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Mathematical Modeling and PID Controller Design Using Transfer Function and Root Locus Method for Active Suspension System

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Abstract: The high- mobility armoured fighting vehicles are normally fitted with passive suspension systems using torsion bars and shock absorbers to absorb the terrain-induced vibrations and heavy shocks. While moving through different rough terrains at high speed, it was found that these passive suspension systems are incapable of isolating the vehicle from road vibrations and shocks. Hence, hydro-pneumatic suspension system with variable damping an alternate substitution to the earlier type of passive suspension systems for further improvement in the ride quality for the modern armoured fighting vehicles. An active suspension system has proved to be an effective alternate to the hydro-pneumatic suspension systems for the futuristic armoured fighting vehicles. Hence, the mathematical modeling of a single station hydro-pneumatic suspension has been developed using a quarter-car dynamic model and analyses have been carried for the open loop and close loop feedback control system with proportional, integral and derivative controllers using transfer function and root-locus methods. The results are analyzed and compared with passive suspension system using MATLAB.

Key words: Mathematical Modeling · PID Controller · Transfer function · Root Locus · Active Suspension System

INTRODUCTION

A good suspension system should have satisfactory road holding ability, while still providing comfort when riding over bumps and holes in the road. Whenever, AFV is experiencing any road disturbance due to uneven road pavement, the vehicle body should not have large oscillations and it should not transmit the shocks and vibrations to the vehicle body. So, the oscillations induced by the road undulations should dissipate quickly. Since the displacement of sprung mass (z_s) to the road disturbance (z_0) is very difficult to measure and the deformation of the tire (z_{us}-z_0) is negligible. Hence, we will use the relative displacement of sprung mass and unsprung mass (z_s-z_{us}) instead of relative displacement of sprung mass and road disturbance (z_s-z_0) as the output in our problem. The road disturbance (z_0) will be simulated by a step input to the quarter car [1] mathematical models of passive suspension system. This step could represent the vehicle is coming out of a bounce of 100mm height. So, we want to design a feedback controller so that the output (z_s-z_{us}) with an overshoot less than 5%, oscillate within the range of +/- 5mm and return to a smooth ride within 5 seconds. In this paper a detailed study was carried out and analyzed by developing a mathematical modeling of a single station Hydro-Pneumatic Suspension [2] System (HPSS) using a quarter-car dynamic model concept and Proportional, Integral and Derivative (PID) controller design using Transfer Function and Root-Locus methods.

Mathematical Modeling of HPSS: A typical quarter car mathematical models of passive, semi-active and active suspension systems [3,4] are shown in the Fig. 1. The passive suspension system consists of non-linear spring (k_s), damper (c_s), road wheel / tyre spring (k_t) and road wheel / tyre damping (c_{us}) is considered as zero. A sinusoidal displacement or step input (z_0) is considered as an input excitation for the model. Designing an active suspension system for tracked vehicles by adding the

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active element of force actuator into the existing passive suspension system for single station is used to simplify the problem to a two dimensional spring-damper system. A quarter car mathematical model of Hydro-pneumatic suspension system is shown in Fig.2:

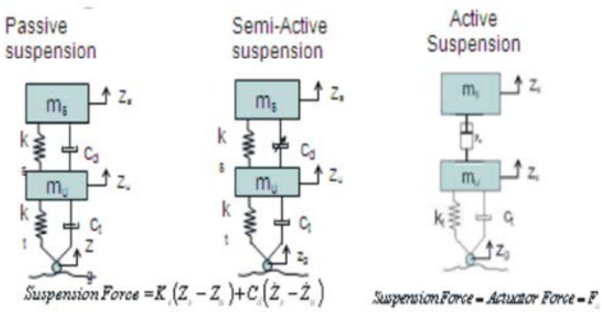

Fig. 1: Quarter-car models of suspension systems

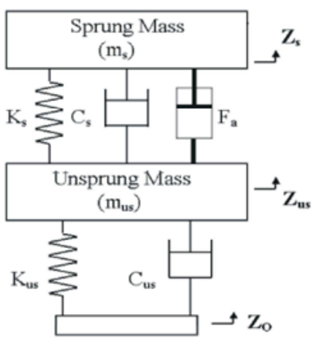

Fig. 2: Quarter-car model of a HPSS

Equations of Motion: The dynamic equations of motion for the sprung and unsprung masses of the quarter car model of hydro-gas suspension system by applying the Newton's law of motion are given below

$$m_s \ddot{z}_s = -k_s(z_s - z_{us}) - c_s(\dot{z}_s - \dot{z}_{us}) + Fa \quad (1)$$

$$m_{us} \ddot{z}_{us} = k_s(z_s - z_{us}) + c_s(\dot{z}_s - \dot{z}_{us}) + k_{us}(z_0 - z_{us}) - Fa \quad (2)$$

Transfer Function: The above dynamic equations (1) and (2) can be expressed in the form of Transfer Function (TF) [4] by taking the Laplace Transform. Therefore the derivations of transfer functions of G1(s) and G2(s) of output (Zs-Zus) and two inputs F_a and z_0 are as follows

$$G_1(s) = \frac{z_s(s) - z_{us}(s)}{F(s)} = \frac{z_s(s) - z_{us}(s)}{\Delta}$$

$$G_1(s) = \frac{(m_{us}s^2 + k_{us}) - (-m_s s^2)}{\Delta} \quad (3)$$

$$G_2(s) = \frac{z_s(s) - z_{us}(s)}{z_0(s)} = \frac{z_s(s) - z_{us}(s)}{\Delta}$$

$$G_2(s) = \frac{-(m_s s^2 * k_{us})}{\Delta} \quad (4)$$

Where,

$$\Delta = (m_s s^2 + c_s s + k_{us}) - (m_{us} s^2 + c_s s + (k_s + k_{us})) - (c_s s + k_{us}) * (c_s s + k_{us})$$

Open Loop Response of Passive Suspension System: The above transfer function equations (3) and (4) can be entered into MATLAB [5] by defining the numerator and denominator in the form of G1(s)=nump/denp for actuator force(F) and G2(s)=num1/den1 for the road disturbance (z_0) as a standard form of transfer function. The original open-loop system (without feedback control) for the unit step actuated force and unit step disturbance as a input to the passive suspension system are shown in the figure given below by using the matlab command of step (nump,denp) and step(0.1*num1,den1)

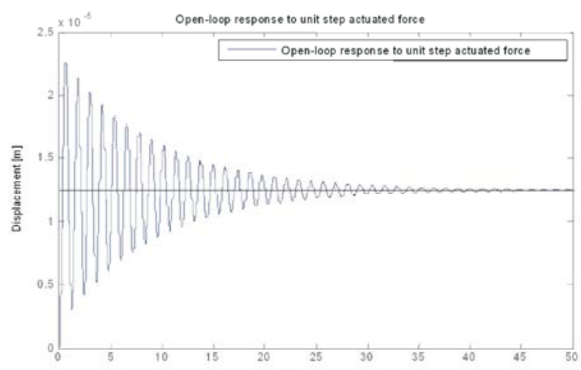

Fig. 2: Open-loop response to unit step actuated force

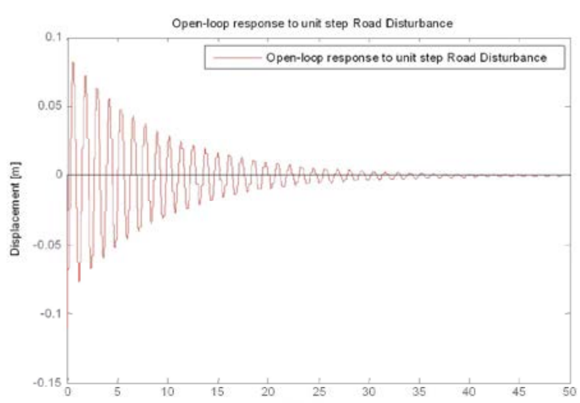

Fig. 3: Open-loop response to 0.1m step disturbance

The open-loop response for a unit step actuated force is shown in Fig.2. It shows that the system is under-damped, with very small amount of oscillation and very long unacceptable time for it to reach the steady state or the settling time is very large. Similarly, the open-loop response for step input of 0.1 m road disturbance is also shown in Fig.3. It shows that the system will oscillate for long time with larger amplitude than the initial impact. The high overshoot (from the impact itself) and the slow settling time will cause damage to the suspension system. The solution to this problem is to add a feedback controller into the system to improve the performance.

Closed Loop Feedback Controller System: The basic operations of the closed loop feedback controllers are to compare the measured value with the set point value and produce the error signal. On the basis of error signal, the controller algorithm will decide what parameter has to be controlled thereby eliminating the need for continuous operator attention. The adjustment chosen by the control algorithm is applied to some adjustable variable and eliminates the error signal to bring the measured quantity to its required value or set-point. The schematic block diagrams of the closed-loop feedback controller of the suspension system due to actuator force and road disturbance and its transfer functions are shown below.

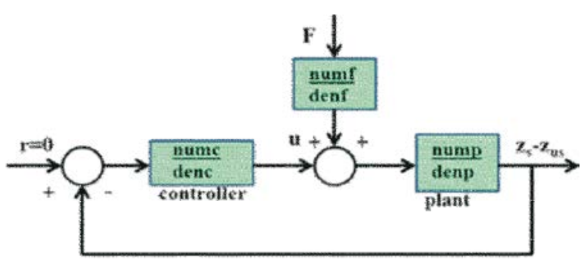

Fig. 4: Close-loop feedback controller system for Force

$$plant = \frac{nump}{denp}$$

$$F \cdot plant = \frac{num1}{den1}$$

$$F = \frac{num1}{den1 \cdot plant}$$

$$F = \frac{num1 \cdot denp}{den1 \cdot nump} = \frac{num1}{nump} = \frac{numf}{denf}$$

where,

denp=den1 , numf-num1 &nump = denf (5)

The schematic block diagram of close loop feedback controller due to road disturbance is shown in Fig.5. We can find the transfer function from the road disturbance z_0 to the output (z_s-z_{us}) is as given below.

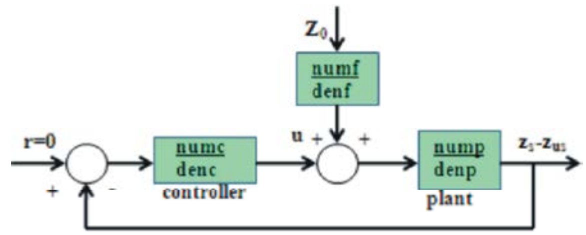

Fig. 5: Close-loop feedback controller system for Displacement

$$\left(\frac{numf}{denf}z_0 - \frac{numc}{denc}(z_s - z_{us})\right)\frac{nump}{denp} = (z_s - z_{us})$$

$$\left(\frac{numf}{denf}z_0 - \frac{numc}{denc}(z_s - z_{us})\right) = \frac{denp}{nump}(z_s - z_{us})$$

$$\frac{numf}{denf}z_0 = \left(\frac{denp}{nump} + \frac{numc}{denc}\right)(z_s - z_{us})$$

$$\frac{(z_s - z_{us})}{z_0} = \frac{nump * numf * denc}{denf(denp * denc + nump * numc)} \quad (6)$$

The above transfer functions for the force and road disturbance of the closed loop feedback controller system can be represented in Matlab by adding the following code into m-file:

```
numa=conv(conv(numf,nump),denc);
dena=conv(denf,polyadd(conv(denp,denc),conv(nump,
numc)));
nump=[(ms+m2) b2 k2]
denp=[(m1*m2) (m1*(b1+b2))+(m2*b1)
(m1*(k1+k2))+(m2*k1)+(b1*b2) (b1*k2)+(b2*k1) k1*k2]

num1=[-(m1*b2) -(m1*k2) 0 0]
den1=[(m1*m2) (m1*(b1+b2))+(m2*b1)
(m1*(k1+k2))+(m2*k1)+(b1*b2) (b1*k2)+(b2*k1) k1*k2]

numf=num1;
denf=nump;
```

Design of PID Algorithm: The PID algorithm is the most popular feedback controller algorithm used. It is a robust easily understood algorithm that can provide excellent control performance despite the varied dynamic characteristics of processes. As the name suggests, the

PID algorithm consists of three basic modes namely the Proportional mode, the Integral mode and the Derivative mode. When utilizing the PID algorithm, it is necessary to decide which modes are to be used (P, I or D) and then specify the parameters (or settings) for each mode used. Generally, three basic algorithms P, PI or PID are used to automatically adjust some variable to hold a measurement (or process variable) to a desired variable (or set-point)

The schematic representation of the PID controller and its transfer function has given below,

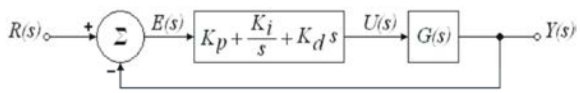

Fig. 6: Schematic block diagram of PID Controller

$$U(s) = \left(K_p + \frac{K_i}{s} + K_d s \right) E(s)$$

$$\frac{U(s)}{E(s)} = \left(K_p + \frac{K_i}{s} + K_d s \right)$$

$$= \frac{K_d s^2 + K_p s + K_i}{s} \tag{7}$$

Where, k_p is the proportional gain, k_i is the integral gain and k_d is the derivative gain. To begin the simulation, start with guessing a gain for each gain variables and can be implemented into Matlab by adding the following code into m-file:

```
numc=[kd,kp,ki];
denc=[1 0];
```

The transfer function from the input of road disturbance (z_0) to the output of suspension travel (z_s-z_{us}) is given below and it can be modeled in Matlab by adding the following code into your m-file:

```
numa=conv(conv(numf,nump),denc);
dena=conv(denf,polyadd(conv(denp,denc),conv(nump,
numc)));
```

Effects of PID Controller:

- A proportional controller (P) *reduces error responses to disturbances*, but *still allows a steady-state error.*
- When the controller includes a term proportional to the integral of the error (I), then the *steady state error to a constant input is eliminated*, although typically *at the cost of deterioration in the dynamic response.*
- A derivative control typically *makes the system better damped and more stable.*

Tuning of PID Controller by Gain Variables: The closed-loop feedback control system transfer function created in Matlab represents the plant, the disturbance, as well as the controller. The closed-loop step response due to the disturbance of (0.1m) is simulated by multiply the numerator by 0.1 and adds the following commands to the m-file step (0.1*numa, dena, t) with time (t) = 0:0.05:5 is shown below. The Fig.7 shows that the system has larger damping than the requirement, but settling time is very short. Hence, fine tuning of PID controller [6] will yield reasonable output and better responses by controlling the system is simply a matter of changing the gain variables of k_p, k_i and k_d as per the characteristics of PID controller listed above. After tuning, the close-loop step response of road disturbance [7] for (0.1m) with PID is shown in the Fig.8.

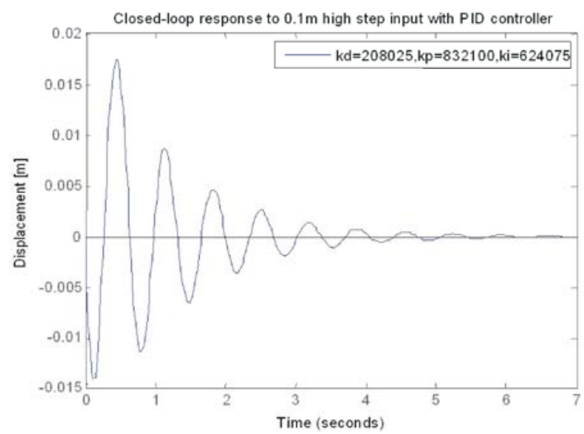

Fig. 7: Close-loop step response with PID for 0.1m step Disturbance (before tuning)

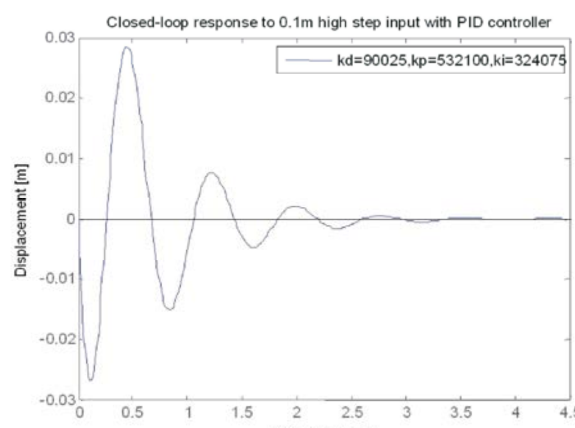

Fig. 8: Close-loop step response with PID for 0.1m step (after tuning)

Tuning of PID Controller by Root Locus: The percentage of overshoot is shown in the above Fig.7 is 9%, which is higher than the requirement of 5%, but the settling time is below 5 seconds. To choose the proper gain that will give the reasonable output by choosing a pole and two zeros for PID controller. A pole of this controller must be at zero and one of the zeros has to be very close to the pole at the origin, at 1. The other zero, we will put further from the first zero, at 3, actually we can adjust the second-zero's position to get the system to fulfill the requirement. Add the following command in the Matlab m-file, so we can adjust the second-zero's location and choose the gain to have an idea, what gain we should use for the gain variables of k_p, k_i and k_d.

```
z1=1;
z2=3;
p1=0;
numc=conv([1 z1],[1 z2])
denc=[1 p1]
num2=conv(nump,numc);
den2=conv(denp,denc);
rlocus(num2,den2)
[K,p]=rlocfind(num2,den2)
```

The closed-loop poles and zeros on the s-plane are shown in the figure given below. The gain and dominant poles can be chosen on the graph by manually and the gain values are given below

Select a point in the graphics window in Fig.9 and the selected point = -6.3152 - 3.2609i is shown in Fig.10.

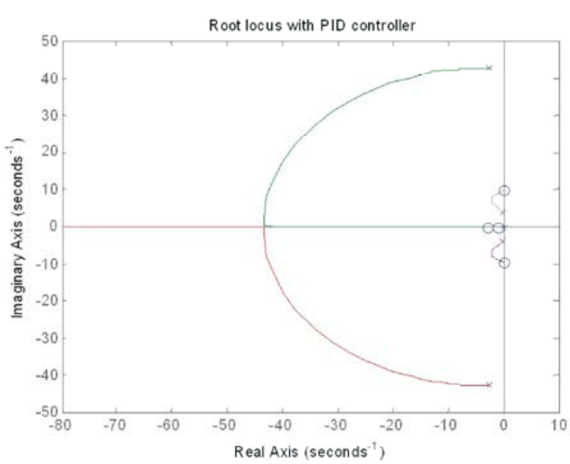

Fig. 9: Root Locus with PID controller

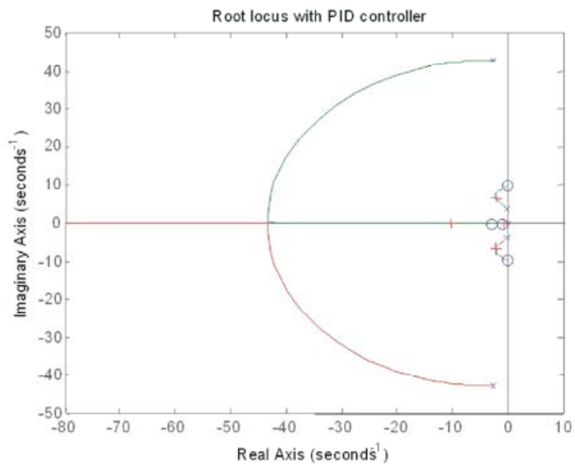

Fig. 10: Root Locus with PID controller with gain values

CONCLUSIONS

The open-loop response of passive suspension system shows that the system is under-damped, with very small amount of oscillation and the steady-state error about 0.013 mm. Moreover, it takes very long time for it to reach the steady state for a unit step actuated force. Similarly, when the road wheel passes a 10 cm high bump on the road pavement, it will oscillate for an unacceptable long time of 50 seconds with larger amplitude of 13 cm, than the initial impact. The high overshoot and the slow settling time will cause damage to the suspension system. Hence, solution to this problem is addressed by adding a close-loop feedback control system with robust and adequately used PID controller to improve the performance of the active suspension system. The system has yielded a reasonable output and better response by controlling the system by changing gain variables of k_p, k_i and k_d as per the characteristics of PID controller. The results after fine tuning of PID controller, shows that the percentage of overshoot (3%) is less than 5% of the input's amplitude and low settling time of 2 seconds Hence, it meets the design requirements of the system.

Nomenclature

m_s: Single station sprung mass, (2500kg)
m_{us}: Unsprung mass, (320kg)
c_s: Damping coefficient, (1000N/m/s)
c_s: Tyre damping coefficient, (0N/m/s)
k_s: Spring constant of suspension system (85,000 N/m)
k_{us}: Spring constant of wheel (475,000 N/m)

k_d: Derivative gain
k_i: Integral gain
k_p: Proportional gain
F_a; Actuator force, (kN)
F_a; Actuator force, (kN)
z_0: road profile, (m)
s: Laplace operator
z_s: Sprung mass vertical displacement, (m)
z_{us}: Unsprung mass vertical displacement, (m)
ù: Waviness of road,
t: time, (sec),

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