

Effect of Heat Transfer on Free Surface Flow of a Jeffrey Fluid over a Deformable Permeable Bed

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Abstract: Free surface flow of a conducting Jeffrey fluid in a channel is investigated. The channel is bounded below by a finite deformable porous layer. The governing equations are solved in the free flow region and deformable porous layer. The expressions for the velocity field and solid displacement are obtained. The effects of the Jeffrey parameter, viscosity parameter, the volume fraction component of the fluid on the velocity displacement and mass flux are discussed. It is found that the velocity increases with the increase in the non-Newtonian Jeffrey parameter.

Key words: Free surface flow • Jeffrey fluid • Porous layer • Permeable bed

INTRODUCTION

The study of flow through and past porous media has attracted the attention of many research workers because of its potential applications in industrial, physical biological and hydrological problems. The pumping of petroleum from oil wells can be improved with the knowledge of physics of flow through porous media. Also when we deal with blood flow in an artery, it will be appropriate to model the tissue region as finite porous layer. Hence mathematical modelling of porous flows plays vital role in understanding practical situations.

Rajesekhara [1] has investigated plane Couette flow in the presence of a pressure gradient and found slight deviation between his theoretical and experimental results. Channabasappa *et al.* [2] investigated analytically the effect of the thickness of the porous material on the parallel plate channel flow when the walls are provided with non-erodible porous lining. Chikh *et al.* [3] performed an analytical study on fully developed forced convection in an annular duct partially filled with a porous medium. Some important studies on the forced convection heat transfer in a parallel plate channel partially filled with porous media are made by Kuznetsov [4, 5, 6, 7]. Morosuk [8] investigated entropy generation due to flow in a pipe and parallel plate channel partially filled with porous medium. All these works are done assuming the porous media to be non-deformable.

The study of flow through deformable porous materials was initiated by Terzaghi [9] and later continued by Biot [9, 10] into a successful theory of soil consolidation and acoustic propagation. Atkin and Craine [11], Bowen [12] and Bedford and Drumheller [13] made important contributions to the theory of mixtures. Similar theory was proposed by Mow *et al.* [14] for the study of biological tissue mechanics. Water transport in the artery wall is studied by Jayaraman [15] using theory of deformable porous media. Sreenadh *et al.* [16] analyzed the Couette flow of a viscous fluid in a parallel plate channel in which a finite deformable porous layer is attached to the lower plate. Sreenadh *et al.* [17] discussed the free convection flow of a Jeffrey fluid through a vertical deformable porous stratum.

Jeffrey model is one of the best non-Newtonian fluid models used by researchers to explain the biological fluid flow in living organisms. Nadeem *et al.* [18] examined the effects of thermal radiation on the boundary layer flow of a Jeffrey fluid over an exponentially stretching surface. Vajravelu *et al.* [19] studied the influence of heat transfer on peristaltic transport of a Jeffrey fluid in a vertical porous stratum. Hayat *et al.* [20] investigated the boundary layer flow of a Jeffrey fluid with convective boundary conditions. Bhaskara Reddy *et al.* [21] studied the flow of a Jeffrey fluid between torsionally oscillating disks.

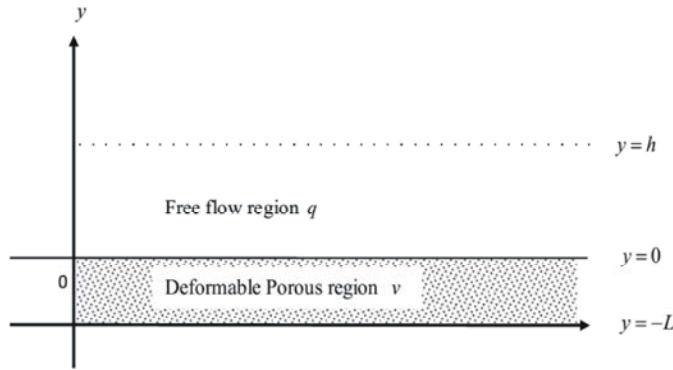


Fig. 1: Physical Model

Motivated by these studies, the effect of heat transfer on free surface flow of a Jeffrey fluid over a deformable permeable bed is investigated. The fluid velocities, the temperature, the displacement of the solid matrix and the mass flux are obtained. The effects of various physical parameters on the flow quantities are discussed through graphs.

Mathematical Formulation: Consider a steady, fully developed free surface flow of a Jeffrey fluid in a channel bounded below by a deformable porous layer of finite thickness L (Fig. 1) the nominal surface of the porous layer is taken as $y = 0$ and the free surface is represented by $y = h$. The temperature at $y = 0$ and $y = h$ are T_2 and T_1 ($<T_2$) respectively. The fluid velocity in the free surface flow region and the porous flow region are assumed to be $(q, 0, 0)$ and $(v, 0, 0)$ respectively. The displacement due to the deformation of the solid matrix is taken as $(u, 0, 0)$. A pressure gradient $\frac{\partial p}{\partial x} = G_0$ is applied, producing an axially

directed flow in the channel.

In view of the assumptions mentioned above, the equations of motion in the deformable porous region and energy equation in the free flow region are [22, 23].

$$\mu \frac{\partial^2 u}{\partial y^2} - \phi^s G_0 + K v = 0, \tag{1}$$

$$\frac{2\mu_a}{1 + \lambda_1} \frac{\partial^2 v}{\partial y^2} - \phi^f G_0 - K v = 0 \tag{2}$$

$$\frac{\mu_f}{1 + \lambda_1} \frac{\partial^2 q}{\partial y^2} = G_0 \tag{3}$$

$$K_T \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{1 + \lambda_1} \left(\frac{\partial q}{\partial y} \right)^2 = 0 \tag{4}$$

The boundary conditions are;

$$\begin{aligned} \text{at } y = -L; v = 0, u = 0 \\ \text{at } y = 0: q = \phi^f v \end{aligned} \tag{5a}$$

$$\begin{aligned} \phi^f \mu_f \frac{dq}{dy} = 2\mu_a \frac{dv}{dy} \\ \mu_f \frac{dq}{dy} = \frac{\mu}{\phi^s} \frac{du}{dy} \\ T = T_2 \end{aligned} \tag{5b}$$

$$\begin{aligned} \text{at } y = h: \frac{dq}{dy} = 0 \\ T = T_1 \end{aligned} \tag{5c}$$

Non-Dimensionalization of the Flow Quantities: It is convenient to introduce the following non-dimensional quantities.

$$y = h\hat{y}, u = -\frac{h^2 G_0}{\mu} \hat{u}, v = -\frac{h^2 G_0}{\mu_f} \hat{v}, q = -\frac{h^2 G_0}{\mu_f} \hat{q}, \varepsilon = \frac{L}{h}, \theta = \frac{\hat{T} - T_1}{T_2 - T_1}$$

In view of the above dimensionless quantities, the equations (1) – (5) takes the following form after neglecting the hats (^) are neglected.

$$\frac{d^2 u}{dy^2} = -\phi^s - \delta v \tag{6}$$

$$\frac{d^2 v}{dy^2} - \delta \eta (1 + \lambda_1) v = -\phi^f (1 + \lambda_1) \eta \tag{7}$$

$$\frac{d^2 q}{dy^2} = -(1 + \lambda_1) \tag{8}$$

$$\frac{d^2 \theta}{dy^2} + \frac{Ec Pr}{1 + \lambda_1} \left(\frac{dq}{dy} \right)^2 = 0 \tag{9}$$

where,

$$\delta = \frac{Kh^2}{\mu_f}, \quad \hat{G} = \frac{G}{G_0}, \quad \eta = \frac{\mu_f}{2\mu_a}, \quad G_0 = \frac{dp}{dx}, \quad Ec = \frac{\left(\frac{h^2 G_0}{\mu_f}\right)^2}{c_p(T_1 - T_0)}, \quad Pr = \frac{\mu c_p}{K_T}$$

we note that δ is a measure of the viscous drag of the outside fluid relative to drag in the porous medium and η is the ratio of the bulk fluid viscosity to the apparent fluid viscosity in the porous layer.

The boundary conditions are;

$$\begin{aligned} \text{at } y = -\varepsilon: v = 0, u = 0 \\ \text{at } y = 0: q = \phi^f v \end{aligned} \tag{10a}$$

$$\begin{aligned} \frac{dq}{dy} &= \frac{1}{\eta\phi^f} \frac{dv}{dy} \\ \frac{dq}{dy} &= \frac{1}{\phi^s} \frac{du}{dy} \\ \theta &= 1 + m \end{aligned} \tag{10b}$$

$$\begin{aligned} \text{at } y = 1: \frac{dq}{dy} = 0 \\ \theta = 1 \end{aligned} \tag{10c}$$

Solution of the Problem: Equations (6) - (9) are coupled differential equations that can be solved by using the boundary conditions (10). The solid displacement in the deformable porous region and fluid velocities, temperature in the free surface flow region are obtained as:

$$u(y) = -\frac{\delta c_1 e^{ay}}{a^2} - \frac{\delta c_2 e^{-ay}}{a^2} - \frac{y^2}{2} + c_5 y + c_6 \quad (-\varepsilon \leq y \leq 0) \tag{11}$$

$$v(y) = c_1 e^{ay} + c_2 e^{-ay} + \frac{\phi^f}{\delta} \quad (-\varepsilon \leq y \leq 0) \tag{12}$$

$$q(y) = c_3 + c_4 y - \frac{y^2(1 + \lambda_1)}{2} \quad (0 \leq y \leq 1) \tag{13}$$

$$\theta(y) = c_7 + c_8 y - Ec Pr (1 + \lambda_1) \left(\frac{y^2}{2} + \frac{y^4}{12} - \frac{y^4}{2} \right) \quad (0 \leq y \leq 1) \tag{14}$$

where $a = \sqrt{\delta\eta(1 + \lambda_1)}$. The constants $c_1, c_2, c_3, c_4, c_5, c_6, c_7$ and c_8 are;

$$\begin{aligned} c_1 &= \frac{1}{e^{a\varepsilon} + e^{-a\varepsilon}} \left[\frac{e^{a\varepsilon}\eta\phi^f(1 + \lambda_1)}{a} - \frac{\phi^f}{\delta} \right], c_2 = -\frac{\left[c_1 e^{-a\varepsilon} + \frac{\phi^f}{\delta} \right]}{e^{a\varepsilon}}, c_3 = (c_1 + c_2)\phi^f + \frac{(\phi^f)^2}{\delta} \\ c_4 &= 1 + \lambda_1, c_5 = (1 - \phi^f)c_4 - \frac{\delta(c_2 - c_2)}{a}, c_6 = \frac{\delta \left[c_1 e^{-a\varepsilon} + c_2 e^{a\varepsilon} \right]}{a^2} + \frac{\varepsilon^2}{2} + c_5 \varepsilon, c_7 = 1 + m \text{ and} \\ c_8 &= 1 - c_7 + \frac{Ec Pr (1 + \lambda_1)^2}{12} \end{aligned}$$

The dimensionless total mass flow rate M per unit width of the channel in the deformable porous flow region and free flow region is given by,

$$M = \int_{-\varepsilon}^0 v dy + \int_0^1 q dy = \frac{c_1(1 - e^{-a\varepsilon}) - c_2(1 - e^{a\varepsilon})}{a} + \frac{\phi^f \varepsilon}{\delta} + \frac{c_4}{2} - \frac{(1 + \lambda_1)}{6} + c_3 \quad (15)$$

RESULTS AND DISCUSSIONS

In this paper free surface flow of a Jeffrey fluid past a deformable finite porous permeable bed is investigated. The solutions for the fluid velocities q , v and temperature θ in the free flow region and deformable porous region and solid displacement of solid matrix u are evaluated numerically for different values of physical parameters such as the volume fraction of component ϕ^f , the viscous drag parameter δ , the viscosity parameter η and the thickness of lower wall ε , Jeffrey parameter λ_1 , Eckert number Ec and Prandtl number Pr .

The variation of fluid velocities q , v solid displacement u and temperature θ in the channel is calculated from equation (11) – (14) for different values of Jeffrey parameter λ_1 and is shown in Figures 2, 3, 4 and 5 for fixed $\delta = 2.0$, $\eta = 0.5$, $\phi^f = 0.5$, $\varepsilon = 0.2$, $Ec Pr = 2$ and $m = 2$. We observe that the velocities q , v and solid displacement increases whereas the temperature θ decreases with the increase Jeffrey parameter λ_1 .

The variation of temperature θ in the channel is calculated from equation (14) for different values of m and is shown in Figure 6 for fixed $\lambda_1 = 1.0$ and $ECPr = 2$. We observe that the temperature θ increases with the increase m .

The variation of temperature θ , in the channel is calculated from equation (14) for different values of $EcPr$ and is shown in Figure 7 for fixed $\lambda_1 = 1.0$ and $m = 2$. We observe that the temperature θ decreases with the increase $EcPr$.

The variation of fluid velocities in the channel q, v is calculated from equation (12) – (13) for different values of viscosity parameter η and is shown in Figures 8 and 9 for fixed $\delta = 2.0$, $\phi^f = 0.5$, λ_1 and $\varepsilon = 0.2$. We observe that the velocities q , v increases with the increase in viscosity parameter η .

The variation of fluid velocities in the channel q , v and solid displacement u in the channel is calculated from equation (11) – (14) for different values of volume fraction of component ϕ^f and is shown in Figures 10, 11 and 12 for fixed $\delta = 2.0$, $\eta = 0.5$, λ_1 and $\varepsilon = 0.2$. We observe that the velocities q , v increase with the increasing ϕ^f whereas the solid displacement u decreases with the increase in ϕ^f .

The variation of total mass flow rate for M in the free flow and deformable region is calculated from equation (15) for different values of Jeffrey Parameter λ_1 are shown in Figure 13 for fixed $\delta = 2.0$, $\eta = 0.5$ and $\varepsilon = 0.2$. We observe that the mass flow rate increases with increase in the Jeffrey parameter.

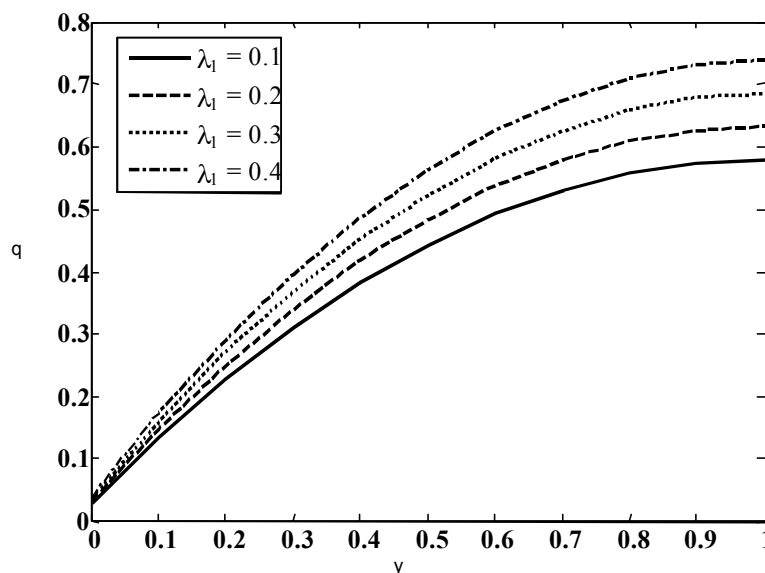


Fig. 2: Velocity profiles in free flow surface for different values of λ_1 for fixed values of $\phi^f = 0.5$, $\eta = 0.5$, $\delta = 2$ and $\varepsilon = 0.2$

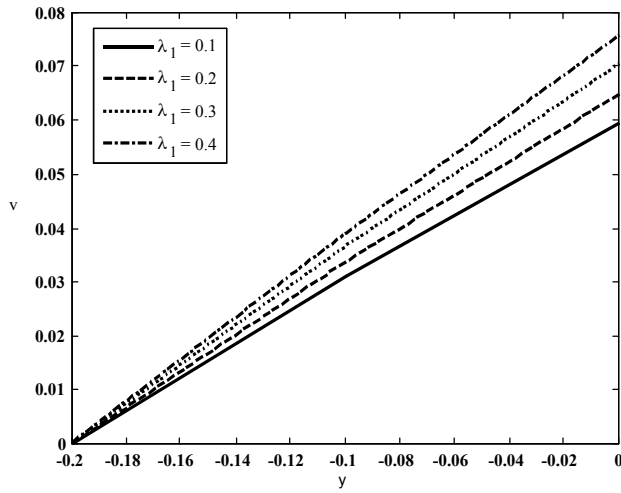


Fig. 3: Velocity profiles in deformable porous region for different values of λ_1 for fixed values of $\phi' = 0.5$, $\eta = 0.5$, $\delta = 2$ and $\varepsilon = 0.2$

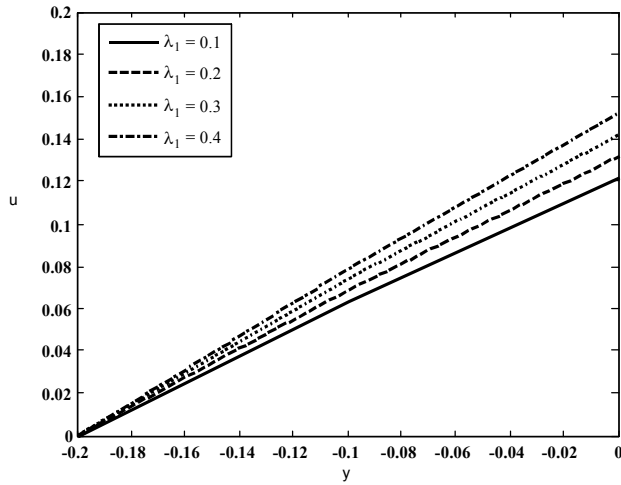


Fig. 4: Displacement profiles in deformable porous region for different values of λ_1 for fixed values of $\phi' = 0.5$, $\eta = 0.5$, $\delta = 2$ and $\varepsilon = 0.2$

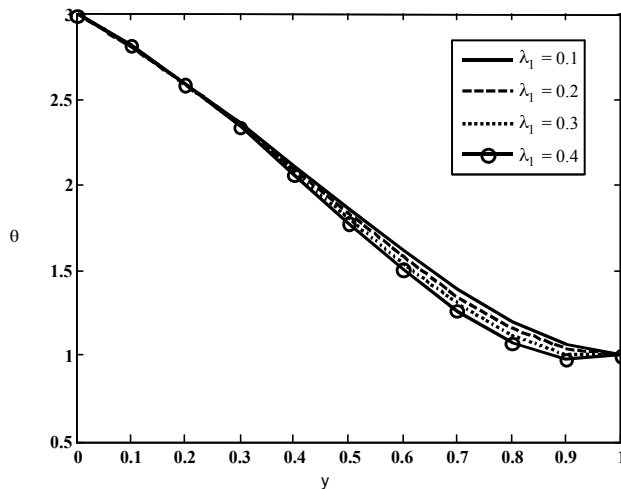


Fig. 5: Temperature profiles for different values of λ_1 for fixed values of $EcPr = 2$ and $m = 2$

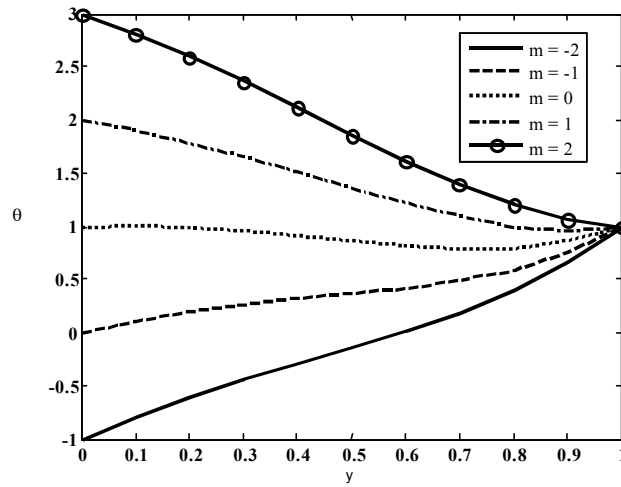


Fig. 6: Temperature profiles for different values of m for fixed values of $\lambda_1 = 0.1$ and $EcPr = 2$

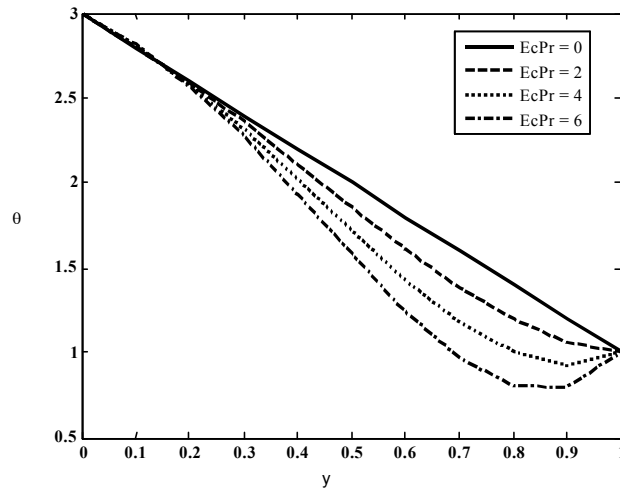


Fig. 7: Temperature profiles for different values of $EcPr$ for fixed values of $\lambda_1 = 0.1$ and $m = 2$

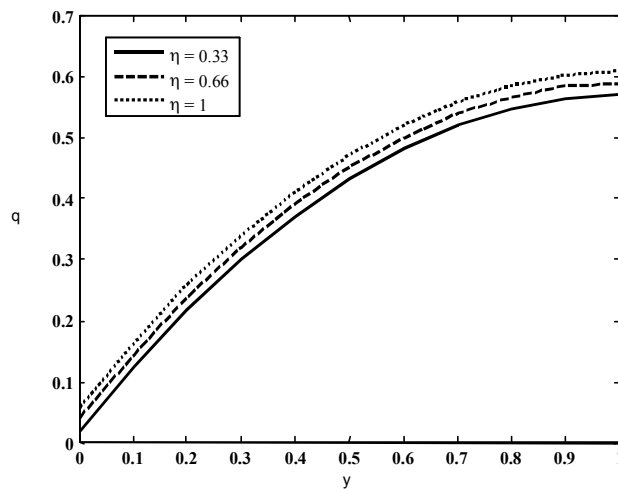


Fig. 8: Velocity profiles in free flow surface for different values of η for fixed values of $\phi = 0.5$, $\lambda_1 = 0.1$, $\delta = 2$ and $\epsilon = 0.2$

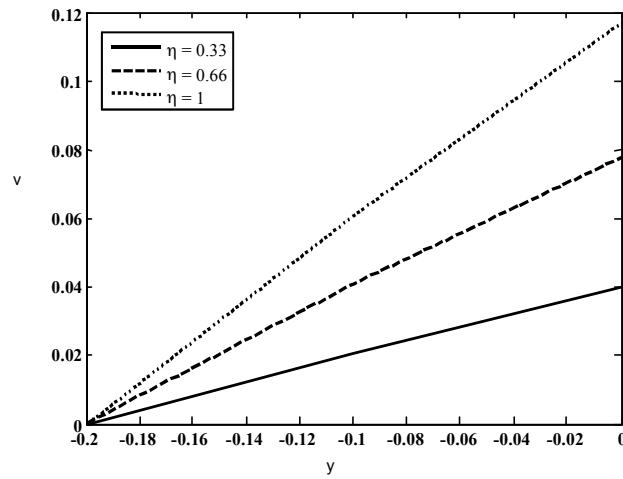


Fig. 9: Velocity profiles in deformable porous region for different values of η for fixed values of $\phi^f = 0.5$, $\lambda_1 = 0.1$, $\delta = 2$ and $\varepsilon = 0.2$

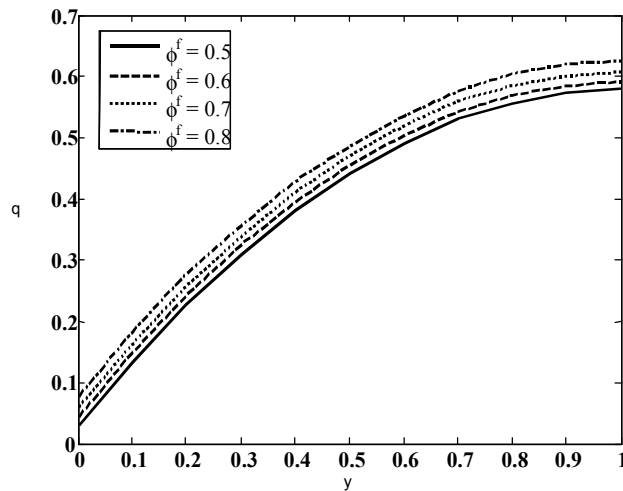


Fig. 10: Velocity profiles in free flow region for different values of ϕ^f for fixed values of $\eta = 0.5$, $\lambda_1 = 0.1$, $\delta = 2$ and $\varepsilon = 0.2$

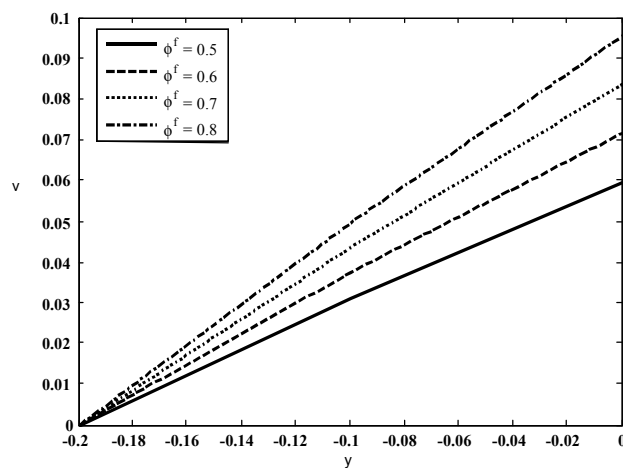


Fig. 11: Velocity profiles in deformable porous region for different values of ϕ^f for fixed values of $\eta = 0.5$, $\lambda_1 = 0.1$, $\delta = 2$ and $\varepsilon = 0.2$

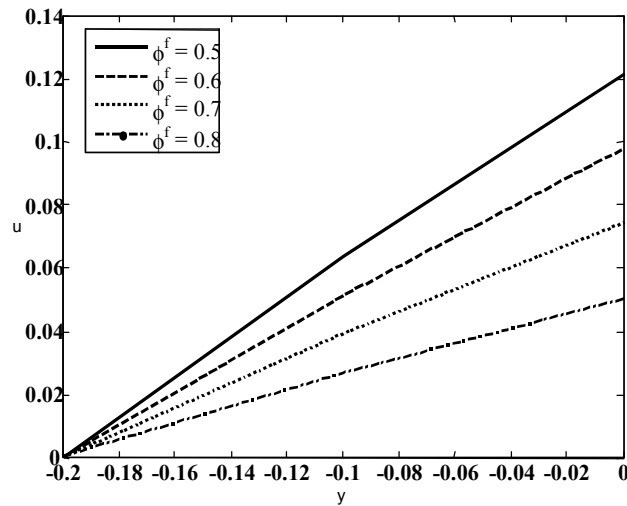


Fig. 12: Displacement profiles in deformable porous region for different values of ϕ^f for fixed values of $\eta = 0.5$, $\lambda_1 = 0.1$, $\delta = 2$ and $\varepsilon = 0.2$

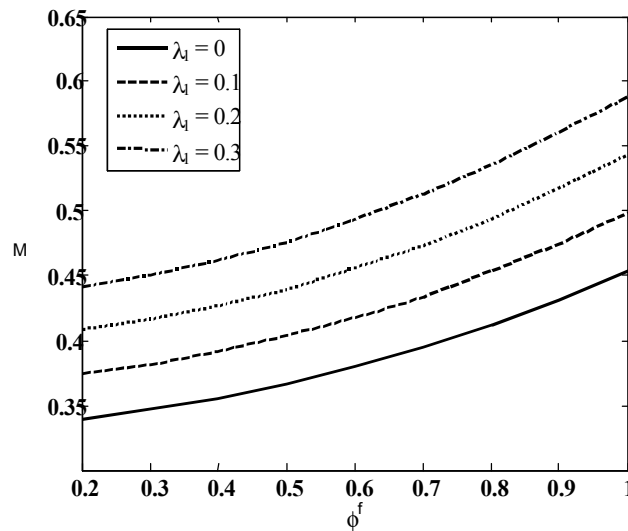


Fig. 13: Total mass flux for different values of λ_1 for fixed values of $\eta = 0.5$, $\delta = 2$ and $\varepsilon = 0.2$

CONCLUSIONS

The present study deals with free surface flow of a Jeffrey fluid over a deformable porous layer. The results are analyzed for different values of the pertinent parameters, namely, Jeffrey parameter, viscosity parameter, volume fraction component, Prandtl number, Eckert number, temperature parameter and the mass flux. The findings of the problem find applications in understanding the blood (modelled as Jeffrey fluid) flow behavior near the tissue layer (modelled as a deformable porous layer). Some of the interesting findings are as follows:

- The velocity of the fluid in the free flow region and the deformable porous layer and solid displacement increases with increase Jeffrey parameter whereas the temperature decreases with increasing Jeffrey parameter.
- The temperature in the free flow region increases with the increase in m whereas it will be decreases with increasing $EcPr$.
- An increase in the volume fraction component ϕ^f is to enhance in free flow fluid velocity between the parallel plates. But opposite behavior is observed in the case of deformable fluid flow velocity and solid displacement.

- The total mass flux in the free flow and porous flow region increases with an increase in the Jeffrey parameter.

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Nomenclature:

μ_a Apparent viscosity of the fluid in the porous material.
 K Drag coefficient.
 μ Lamé constant.
 μ_f Coefficient of viscosity.
 q Fluid velocity in the free flow region
 u Displacement in x -direction.
 G_0 Typical pressure gradient.
 ϕ^β Volume fraction of component β and $\beta = s, f$ for the binary mixture of solid and fluid phases with $\phi^s + \phi^f = 1$.
 T_0 Temperature of the ambient fluid
 m Temperature parameter $(T_2 - T_1)/(T_1 - T_0)$
 M_d Mass flow rate in the deformable porous layer.
 M_r Mass flow rate in the non deformable porous layer.
 M Mass flow rate in the channel
 v Velocity of the fluid in the deformable porous layer.
 δ Viscous drag.
 η Viscosity parameter in porous layer.
 λ_1 Jeffrey parameter.
 ε Porous layer thickness.
 K_T Thermal conductivity
 T Temperature

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