Middle-East Journal of Scientific Research 23 (Sensing, Signal Processing and Security): 249-256, 2015
ISSN 1990-9233

DOI: 10.5829/idosi.mejsr.2015.23.ssps.104

QFT Based Controller Design for Class of Nonlinear Uncertain System

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Abstract: Most of the practical systems are having uncertainty which makes it difficult to maintain stability and better performance. In this paper a robust controller design, based on the quantitative feedback theory (QFT) is proposed for a nonlinear process like Spherical tank system and bioreactor. The QFT controller is designed to ensure the robust stability and performance (input and output disturbance rejection, set point tracking and stability specification) in the presence of parametric and non-parametric uncertainty. The effectiveness of the method is experimentally verified with the spherical tank setup. From the reference tracking, input disturbance rejection and output disturbance rejection plot, it is observed that the proposed controller is able to control the process with satisfied performance compared with other controller tuning methods available in the literature.

Key words: Quantitative Feedback Theory · Spherical Tank System · Bioreactor · Loop Shaping · Robust Control.

INTRODUCTION

Quantitative Feedback theory (QFT) proposed by Isaac Horowitz [1] is a frequency response approach for controller design for the system with structured uncertainties. Horowitz's work sensitively analyzed in classical frequency response analysis involving Bode plot and template formation and Nichols Charts (NC). Template is nothing but, the area in which the response of the system for the frequency, when the process parameter varies with in the range. It incorporates the effects of plant uncertainty and gives the required performance specifications. In this approach, the plant dynamics may be described by frequency domain data or by transfer function with mixed (parametric and non-parametric) uncertainty models called interval model. QFT is used to design the controller with guaranteed performance for all linear models such an uncertainty model set. Constantine H. Houpis [2] *et al*. explained the fundamentals of QFT based controller design with its own merits and demerits. Rueda [3] *et al*. discussed the detailed procedure to design the controller using QFT for the control of the course changing of a ship. P.S.V. Nataraj [4] briefs the application of QFT for MIMO systems. Veronica Olesen [8] *et al*. used the idea of treating the nonlinearities as uncertainties and disturbances to show that only a few linear controllers are needed in gain scheduling control of the temperature in an exothermic tank reactor. In this paper bioreactor and spherical tank system are considered for the analysis. In bioreactor, when the substrate concentration alone varies, what kind of effect is in output and substrate feed rate alone varies what kind of effect in output or both varies at a time, what is the effect, are considered for the analysis.

Bioreactors [9-11] are used in many applications including industries concerned with food, beverages, pharmaceuticals, waste water treatment and alcohol fermentation. They are inherently nonlinear. New cells are produced by the reactor by consuming substrate. In this process most of the parameters like rate of substrate consumption, substrate feed and substrate concentration are varied due to various environmental changes like surrounding temperature, pressure etc.

Modeling and control of a spherical tank system is a benchmark non-linear system attempted by numerous researchers. Faccin and Trierweiler [14] proposed a simple model identification technique using the first principle analysis for the similar kind of systems. Madhavasarma and Sundaram [15] proposed a model based controller tuning for the non linear spherical tank system. They developed an approximate stable first order plus dead time (FOPDT) model around the operating region

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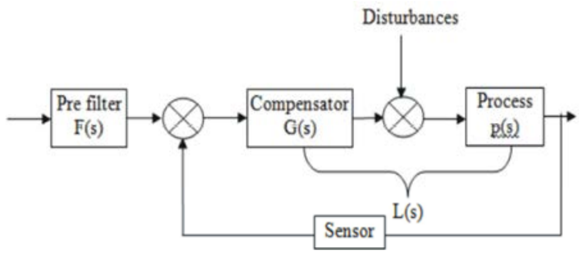

Fig. 1: Block diagram of QFT

and proposed an IMCPID controller. In this work, to evaluate the effectiveness of the proposed algorithm, a nonlinear spherical tank system is considered. The objective is to maintain the fluid level (h) when the inlet flow rate (F_{in}) and other process parameter are varied within range.

This paper is organized as, followed by the introduction, chapter 2 gives the overview of Quantitative Feedback Theory (QFT) based Controller design, chapter 3 explained that the controller design for bioreactor for both stable and unstable mode of operation. Chapter 4 explained the controller design methodology for simulated model of spherical tank system followed by it is verified in spherical tank real time setup. Chapter 5 gives the conclusion and future scope of the work.

Quantitative Feedback Controller: It is a controller or compensator, designed to control any uncertain system with its linear model in interval structure. Interval structure [12] is created due to the structured parametric uncertainty in the process parameter. It is a frequency response based controller design technique, uses Nichols chart to design the controller/compensator parameter. Fig. 1 shows the typical block diagram of the QFT controller. From the diagram, it is observed that, the controller (compensator) is placed in feedback path and a pre filter is placed in the input path. Both the controller/compensator and pre filter is designed using loop shaping method via pole placement technique.

QFT employs a two degree of freedom control configuration that includes a feedback controller and a pre filter in the feed-forward path. In this diagram p(s) is uncertain plant belongs to a set of P(s). The design methodology is quite transparent; allowing the designer to see the necessary changes to achieve both in time response and frequency response of closed loop system specifications. The design of lower order controller/compensator is a challenge task, which depends on the ability of the designer.

QFT Controller Design Procedure: Design procedure of QFT controller is having six systematic steps. Firstly Plant model should be entered either transfer function model or state space model. Then the template of nominal plant for the range of operating frequency has to be generated. Afterwards required performance specifications like stability margin, input disturbance rejection, output disturbance rejection etc. sensor noise attenuation has to be given for the bound generation. Which results frequency bound, then using loop shaping method by adding a real or/and complex pole / zero, integrator/differentiator, lead-lag compensator required shape is obtained. Detailed procedure for each step is explained below.

Plant Model (With Uncertainty), Templates Generation and Nominal Plant Selection: The creation of the templates is at specified frequencies which graphically describe the parametric uncertainty area of the process on Nichols chart. The characteristics (gain and phase) of the considered models are represented on Nichols chart for every frequency value. These N points define a closed contour, named template which limits the variation range of parametric uncertainty.

Performance Specifications: In this step, objective can be selected, like stability specification, sensor noise attenuation, output disturbance rejection, input disturbance rejection, control effort limitation, reference tracking and other user required specifications. For an example, if the output should be within 0.1percentage deviation from the reference, it can be set with the reference tracking tack, likewise for input disturbance rejection gain etc. use the concern tack.

Bounds Creation: These bounds, typically displayed on a Nichols chart-like plot, then serve as a guide for shaping the nominal loop transfer function which involves the manipulation of gain, poles and zeros. In this step QFT creates the bounds of open loop system response for various frequencies. These curves are the objects which define the bounds of the regions prohibited for the adjustment of the controller If the transfer function of the controller is denoted as G(jω) and the transfer function of the nominal plant as P0(jω), then the bounds are those regions that the open loop frequency response L0(jω), must be avoid in order to guarantee the fulfillment of the design specifications for the whole set of plants P(jω). In order to use the QFT method, the bounds need to be defined in the frequency range.

$$T(s) = \frac{F(s)G(s)P(s)}{1+G(s)P(s)} = \frac{F(s)L(s)}{1+L(S)} \quad (1)$$

$$T(j\omega) = \frac{F(j\omega)G(j\omega)P(j\omega)}{1+G(j\omega)P(j\omega)} = \frac{F(j\omega)L(j\omega)}{1+L(j\omega)} \quad (2)$$

Loop Shaping (Controller Design): In order to ensure the smallest possible order compensator (Controller), Loop shaping process starts from the nominal plant by adding zeros and poles successively, till obtain the required loop shape.

Pre-filter Synthesis: It is the additional step, to ensure the Loop shaping process starts from the nominal plant by adding zeros and poles successively, till obtain the required loop shape.

Simulation and Design Validation: It is the final step in the controller design process, where the closed loop response with controller and pre filter of the system is good enough to meet the performance.

Pid Controller Configurations: A PID controller consists of the three terms: proportional (P), integral (I) and derivative (D). Its behavior can be roughly interpreted as the sum of the three term actions. The P term gives a rapid control response and a possible steady state error, the 'I' term eliminates the steady state error and the 'D' term improves the behavior of the control system during transients [5]. It can be configured either series or in parallel form.

Bioreactor (Simulation): Bioreactors [9] are used in many applications including industries concerned with food, beverages, pharmaceuticals, waste treatment and alcohol fermentation. Fig. 2 shows the functional block diagram of bioreactor. In this setup substrate is pumped into the tank, where air is mixed and stirred using stirrer. Substrate flow rate is controlled using a control valve. Similarly final product is taken out from the reactor continuously.

After adding living cells to a reactor containing substrate, micro organisms will start to grow up. The growth of micro organism depends on many factors like substrate feed, dilution rate and biomass concentration etc. Cell growth may take place with number of phases [10] like, lag phase (no cell growth), exponential growth phase (cell growth is too high, but at some stage, amount of substrate will be limiting for the growth),

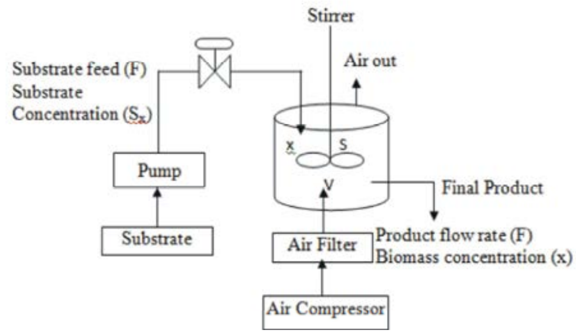

Fig. 2: Functional Block diagram of a bioreactor

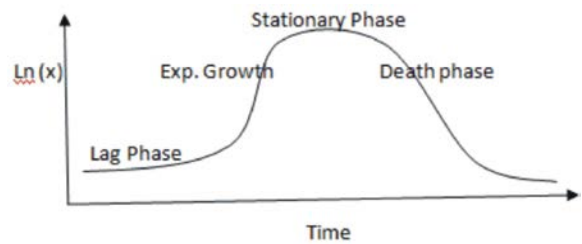

Fig. 3: Typical bacterial growth curve in a reactor after adding substrate

stationary phase (cell count neither increase nor decrease) and death phase (The number of cells decreases due to food shortage.). Fig. 3 shows different phases in cell growth.

The influent (substrate) flow rate is equal to the effluent (output) flow rate Q. Hence, the volume V is constant. The rate of accumulation of biomass is obtained from a mass balance equation. Model is derived with various assumptions, like biomass has a specific growth rate, the total amount of produced biomass per unit time in a reactor with volume V is V_x, compare with (1). Since the reactor is completely mixed, the outflow concentration of biomass is equal to the concentration in the tank.

Uncertainty in the measurement of above manipulated variable creates the deviation between the expected output and actual output. A main drawback in biotechnological process control is the problem to measure physical and biochemical parameters with higher rate of accuracy.

Process Modeling: Mathematical model is a set of equations by which the characteristics of the plant can be analyzed using various techniques. When the model is derived, certain assumptions are accounted. In this paper bioreactor [11] is modeled with the following assumption.

- Reactor contents are perfectly mixed
- Reactor operating at a constant temperature
- The feed is sterile
- The feed stream and reactor contents have equal and constant density
- The feed and product stream have the same flow rate
- The microbial culture involves a single biomass growing on single substrate

When the process parameter goes beyond the assumptions, the output will vary beyond the level. Hence a tolerance of process parameter value has to be quantified using suitable method.

The dynamic model of a bioreactor using mass balance equation is;

$$\frac{dx_1}{dt} = (\mu - D)x_1 \qquad (3)$$

$$\frac{dx_2}{dt} = D(x_{2f} - x_2) - \frac{\mu_{x_1}}{y} \qquad (4)$$

where,

x_1 = Biomass (cell) concentration (mass of cells/volume)

x_2 = Substrate concentration (mass of substrate/volume)

D = Dilution rate (Volumetric flow rate/reactor volume) – manipulated input.

μ = specific growth rate

and y = yield (biomass of cells formed / mass of substrate consumed)

$$\mu_s = \frac{\partial \mu}{\partial x_{2s}} = \frac{\mu_{\max} k_m}{(k_m + x_{2s})^2} \qquad (5)$$

where,

μ_{max} is maximum specific growth rate.

K_m is limiting substrate concentration.

Linearised model of a bioreactor is

$$A = \begin{bmatrix} \mu_s - D_s & x_{1s}\mu_s \\ -\frac{\mu_s}{Y} & -D_s - \frac{\mu_s x_{1s}}{Y} \end{bmatrix},$$

$$B = \begin{bmatrix} -x_{1s} \\ x_{2fs} - x_{2s} \end{bmatrix}, C = [1\ 0], D = [0] \qquad (6)$$

Bioreactor can be operated in different Equilibrium modes, like 1, 2 and 3. In Equilibrium mode 1, all the cells are washed out from the reactor. Hence usually reactors will not be operated in this mode. Equilibrium mode 2, produce cells but it is a unstable mode, but reactor can be operated with some constraints with a suitable controller or compensator. Equilibrium mode 3, reactor produce cells in stable mode, hence reactor can be operated and produce the cell according to the requirement. Table 1 show that the operating conditions of a bioreactor for various modes of operation.

Nominal values for the reactor variables are μ_{max} = 0.53hr^{-1}, km= 0.12g/liter, k_1=0.4545, Y=0.4, Dilution rate D_s=0.3hr^{-1}, feed substrate concentration $x_2$fs=4.0 g/liter.

Since the reactor is unstable, to operate the reactor, a controller has to be designed. In this paper QFT based controller design is attempted to determine the controller parameter for equilibrium mode 2 and 3.

Controller Design (Stable Mode): The linearised model at the equilibrium state 3 (stable mode) is:

$$A = \begin{bmatrix} 0 & 0.9056 \\ -0.75 & -2.5640 \end{bmatrix}, B = \begin{bmatrix} -1.5301 \\ 3.8255 \end{bmatrix}, \qquad (7)$$
$$C = [1\quad 0], D = [0]$$

Fig. 4 is the template, which is drawn for bioreactor in stable mode of operation with operating frequency range from. 001 to 1000 rad/sec. It is the preliminary step for the controller design using QFT. It is drawn between open loop phase and open loop magnitude of the system. It gives the boundary of response for the various operating frequency range. The range of frequency is selected, based on the operating frequency of the system.

Fig. 5 shows the Controller tuning diagram using Loop shaping method. It is the important step in Controller tuning, by adding adequate number of real and/or Complex poles and/or Zeros, shape of the tuning curve can be modified. The Controller designer has to design the possible lower order controller for the given process. During the Loop shaping process, simultaneously time response and frequency response plot for the closed loop system can be seen in the adjacent window. The controller is designed so that required performance specifications are meet out.

In this work, the compensator having 1 real pole at 1.2 and 1 real zero at 2.4883 and integrator with a gain of -4.479. The Controller is designed, which gives suitable time response and frequency response. The controller transfer function is

$$G_c(s) = \frac{-4.479(s + 2.4883)}{s(s + 1.2)} \qquad (8)$$

Table 1: Bioreactor operating mode

Sl. No	Steady state	Biomass Concentration	Substrate Concentration	Stability
1	mode 1	X1s=0	X2s=4	Stable
2	mode 2	X1s=0.99510	X2s=1.51224	Unstable
3	mode 3	X1s=1.53016	X2s=0.17459	Stable

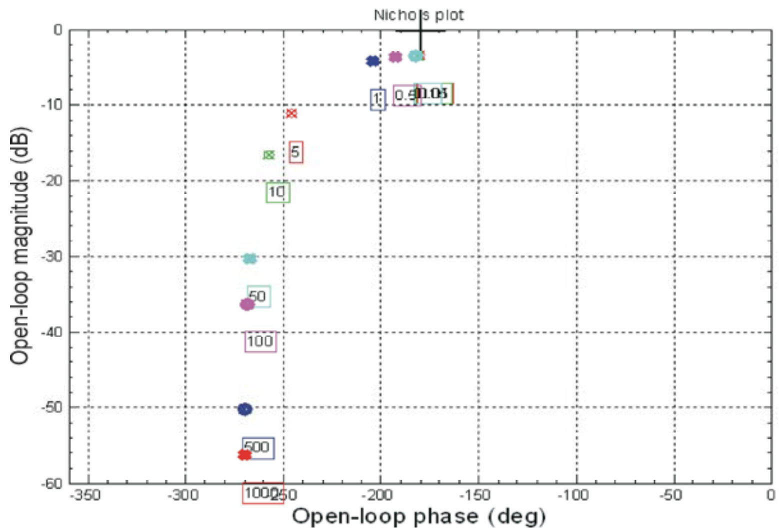

Fig. 4: Template for the bioreactor various frequencies from 0.001 to 1000 rad/Sec. (stable mode)

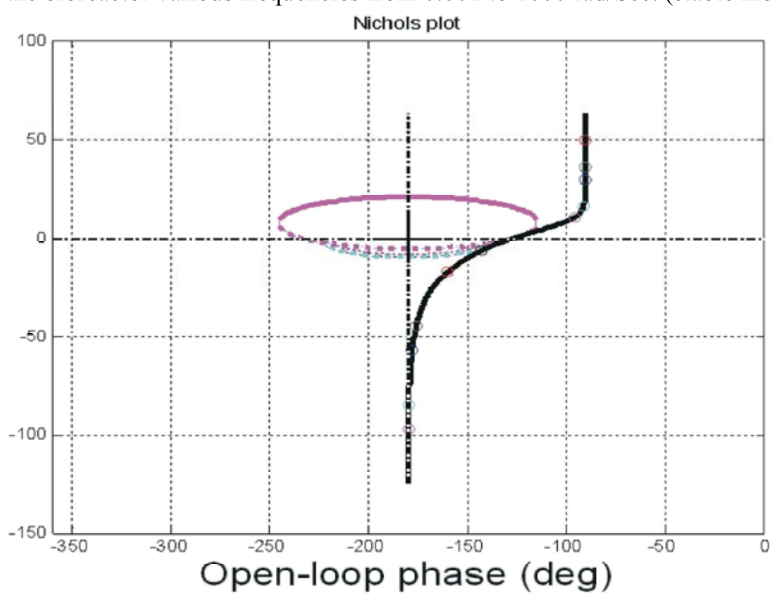

Fig. 5: Loop shaping for Controller parameter tuning (stable mode)

Controller Design (Unstable Mode): Similarly a controller was designed for unstable mode with the linearised model

$$A=\begin{bmatrix}0 & -0.0679\\ -0.75 & -0.1302\end{bmatrix} B=\begin{bmatrix}-0.9951\\ 2.4878\end{bmatrix}$$
$$C=\begin{bmatrix}1 & 0\end{bmatrix} D=\begin{bmatrix}0 & 0\end{bmatrix} \quad (9)$$

Fig. 6 Shows the Loop shaping diagram for unstable mode of bioreactor, It's also drawn between open loop phase and open loop gain. The minimum order controller / compensator is identified using Loop shaping method. The desired performance of the system is the phase margin atleast 45 degree and gain margin is 5. A compensator has been designed with a pole at 2.076 and a zero at 1.5925 with a gain of -34.925.

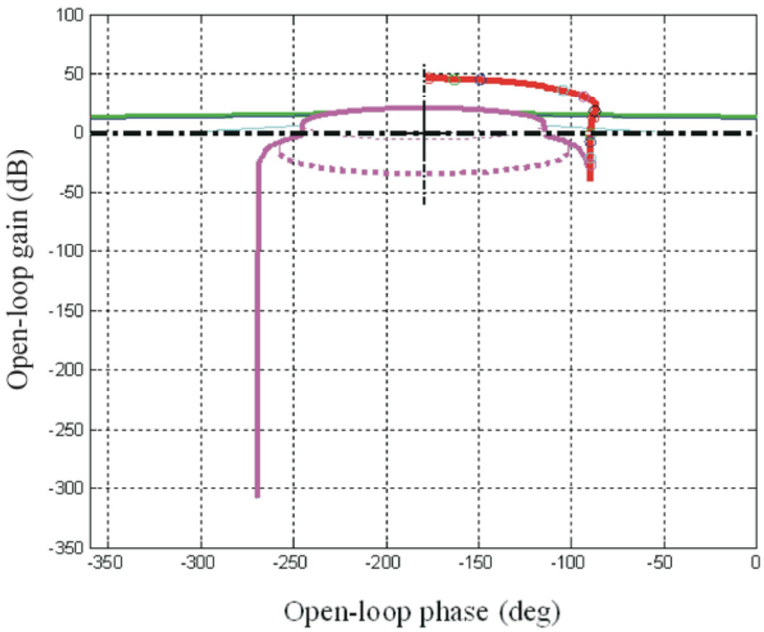

Fig. 6: Loop shaping for Controller parameter tuning (Unstable mode)

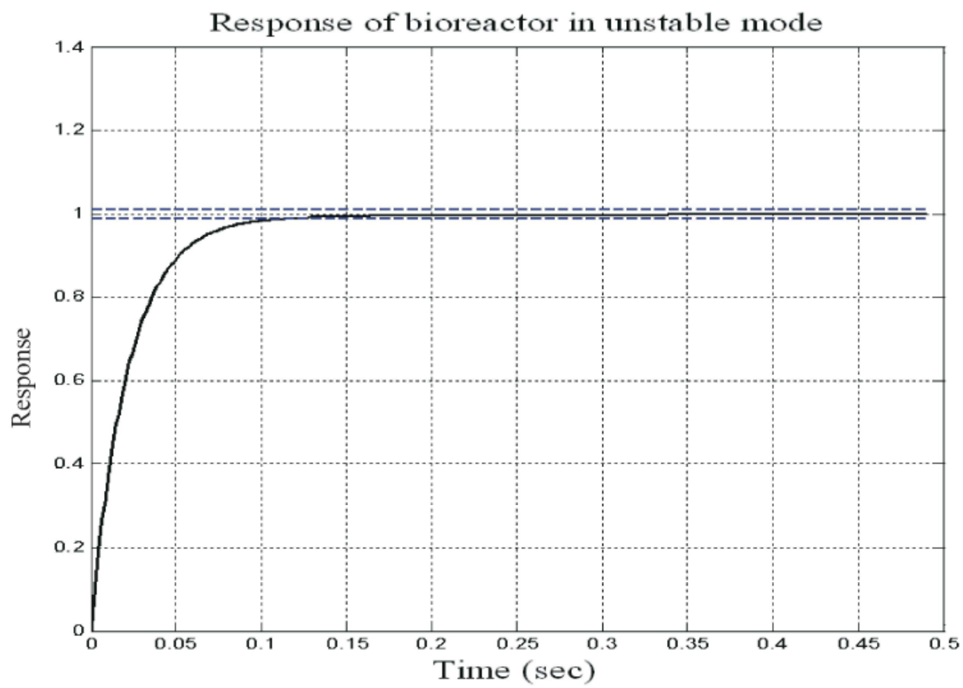

Fig. 7: Reference tracking curve with QFT Controller

$$G_c(s) = \frac{-34.925(s+1.5925)}{s+2.076} \quad (9)$$

Spherical Tank System: Figure 8 shows the real time experimental setup of the spherical tank system. It has a process tank (50 cm in diameter), water reservoir, pump, control valve with positioner, air to open valve to produce a load disturbance, rotameter (1.667-16.67 Lpm),

The modeling of the tank can be done with the mass balance equation:

$$\frac{dV}{dt} = F_{in} - F_{out} \quad (10)$$

$$A\frac{dh}{dt} = F_{in} - F_{out} \quad (11)$$

Fig. 8: Experimental setup of a spherical tank system

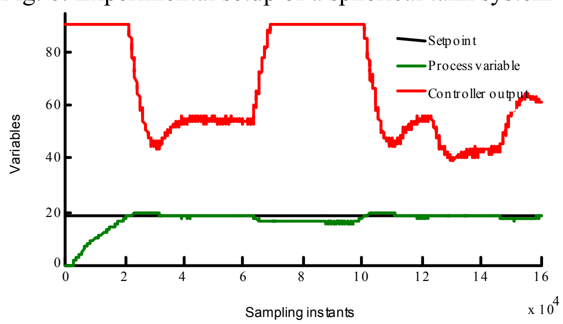

Fig. 9: Experimental output of spherical tank system

$$\frac{dh}{dt} = \frac{F_{in} - \beta * \sqrt{h}}{\Pi * h * (D - h)} \tag{12}$$

where,
F_{in} = inlet flow rate (Lpm), F_{out} = outlet flow rate (Lpm), V = tank volume (Lit),nA = area of tank (m^2), h = head, D = diameter (m) of the tank based on the head and β = outlet flow capacity coefficient.

The linearised transfer function of the system around the operating point can be developed by neglecting the wall thickness of the tank. The 'è' time delay was included to consider the delays produced by the valve and sensor dynamics. The total volume of the tank is 65.45 lit. The model parameters are F_{in}= 9 Lpm, h = 18cm and β = 2.121L.min^{-1}.cm$^{0.5}$, at this operating region, 29.549% of tank is filled with the liquid (19.34 Lit) and 70.451% is with the air and the model identified around this operating point is:

$$G_m(s) = \frac{3.6215\ e^{-11.7s}}{330.46s + 1} \tag{13}$$

A practical transfer function model is also developed based on the black box model proposed by Nithya [14] *et al*. The stable FOPDT model is identified at h = 18cm and F_{in} = 9 Lpm is.

$$G_p(s) = \frac{4.\ 185\ e^{-11.74s}}{604.30s + 1} \tag{14}$$

With experimental result, the delay by the control valve and the DPT level transmitter is accounted as 11.74sec. Online monitoring and the controller parameter tuning are performed with MATLAB software. The necessary monitoring and control program is developed in Simulink with ode45 solver. The program is interfaced with the real time process system through DAQ. It is enabled with National Instruments VISA serial communication interface module. The module supports ASCII data format with a sampling time of 0.01sec and a baud rate of 38400. Monitoring and control of the process can be established with computer control.

Fig.9 shows the response of spherical tank system when the reference level is 18 cm, obtained between the sampling instants and height of the tank, controller output and reference. Process variable follow the reference when the pertubation in input (inflow variation) and output (outflow variation by valve adjustments) with in its tolernce of variation.

Conclusion and future scope: Quantitative feedback theory (QFT), a theory for linear uncertain systems, has been used for the controller design for bioreactor and spherical tank system. parameters 1were determined, using QFT satisfy for all the stability and performance bounds. The designed QFT controllers are implemented through Real Time on a Spherical tank experimental setup. Controller design using QFT is mainly based on the uncertainty presents in each component. In this work uncertainty has been quantified using Monte Carlo and multiple experiment method that ensures the QFT design is valid for the system. At present there is no specific way to select the model of the controller, and its order. Based on the experience only order of the controller, type of the controller etc selected.

A major limitation of the method is the designed cotroller pole/zero should not be cancelled by the plant transfer function. If it is cancelled, a small increment in the controller/ compensator pole/zero can be avoid the lose of properties due to the cancellation of poles and zeros.

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