag{8}$$

Next we introduce the dimensionless variable for

$$\left.\begin{aligned}&\psi(x,y)=x f(\eta)\sqrt{\frac{\nu U_0}{1-\gamma t}},\eta=y\sqrt{\frac{U_0}{\nu(1-\gamma t)}}\\ &u=\frac{\partial\psi}{\partial y}=\frac{U_0 x}{1-\gamma t}f'(\eta),\upsilon=-\frac{\partial\psi}{\partial x}=-f(\eta)\sqrt{\frac{\nu U_0}{1-\gamma t}}\end{aligned}\right\} \tag{9}$$

The wall mass transfer velocity is defined as $\Psi(x, y)$ as

$$\upsilon_w=-f(0)\sqrt{\frac{\nu U_0}{1-\gamma t}} \tag{10}$$

Using the above transformation defined in equation (9), the momentum equation takes the form

$$\frac{1}{1+\lambda_1}f'''-\left(M^2+\beta\right)f'-\frac{\beta}{2}\eta f''+ff''-f'^2=0 \tag{11}$$

Table 1: Skin friction coefficient $\frac{f''(0)}{1+\lambda_1}$ for different values of β with S = 1, M = 2, λ_1 and δ = 0.

β	Nadeem *et al.* [11]	Present study
0	2.30277	2.302782
1	2.48888	2.488882

Table 2: Skin friction coefficient $\frac{f''(0)}{1+\lambda_1}$ for different values of S with β = 3, M = 1, λ_1 = 0 and δ = 0

S	Nadeem *et al.* [11]	Present study
1	2.05352	2.053521
3	3.50774	3.507732

The corresponding boundary conditions are

$$\left.\begin{aligned} &f = S, f' = -1 + \delta f'' \; at \; \eta = 0 \\ &f' \to 0 \; as \; \eta \to \infty \end{aligned}\right\} \tag{12}$$

where $M^2 = \frac{\sigma B_0^2}{\rho U_0}$ is Hartmann number, $\beta = \gamma / U_0$ is dimensionless unsteady parameter, $S = \frac{-\upsilon_w}{\left(\sqrt{\nu U_0/(1-\gamma t)}\right)}$ is suction parameter, $\delta = L\left(\frac{U_0}{\nu(1-\gamma t)}\right)^{1/2}$ is the velocity slip parameter and λ_1 is the Jeffrey parameter.

Local Skin Friction: The physical quantities of interest are the local skin friction coefficient C_f. Which is defined as

$$C_f = \frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\frac{\rho U_w^2}{2}} \tag{13}$$

which in the present case, can be expressed in the following form

$$C_f = \frac{2}{\sqrt{\mathrm{Re}}}\frac{f''(0)}{1+\lambda_1} \tag{14}$$

where $\mathrm{Re} = \frac{U_w x}{\nu}$

Numerical values of the function $\frac{f''(0)}{1+\lambda_1}$ which represent the wall shear stress at the surface respectively for various values of the parameter are presented in Table 1 and Table 2.

RESULTS AND DISCUSSION

The numerical computations are performed for several values of dimensionless parameters involved in the equations via, the Hartmann number M, the mass suction parameter S, unsteady parameter β, Jeffrey parameter λ_1 and velocity slip parameter δ. To illustrate the computed results some figures are plotted and physical explanations are given.

We calculate the numerical values of skin friction coefficient for various values of emerging parameter such as unsteady parameter β, suction parameter S for λ_1 = 0 and δ = 0. The skin friction coefficient $\frac{f''(0)}{1+\lambda_1}$ are compared with the existed results of Nadeem *et al*. [11] are shown in Table 1 and Table 2 and have found in excellent agreement.

First, for the verification of the accuracy of the applied numerical method we compare our results corresponding to the velocity profiles for λ_1 = 0 and δ = 0 (i.e. in the absence of the Jeffrey parameter and slip at the boundary) with the available published results of Nadeem *et al*. [11] in Fig. 1, which are found to be in excellent agreement.

Variation in velocity field for several values of suction parameter S is shown in Fig. 2 for both presence of Jeffrey parameter λ_1, velocity slip parameter δ and absence of Jeffrey parameter λ_1, velocity slip parameter δ. We observed that the velocity $f'(\theta)$ increases with increasing values of suction parameter S and consequently the thickness of the boundary layer decreases.

Influence of an unsteady parameter β on the velocity profiles in the presence of Jeffrey parameter, slip and in the absence of Jeffrey parameter, slip at the boundary. Fig. 3 shows that the velocity $f'(\theta)$ increases with increasing values of an unsteady parameter β under the both cases and consequently the boundary layer thickness decreases with increase of β.

The impacts of the Hartmann number M on the velocity $f'(\theta)$ is very significant in practical point of view. In Fig. 4 shows that the velocity $f'(\theta)$ increases with increasing values of M for both presence of Jeffrey parameter λ_1, velocity slip parameter δ and absence of Jeffrey parameter λ_1, velocity slip parameter δ at the boundary. Accordingly, the thickness of the momentum boundary layer decreases. This happens due to the Lorentz force arising from the electric fields during the motion of the electrically conducting fluid. To reduce

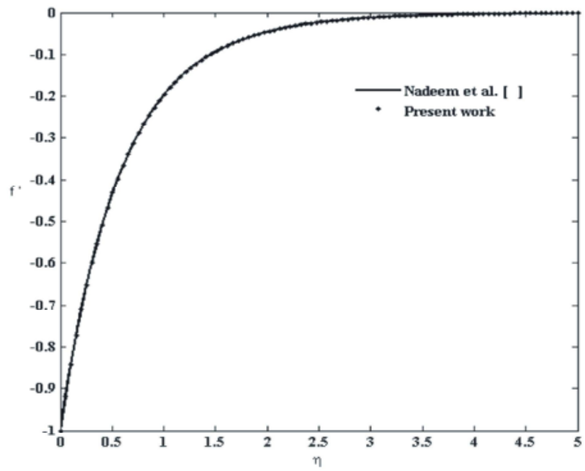

Fig. 1: Velocity profiles for $\lambda_1 = 0$ and $\delta = 0$ with $S = -0.5$, $\beta = 1$ and $M = 2$.

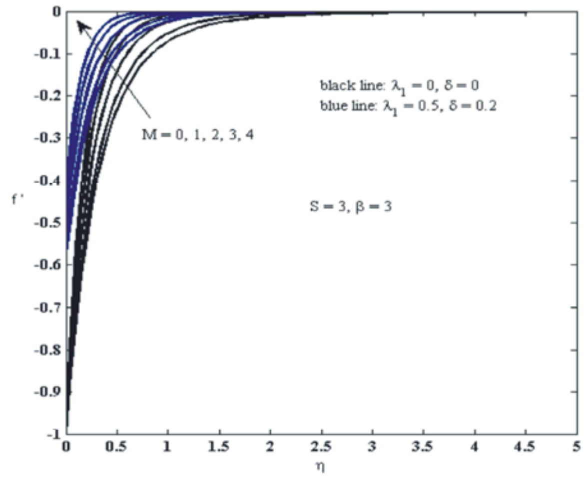

Fig. 4: Velocity profiles for different values of M.

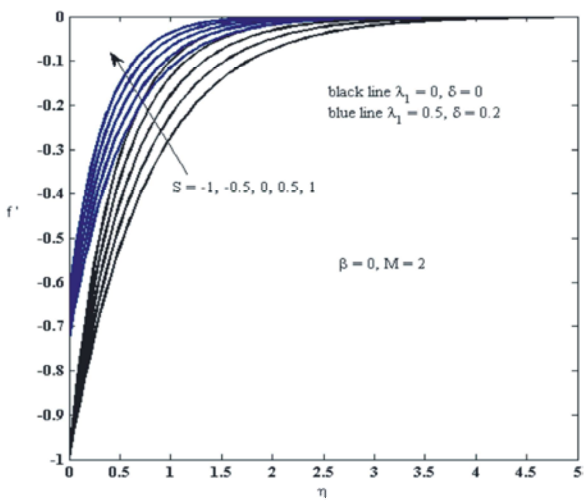

Fig. 2: Velocity profiles for different values of S.

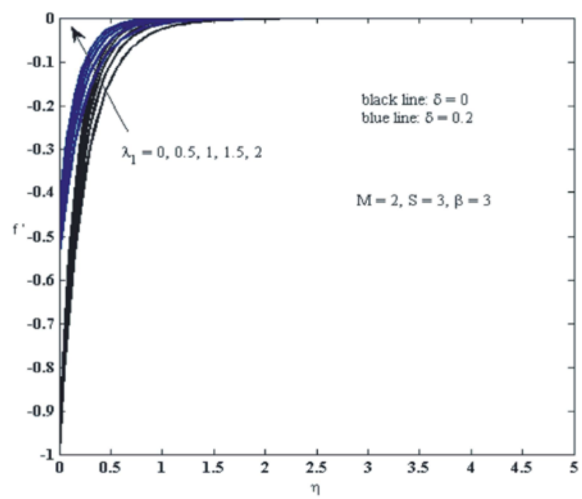

Fig. 5: Velocity profiles for different values of λ_1.

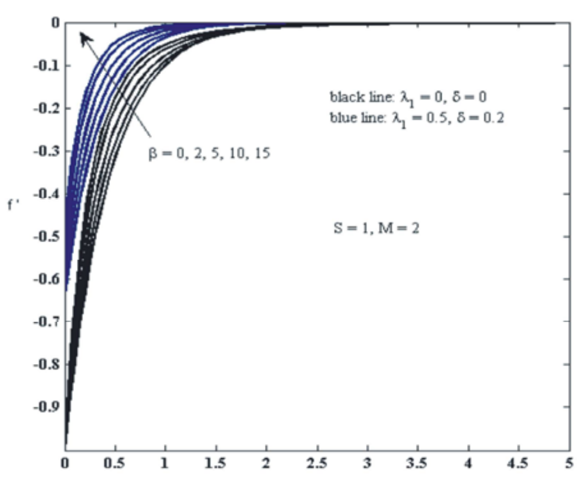

Fig. 3: Velocity profiles for different values of β.

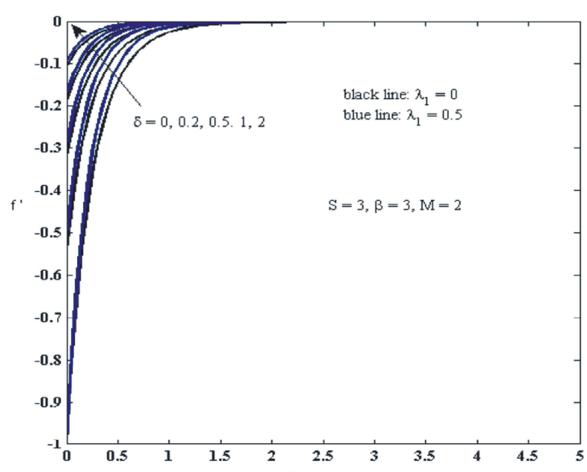

Fig. 6: Velocity profiles for different values of δ.

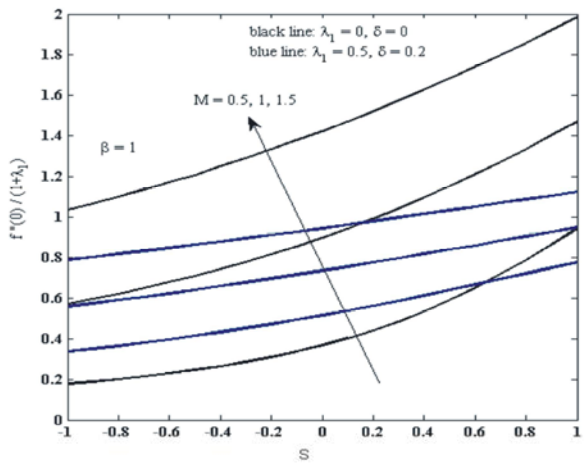

Fig. 7: Skin friction coefficient profiles against S for several values of M.

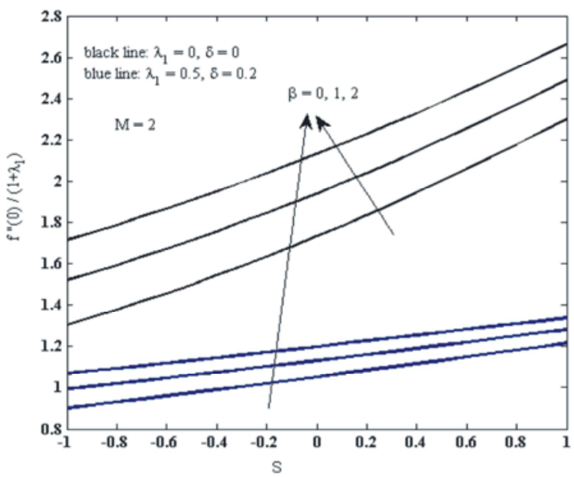

Fig. 8: Skin friction coefficient profiles against S for several values of β.

momentum boundary layer thickness the generated Lorentz force enhances the fluid motion in the boundary layer region.

Influence of the Jeffrey parameter λ_1 on the velocity $f'(\theta)$ distribution is demonstrated in Fig. 5. For both slip and no slip cases we observe that the velocity $f'(\theta)$ increases with increasing values of Jeffrey parameter λ_1 and consequently the thickness of the boundary layer decreases.

Variation in velocity field for several values of velocity slip parameter δ is shown in Fig. 6. We observed that the velocity $f'(\theta)$ increases with the increasing values of δ and accordingly the thickness of the boundary layer decreases under the both Newtonian and non Newtonian fluid cases.

Skin friction coefficient $\frac{f''(0)}{1+\lambda_1}$ is plotted in Figs. 7 and 8 against the suction parameter S for several values of M and unsteady parameter β. We observe that from Fig. 7 with an increase of S skin friction increase gradually for each value of Hartmann number M. For increasing values of unsteady parameter β and suction parameter S same increasing behavior can be obtained for skin friction in Fig. 8 under the both Newtonian, no slip and non Newtonian, slip cases.

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