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Analysis and Optimization of PID Controller Parameters for a Second-Order Transfer Function

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Abstract: Proportional-integral-derivative (PID) controllers are the most widely-used type of controllers for industrial and many other applications. Thus, for the purpose of analysis of the proposed PID controller, a model with a second-order transfer function plant is built to optimize the plant model parameters (coefficients). In order to define a relation between PID controller parameters and transfer functions parameters, three methods of PID tuning were used and simulated. Implementation of the values of PID controller parameters in the modelmay lead to reasonable steady state response. Steady state time was also included as a parameter to obtain the actual values of PID controller parameters.

Key words: PIDcontroller · Tuningparameters · Steady state response · Output constraint · Actuator constraint · Step response specification

INTRODUCTION

PID controllers find a great deal of interest and employed inmany industrial applications. They are structurally simple and exhibit robust performance over a wide range of operating conditions. In the absence of complete knowledge of the process these types of controllers are the most efficient choices. In these controllers, three main parameters are involvedsuch asproportional (P), integral (I) and derivative (D). The proportional part is responsible for following the desired set point (so called reference tracking), while the integral and derivative parts account for the accumulation of past errors and the rate of change of error in the process, respectively.

Among all controlling methods, PID controllersare more popular thanother controlling methods. Even it is simply known as"bread and butter" of control engineering. It is an important component in every control engineer’s toolbox. In addition, it isthe most common form of feedback and is the most flexible and simple method. In the process control, more than 95% of control loops are PID type. One can say that the most loops are actually PI control. PID controllers are today found in all areas where control is used and needed. The controllers come in many different forms. A proportional–integral–derivative (PID) controller is a three-term controller that has a long history in the automatic control field, starting from the beginning of the last century [1-6]. Thegeneral form of a PID controller is given byEquation 1.

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{de(t)}{dt}$$

In order to achieve optimized overall response, tuning of a PID controller refers to the adjustment (or update) of its various parameters (P, I and D). The basic requirements of the responsearestability, desired rise time, peak time and overshoot.

Different processes have different requirements of these parameters which can be achieved by meaningful tuning of the PID parameters. If the system can be taken offline, the tuning method involves analysis of the step-response of the system to obtain different PID parameters. But in most of the industrial applications, the system must be online and tuning is achieved manually which requires veryexperienced personnel. There is always uncertainty due to human error. Another method of tuning can be Ziegler-Nichols method [1, 2]. While this method is good for online calculations, it involves some trial and error which is not very suitable in some cases.

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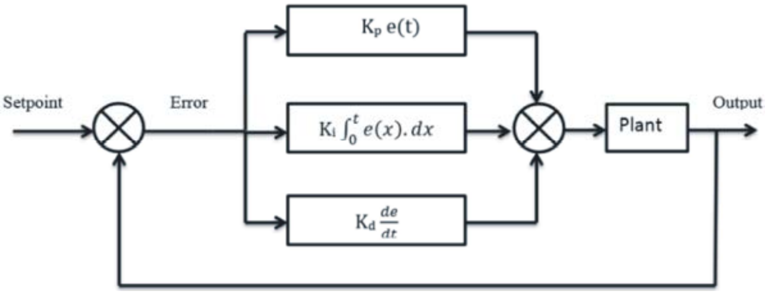


Fig. 1: Schematic diagram of PID controller

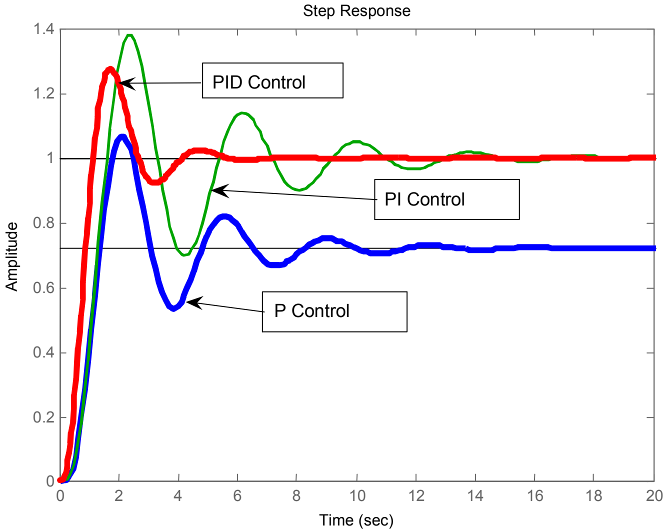


Fig. 2: Reasonable steady state response

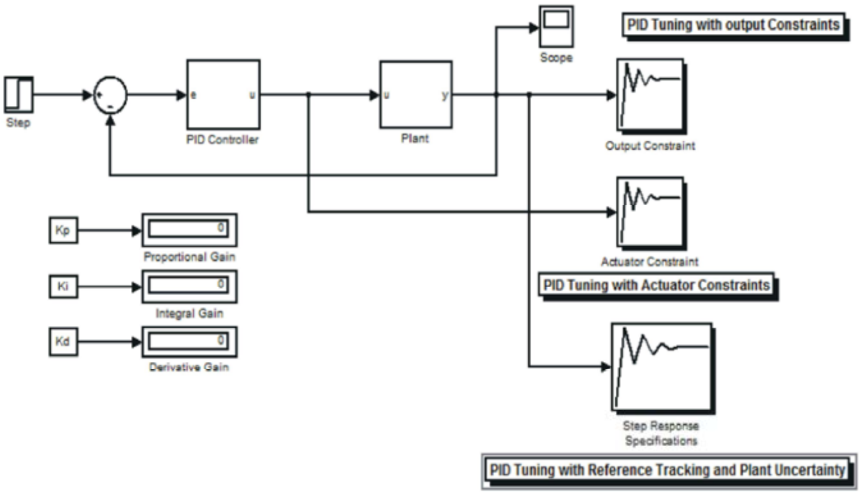


Fig. 3: A proposed model for PID controller

Equation1illustratesthe output function based on setpoint. Figure 1 shows the structure of aPID controller [3-5]. PID tuning meansthe application of a method of optimization in order to calculate PID controller parameters (P, I and D) [6-8]. Thus, use of these parameters to build a model which includes PID controller. Simulating this model must lead to a reasonable steady state response (steady state amplitude equal to 1 unit, without any error (0%), overshoot (0%) and minimum steady state time [9,10]. Reasonable steady state response is shown in Figure 2.

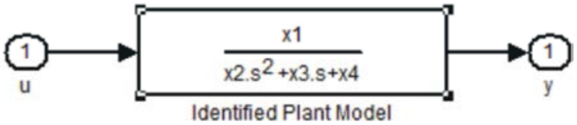


Fig. 4: Plant modelwith second order transfer function

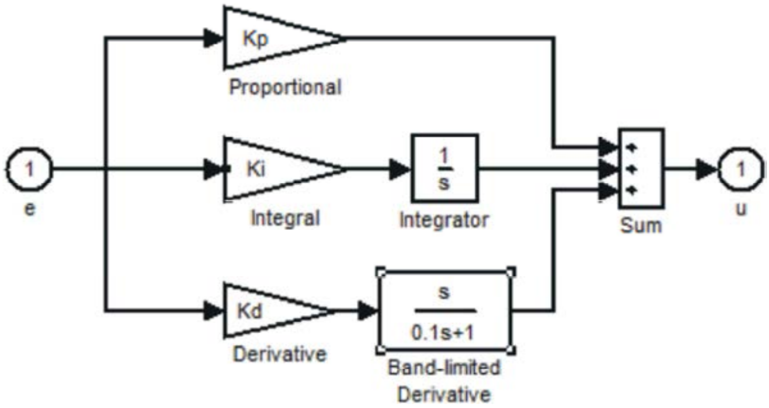


Fig. 5: Schematic diagram of PID controller model

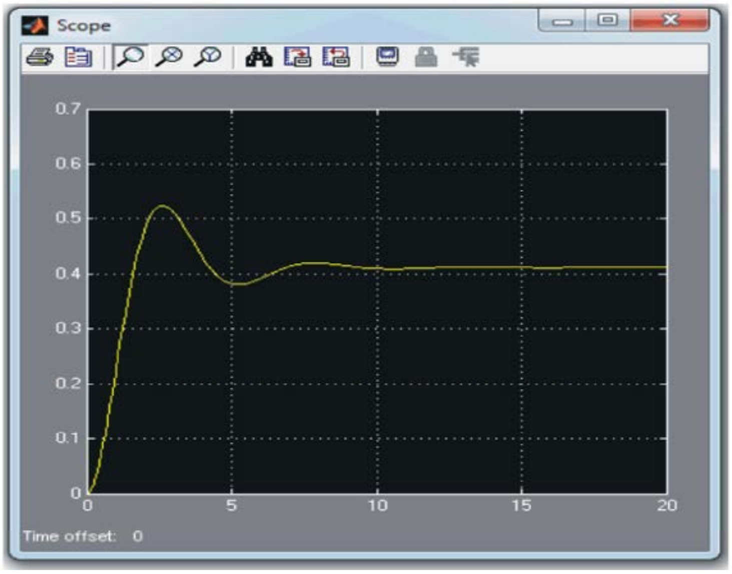


Fig. 6: Model system response

The Proposed Model: A PID controller is to be added to a plant model as shown in Figure 3.

The controlled plant is specified by a second order transfer function as shown in Figure 4. Figure 5 shows the PID controller structure.

The proposed model containsthree methods of PID tuning optimization:

- Output constraint
- Actuator constraint
- Step-response specification

After giving some initial values for X_1, X_2, X_3 and $X_4$and running the model,theobtainedstep-response isshown in Figure 6.

Now at this stage applying tuning method;one can obtain the optimized values for PID controller parameters. Using the new values of PID and running the model again we can obtain a reasonable response as shown in Figure 7.

Experimental Results: We conducted experimental runs; the output actuator method of optimizationusing the

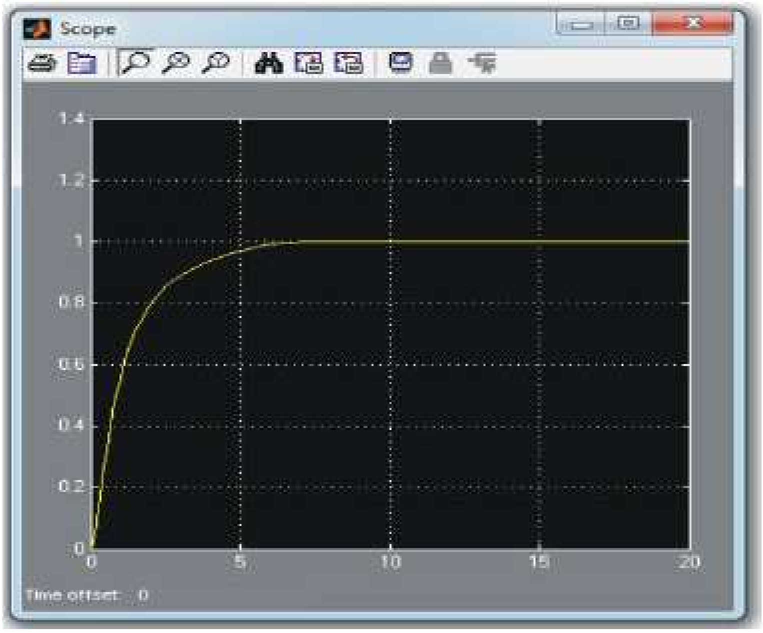


Fig. 7: Reasonable system'sstep-response

Table 1: Experimental results for output actuator method

X_1	X_2	X_3	X_4	K_p	K_i	K_d	Steadystate time(s)	Iterations
1	1	1	1	1.934	0.2253	0.224	10	128
2	1	1	1	2.2177	0.7206	0.198	10	10
3	1	1	1	1.4597	0.5630	0.2542	10	8
4	1	1	1	0.9327	0.5981	0.2977	5	7
5	1	1	1	0.9327	0.5981	0.2977	4	2
6	1	1	1	0.918	0.5669	0.2967	3	2
7	1	1	1	0.2699	0.2097	0.3576	5	8
6	2	1	1	0.5036	0.3159	0.5036	10	102
6	3	1	1	0.1221	0.1287	0.3620	5	6
6	4	1	1	0.1021	0.1073	0.3823	5	102
6	5	1	1	0.1022	0.0984	0.3903	15	102
6	0.1	1	1	2.579	0.6991	0.0961	7	10
6	0.2	1	1	1.8932	0.6048	0.1047	5	10
6	0.3	1	1	1.8009	0.5927	0.2073	5	10
6	0.4	1	1	1.5805	0.5544	0.2305	5	9
6	0.5	1	1	1.4401	9.5275	0.2320	5	7
6	0.6	1	1	1.3661	0.5202	0.2544	5	3
6	0.6	2	1	2.1893	0.5786	0.1207	5	10
6	0.6	3	1	1.2238	0.3160	-0.174	5	8
6	0.6	4	1	2.0311	0.3491	-0.2019	5	8
6	0.6	5	1	1.4717	0.3328	-0.2410	5	9
6	0.6	0.1	1	0.3287	0.3658	0.1980	7	8
6	0.6	0.2	1	0.5019	0.3260	0.2799	7	7
6	0.6	0.3	1	0.7426	0.4016	0.3167	7	7
6	0.6	0.4	1	0.5681	0.3643	0.2167	7	7
6	0.6	1	2	0.6346	0.5275	0.3275	7	9
6	0.6	1	3	0.8605	0.7171	0.2838	7	8
6	0.6	1	5	0.9533	1.0756	0.2753	7	8
6	0.6	1	0.1	0.7262	0.8246	0.3184	5	3
6	0.6	1	0.2	0.7282	0.8246	0.3184	5	3

Table 2: Experimental results for step response specification method

X_1	X_2	X_3	X_4	K_p	K_i	K_d	Steady-state time (sec)	Iterations
1	1	1	1	3.8773	1.5104	12.3365	20	10
2	1	1	1	1.6325	0.2231	0.8019	38	7
3	1	1	1	0.3220	0.1461	2.2114	35	8
4	1	1	1	0.0846	0.1189	0.1366	18	9
5	1	1	1	0.5832	0.1398	0.8868	18	102
6	1	1	1	0.4975	0.2109	0.6074	10	13
7	1	1	1	0.3838	0.3009	1.7150	20	13
6	2	1	1	1.3990	0.4771	5.5334	20	15
6	3	1	1	1.9636	0.7653	3.5150	15	63
6	4	1	1	0.9378	0.4158	2.2082	15	102
6	0.1	1	1	0.5092	0.4435	1.1202	16	7
6	0.6	1	1	0.4893	0.2071	0.9282	16	16
6	0.6	4	1	0.2566	0.0809	1.0496	40	15
6	0.6	0.4	1	0.1224	0.1937	0.2079	15	102
6	0.6	1	5	0.5453	0.8578	0.3563	5	16
6	0.6	1	0.2	0.2649	0.0985	1.9426	40	12

model isshown in Figure 3. Various values of transfer function parameterswere applied. In order to obtain optimal values for PID controller parameters, we tuned the model using the output constraint method of PID tuning. Table 1 shows the experimental results of tuned values optimization.

Similarly, we used actuator constraint method of optimization to run the model shown in Figure 3 using various values of transfer function parameters. In order to obtain optimal values for PID controller parameters, we tuned the model using actuator constraint method of PID tuning. Here, we obtained the same results as summarized in Table 1. In the presented data, one can observe the number of optimization iterations decreased.

Stepresponse specification methodwas used for the optimization to run the same model using various values of transfer function parameters; each time we tune the model using step response specification method of PID tuning in order to obtain optimal values for PID controller parameters. As the steadystate time is doubled, the values of P, I and D were extremely increased. That caused the number of iterations extremely increased; data are shown in Table 2.

RESULTS AND DISCUSSION

Applying multiple regression based on the data obtained in Table 1, we obtain the relationship between PID controller parameters and the transfer function parameters (Equations 2, 3 and 4).

$$K_p=2.3802-0.2124*X_1-0.3852*X_2+0.2281*X_3-0.0397*X_4 \quad (2)$$

$$K_i=0.769+0.0877*X_1-0.2742*X_2-0.1352*X_3-0.015*X_4 \quad (3)$$

$$K_d=0.2823+0.0062*X_1+0.054*X_2-0.1293*X_3+0.01*X_4 \quad (4)$$

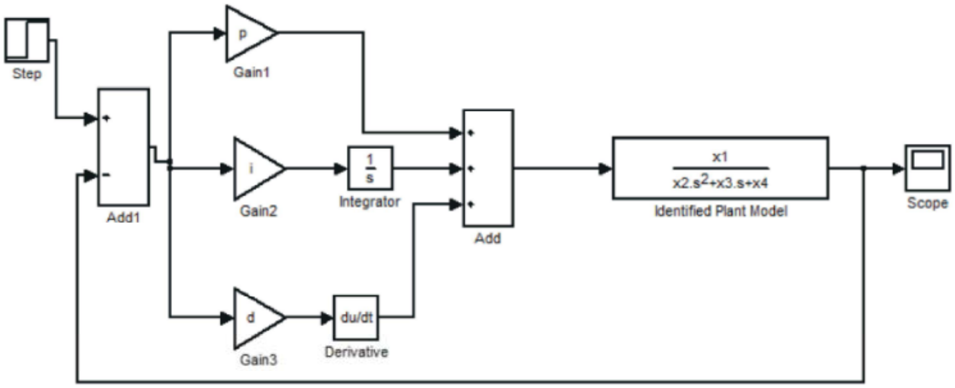
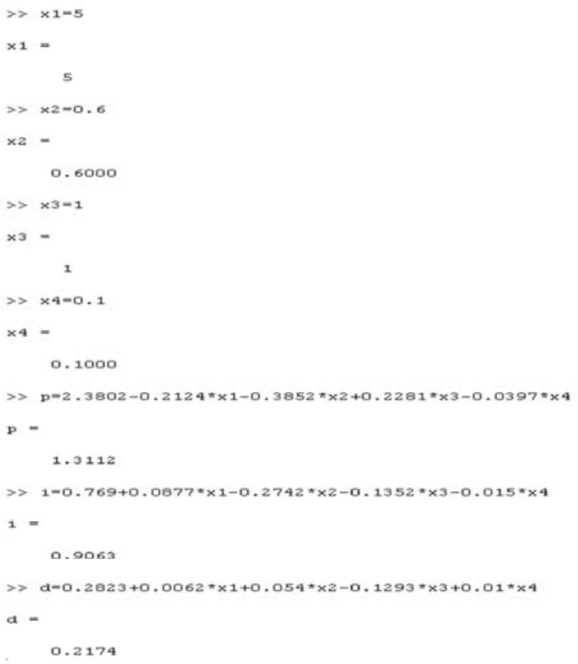


Fig. 8: Fabricated experimental model

These equations were tested as follows:

- A model like in Figure 8 was built.
- The values for X_1, X_2, X_3 and X_4 were initialized.
- P, I and D were calculated using Equations 2,3 and 4.
- The model was run.

Here are some results of calculations:



```
>> x1=5
x1 =
      5
>> x2=0.6
x2 =
    0.6000
>> x3=1
x3 =
      1
>> x4=0.1
x4 =
    0.1000
>> p=2.3802-0.2124*x1-0.3852*x2+0.2281*x3-0.0397*x4
p =
    1.3112
>> i=0.769+0.0877*x1-0.2742*x2-0.1352*x3-0.015*x4
i =
    0.9063
>> d=0.2823+0.0062*x1+0.054*x2-0.1293*x3+0.01*x4
d =
    0.2174
```

Now running the model, we can obtain a reasonable system'sstep-response as shown in Figure 9.

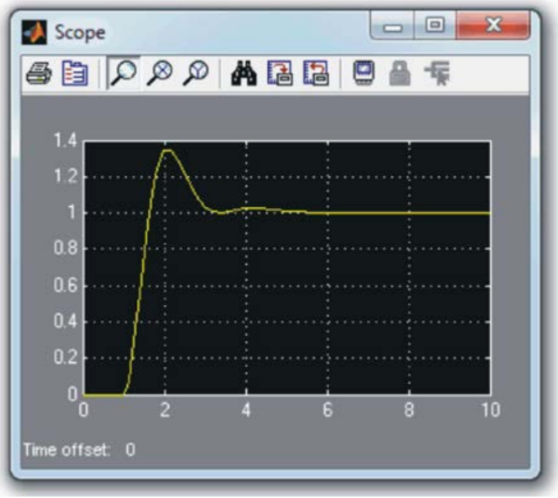


Fig. 9: Model step-response

Now, we include steady state time (SST)as a parameter to get the values of PID controller parameters. Again, applying multiple regressions the following equationsare obtained:

$$Kp=2.0672-0.1862*X_1-0.3608*X_2+0.2384*X_3-0.0555*X_4+0.0327*sst \quad (5)$$

$$Ki=1.2838+0.0448*X_1-0.2173*X_2-0.1521*X_3+0.0110*X_4-0.0537*sst \quad (6)$$

$$Kd=0.3415+0.0012*X_1+0.0606*X_2-0.1313*X_3+0.0130*X_4-0.0062*sst \quad (7)$$

These equations were tested in the model shown in Figure 8 stated as follows:

- The values for X_1, X_2, X_3 and X_4 were initialized
- P, I and D were calculated using Equations 2, 3 and 4
- The model was run

Here are some results of calculations:

```
>>X1=6;
>>X2=0.6;
>>X3=1;
>>X4=1;
>>sst=5;

>>Kp=2.0672-0.1862*X1-0.3608*X2+0.2384*X3-
0.0555*X4+0.0327*sst
Kp = 1.0799
>>Ki=1.2838+0.0448*X1-0.2173*X2-
0.1521*X3+0.0110*X4-0.0537*sst
Ki = 1.0126
>>Kd=0.3415+0.0012*X1+0.0606*X2-
0.1313*X3+0.0130*X4-0.0062*sst
Kd = 0.2358
```

Figure10 shows the system's step response:

Now we apply multiple regressionsto get the relationship between PID controller parameters and transfer function parameters based on the data reported in Table 2. Here we obtained the following equations:

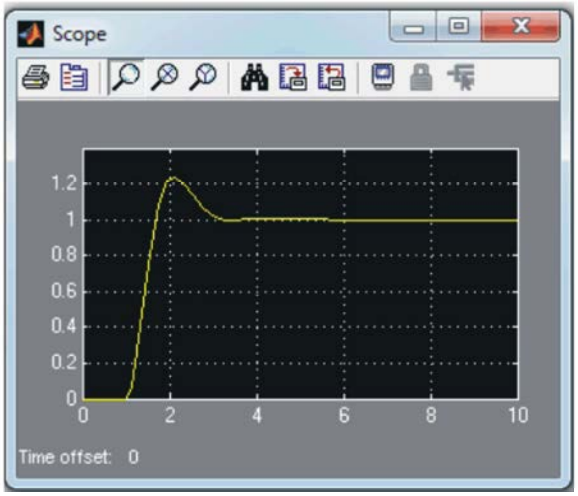


Fig. 10: Model step-response

$$K_p=2.2767-0.3713*X1+0.3617*X2-0.0068*X3+0.0574*X4 \qquad (8)$$

$$K_i=0.6071-0.0958*X1+0.1057*X2-0.0415*X3+0.1602*X4 \qquad (9)$$

$$K_d=6.2940-0.9516*X1+0.7882*X2+0.0801*X3-0.1900*X4 \qquad (10)$$

These equations were tested and they demonstratedgood results as shown below:

```
>>X1=5;
>>X2=0.6;
>>X3=1;
>>X4=0.1;

 >>Kp=2.2767-0.3713*x1+0.3617*x2-0.0068*x3+0.0574*x4
Kp = 0.6362
>>Ki=0.6071-0.0958*x1+0.1057*x2-0.0415*x3+0.1602*x4
Ki = 0.1660
 >>Kd=6.2940-0.9516*x1+0.7882*x2+0.0801*x3-0.1900*x4
Kd = 2.0700
```

The parameters obtained in this study were used in the model shown in Figure 8 and the model delivered reasonable steady state responses in all cases.

CONCLUSIONS

A model for a plant with second-order transfer function was designed.Output constraint, actuator constraint and step response specification methods of optimization of PID tuning were used. A relationship between PID controller parameters and transfer function parameters were calculated. In addition, steady-state time was included as a parameter to get PID controller parameters.

The obtainedequations were tested and the results showed that they can be used to calculate P, I and D parameters for optimization purposes.

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