

A Special Approach of VON Bertalanffy Growth Formula for the Small Sample Size

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Abstract: This study suggest the use of bootstrap method using SAS programming for the better and accurate estimation of Von Bertalanffy growth formula for a small sample size. The length frequency data and the length at age data can be used to estimate the parameters k , L_{∞} and t_0 in the von Bertalanffy equation. However, a large sample size usually is required for a better estimate of the parameters. In some studies where the large sample size are not possible to obtain, this alternative method can be applied instead. In this study, growth parameters were estimated as $L_{\infty} = 12.664$ (SE = 0.6730, CI: 11.3182, 14.0146), $k = 0.118$ (SE = 0.0253, CI: 0.0674, 0.1688), $t_0 = 0.08$ (SE = 0.9282, CI: -1.7795, 1.9395), using the data of length at age of 59 samples. By bootstrapping the data up to 1770 samples, growth parameters were estimated at $L_{\infty} = 12.5849$ (SE = 0.1159, CI: 12.3595, 12.810), $k = 0.0337$ (SE = 0.1654, CI: -0.2907, 0.3582), t_0 (SE = 0.0045, CI: 0.1107, 0.1284). Therefore, useful results and conclusions can be drawn from the analysis. We also provided the algorithm of application of the method discussed by using SAS computer software.

Key words: Von Bertalanffy • Sample Size • Bootstrapping • SAS

INTRODUCTION

The von Bertalanffy growth equation are the most commonly used model to express growth of fish [1-4]. The von Bertalanffy equation in terms of length is:

$$L_t = L_{\infty} \times (1 - \exp[-k(t - t_0)]) \quad (1)$$

(Sources = Beverton and Holt, 1957) [1].

where,

L_t is the length at the age t

L_{∞} is the theoretical maximum or asymptotic length that the species would reach if it had lived indefinitely

k is a growth coefficient which is a measure of the rate at which maximum size is reached

t_0 is the theoretical age at zero length, often has a small negative value [2, 3].

The von Bertalanffy growth parameters for fish can be estimated through various methods such as from the

length frequency distribution, mark recapture experiments or growth checks form in the hard structure of fish such as scales, vertebrae and otolith [2-6]. The estimation of the growth parameters can be estimated from the length frequency data and length at age data using a small sample size, but the large sample size is preferable for more accurate estimates. However, in some studies, the large sample size is difficult to obtain especially when working with critical, endangered or low abundance species where sampling a big population is not possible to obtain or the difficulty in estimating the age from otolith due to its shape or size.

This study investigate the possible use of bootstrap method using SAS programming for better estimate of growth parameters for small length at age data.

MATERIALS AND METHODS

Preliminary works on the age determination of fish species has been carried out to prepare the length at age data. The length at age data is the age of fish for a given

length. In this study, age determination was carried out by investigating the growth rings in the otolith of banded pufferfish, *Marilyna pleurosticta*. From a total of 203 fish samples, only 59 otoliths can be confidently aged due to the small size of the otolith. The ages and corresponding length are analysed using a non-linear regression approach in SPSS where von Bertalanffy growth equation (Equation 1) is fitted to the length at age data to estimate growth parameters; L_{∞} and k . SAS Programming for Bootstrap Bertalanffy Method is given by:

With Bootstrapping Method

```
Title"Von
Bertalanffy";
Data one;
input y x;
Cards;
11.20 13.00 8.80 8.00 /*BOOTSTRAPPING DATA WITH
5.20 5.00 10.90 12.00 A CASE RESAMPLING*/
6.30 7.00 8.40 10.00 odsrtffile='robdunc0.rtf'style=journal;
12.40 18.00 9.30 18.00 procsurveyslectdata=one out=boot1
8.90 15.00 9.30 21.00 method=urssamprate=1outhitsrep=1
11.80 14.00 6.20 6.00 0;run;
5.80 5.00 12.30 28.00
6.20 5.00 5.80 5.00
4.90 4.00 10.80 19.00 /*THIS SAYS TO RETURN THE BEST
5.60 6.00 10.70 13.00 SINGLE FIT, BUT ONLY TRY TO
12.80 22.00 12.30 17.00 FIT THE MODEL 10 TIMES BEFORE
9.10 12.00 8.60 10.00 STOPPING*/procnldata=boot1
10.80 15.00 9.20 14.00 outest=parmest maxiter=10;
5.70 5.00 6.60 7.00
7.50 10.00 11.40 24.00 title'VB Growth';
4.50 4.00 8.40 8.00 parms linf=100 to 105 by 1
12.50 18.00 10.70 12.00 to=0 to 0.2 by 0.1
9.20 14.00 10.60 18.00 k= 0.17;
10.90 19.00 10.90 17.00
10.90 16.00 10.00 17.00 /*BELOW IS THE EQUATION FOR
11.50 22.00 9.50 11.00 THE VONBERTALANFFY GROWTH
9.40 15.00 11.10 21.00 QUATION USING THE
11.60 17.00 10.40 14.00 PARAMETER NAMES GIVEN
9.50 13.00 9.60 16.00 ABOVE*/
9.90 15.00 8.60 10.00 model y=linf*(1-exp(-((k)*(x - to))));
12.80 23.00 11.10 19.00 outputout=c parms=Linf to k p=vbp
10.00 15.00 11.50 23.00 r=yresid;run;odsrtfclose;
12.10 17.00 9.60 15.00
13.60 19.00 12.20 22.00
11.20 15.00;
```

RESULTS AND DISCUSSION

The Von Bertalanffy growth parameters obtained by non linear fitting method is shown in Table 1 and Table 2. The summary values of L_{∞} and k using 59 samples and 1770 samples (using bootstrap with the number of replicate 30) is given by Table 1 and Table 2.

Table 1: Result without Bootstrap

59 samples				

Approximate 95% confidence limit				

Parameter	Estimate	Approx Std error	Lower	Upper
L_{∞}	12.6664	0.6730	11.3182	14.0146
t_0	0.0800	0.9282	-1.7795	1.9395
k	0.1181	0.0253	0.0674	0.1688

Table 2: Result with Bootstrap (Number of replicate 30)

1770 samples				

Approximate 95% confidence limit				

Parameter	Estimate	Approx Std error	Lower	Upper
L_{∞}	12.5849	0.1149	12.3595	12.810
t_0	0.0337	0.1654	-0.2907	0.3582
k	0.1195	0.0045	0.1107	0.1284

Using the normal procedure with 59 samples we obtained $L_{\infty} = 12.664$, $k = 0.118$, $t_0 = 0.08$. using the data of length at age of 59 samples. While using bootstrap methods with number of replicate 30 (1770 samples) the growth parameters were estimated as $L_{\infty} = 12.5849$, $k = 0.118$, $t_0 = 0.08$. We can see clearly the approximation standard error for the error by using bootstrap method is much better compared to the original. The smaller the standard error, the more representative the sample will be of the overall population (small standard error will approach the actual value).

CONCLUSION

This paper explained on how an alternative programming of Von Bertalanffy growth methods using SAS software can be applied for the small sample size which the data is very difficult to collect. By resampling (using bootstrap method), it provides the preliminary comprehensive information and also give the general overview on how the data behaviour eventhough the original data is not enough (small sample size). In our case, smaller standard error of the estimate parameter will tell us how accurate our estimate parameter is likely to be.

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