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# A Study of Three-Level Atom with One-Mode Cavity Field in the Presence of Time-dependent Coupling Parameter and Kerr-like Medium

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**Abstract:** In this paper, we study a model for an atomic system described by a moving three-level Λ-type atom and one-mod radiation field. The model describes multi-photon processes and includes the nonlinearities Kerr-like medium. Also, the coupling parameters are taken in time-dependent. We used the Schrödinger equation in solving this problem. The solution is obtained when the atom is initially in a superposition coherent state. The momentum increment and the field entropy of coherent field of this atomic system are calculated. The effects of the detuning, the Kerr-like medium and the time-dependent coupling on the collapses-revivals and the entanglement phenomena are discussed. The momentum increment as well as the entropy squeezing are considered and it has been shown that existence of the time dependent coupling parameter leads to a time delaying in the interaction which is twice the delay time for the independent case.

**Key words:** Three-level atom . time-dependent coupling parameter . field entropy

## INTRODUCTION

The interaction between electromagnetic fields and matter (atoms) lies at the heart of quantum optics. After more than three decades the Jaynes-Cummings Model (JCM) [1] is still the best model to represent this concept. This model is one of the exactly solvable models describing the interaction between a two-level atom and a single mode cavity field. Also, this model has been realized experimentally [2]. Moreover, it has various extensions such as; an atom has three, four or five effective levels while the field has one or only a few cavity modes. In particular, the formalism of the JCM for a three-level atom under different configurations (Vee (V), Cascade (Ξ) and Lambda (Λ)) interacting with a single or two-mode cavity field has been demonstrated [3-5]. Also, some of these extensions are based on considering the multiphotons transitions [6, 7], the atom field coupling in intensity [8, 9] and the Kerr-like medium [10]. Moreover, it is shown that the Schrödinger equation can be solved exactly in the Rotating Wave Approximation (RWA) [11]. A Kerr-like medium can be modeled by an harmonic oscillator [10, 12]. The Kerr nonlinearity corresponds to a Hamiltonian is quadratic in the photon number operator. Physically, the model with Kerr-like medium may be realized as if the cavity contains two different species of Rydberg atoms of which one behaves like a harmonic oscillator in the field and the other interacts with the field mode.

Over the years, the JCM has also been extended to include the atomic motion along the axis of the cavity [13]. Also, an extension of the standard JCM has been made to include atomic external effects due to quantization of atomic motion, where the center-of-mass motion of an atom is cooled extremely low temperature, so vibrational motion is quantized [14]. The statistics of the atomic external and internal quantities such as the radiation force and atomic momentum for the Raman-coupled JCM are studied [15, 16]. Also, the interaction between a moving three-level Λ-type atom with one and two-mode cavity field containing a Kerr-like medium in this cavity are investigated [17, 18]. These investigations are considered where the atom is initially prepared in a momentum eigenstate and the field is in the squeezed state. The atomic system consists of a moving Rubidium atom interacting with a single mode cavity field is discussed [19]. Moreover, the interaction of a moving N-level ladder type atom and (N-1)-mode cavity field in the resonant case is discussed [20]. Recently, the system consists of a four-level ladder type in a momentum eigenstate coupled with a single mode cavity field in the presence of nonlinearities of both the intensity-dependent coupling is explored [21]. Also, the non-resonant interaction of a moving four-level N-type

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atom with three-mode cavity field in the existence of a nonlinear Kerr-like medium is studied [22]. On the other hand, the phase distribution depends on the coherent field intensity and the detuning parameter is explored [23]. Also, the properties of the entropy and phase of the field in the two-photon JCM with an added Kerr medium are studied [24]. Furthermore, a method to accelerate the revivals, undoing the dynamics by a suitable manipulation of the two-level system, more specifically by a quasi-instantaneous change of its phase has been shown [25].

More recently, the nonclassical properties of the state and dynamics of entropy of a Λ-type three-level atom interacting with a single-mode cavity field with intensity-dependent coupling in a Kerr medium is explored [26]. Also, a model of a three-level atom in the Λ-configuration interacting with a two-mode field under a multi-photon process is considered [27]. A semi-classical versus quantum description of the ground state of three-level atoms interacting with a one-mode electromagnetic field is studied [28]. Moreover, the quantum entanglement and position-momentum entropic squeezing of a moving Lambda-type three-level atom interacting with a single-mode quantized field with intensity-dependent coupling is investigated [29]. The interaction between a Λ-type three-level atom and two quantized electromagnetic fields which are simultaneously injected in a bichromatic cavity surrounded by a Kerr medium in the presence of field-field interaction (parametric down conversion) and detuning parameters is considered [30]. Furthermore, the dynamics of a pair of short laser pulse trains propagating in a medium consisting of three-level Λ-type atoms by numerically solving the Maxwell-Schrödinger equations for atoms and fields is discussed [31]. Now, we turn our attention to give the importance of the three-level quantum systems. The importance lies in that it describes the essential physics of radiation-matter interaction. Some authors discussed the collapses and revivals phenomena [32-35]. Also, the two-photon excitation [36-38] and two-photon laser [39, 40] have been investigated. Moreover, the squeezed light [41-44], chaos [45], coherence trapping [46, 47] and optical communications [48] have been demonstrated. Recently, the entanglement of the atom field and quantum entropy has been discussed [17]. The entanglement has been used in quantum information such as super coding [49] and quantum teleportation [50].

## DESCRIPTION OF THE MODEL AND SOLUTION

The considered system consists of a moving three-level Λ-type atom interacts with a single mode cavity field with frequency $\Omega$. The atom has upper state $|1\rangle$, intermediate state $|2\rangle$ and lower state $|3\rangle$ with energies $\omega_1$, $\omega_2$ and $\omega_3$, respectively. We suppose that the allowed transitions $|1\rangle\leftrightarrow|2\rangle$ and $|1\rangle\leftrightarrow|3\rangle$ while the transition $|2\rangle\leftrightarrow|3\rangle$ is forbidden. The Hamiltonian describing the non-resonant atom-field interaction including the center of mass of the atom beside the presence of the Kerr-like medium is given by

$$\hat{H} = \hat{H}_\chi + \hat{H}_A + \hat{H}_F + \hat{H}_{AF} \qquad (1)$$

where $\hat{H}_\chi$ is the non-linearity term, $\hat{H}_A(\hat{H}_F)$ is the Hamiltonian of the atom (field) and $\hat{H}_{AF}$ is the interaction Hamiltonian, for simplicity, we set $\hbar = 1$. In the RWA these terms are:

$$\hat{H}_\chi = \chi \hat{a}^{\dagger 2}\hat{a}^2$$

$$\hat{H}_A = \frac{\vec{P}^2}{2M} + \sum_{j=1}^{3}\omega_i \sigma_{ii}$$

$$\hat{H}_F = \Omega \hat{a}^\dagger \hat{a} \qquad (2)$$

and

$$\begin{aligned}\hat{H}_{AF} &= \lambda_1(t)\left(\hat{a}^m e^{im\vec{k}\vec{r}}\hat{\sigma}_{12} + \hat{a}^{\dagger m}e^{-im\vec{k}\vec{r}}\hat{\sigma}_{21}\right)\\ &+ \lambda_2(t)\left(\hat{a}^m e^{im\vec{k}\vec{r}}\sigma_{13} + \hat{a}^{\dagger m}e^{-im\vec{k}\vec{r}}\sigma_{31}\right)\end{aligned} \qquad (3)$$

where, $\vec{P},\vec{k}$ and $\vec{r}$ are the momentum, the propagation vector and the position vector, respectively, $\chi$ is the dispersive part of the third-order nonlinearity of the Kerr medium, $\lambda_\ell(t)$ is the effective coupling parameter and taking $\lambda_\ell = \varepsilon_\ell \cos(\delta_\ell t)$, where $\varepsilon_\ell(\ell = 1,2)$ is an arbitrary constant ( the constant coupling parameter), m is multiplicity of photons . In what follows, we shall present some interesting properties of the atom (field) operators of the considered model. The operators $\hat{\sigma}_{ij}$ are the generators of the unitary group satisfying the following commutation relations:

$$\left[\hat{\sigma}_{ij}, \hat{\sigma}_{k\ell}\right] = \hat{\sigma}_{i\ell}\delta_{jk} - \hat{\sigma}_{kj}\delta_{\ell i},\ \left[\hat{a}^m, \hat{\sigma}_{k\ell}\right] = \left[\hat{a}^{\dagger m}, \hat{\sigma}_{k\ell}\right] = 0 \qquad (4)$$

where $\delta_{ij}$ is the Kroneker symbol and $\sigma_{ij}|j\rangle = |i\rangle$. Also, the operators $\hat{a}$ and $\hat{a}^\dagger$ satisfy the canonical commutation relation $\left[\hat{a}, \hat{a}^\dagger\right] = 1$ while $\left[\hat{a}, \hat{a}\right] = \left[\hat{a}^\dagger, \hat{a}^\dagger\right] = 0$. In the general form it is easy to show that:

$$\left[\hat{a}, \hat{a}^{\dagger m}\right] = m\hat{a}^{\dagger(m-1)},\ \left[\hat{a}^\dagger, \hat{a}^m\right] = -m\hat{a}^{(m-1)} \qquad (5)$$

Moreover, the field operators satisfy the following relations:

$$\hat{a}^{m}|n\rangle = \sqrt{\frac{n!}{(n-m)!}}\,|n-m\rangle \quad n > m$$

$$\hat{a}^{\dagger m}|n\rangle = \sqrt{\frac{(n+m)!}{n!}}\,|n+m\rangle \qquad (6)$$

To obtain the wave function $|\psi(t)\rangle$ at any time t>0, we write it as a linear combination of the states $|\vec{P}_0,1,n\rangle$, $|\vec{P}_0 - m\vec{k},2,n+m\rangle$ and $|\vec{P}_0 - m\vec{k},3,n+m\rangle$, where $|\vec{P}_0\rangle$ is the momentum eigenstate, $|j\rangle$ denotes jth atom level and n is the photon number of the field. Thus, we consider

$$|\psi(t)\rangle = \sum_{n=0}^{\infty} q_n \Big\{ A(n,t)e^{-i\gamma_1 t}|\vec{P}_0,1,n\rangle + B(n,t)e^{-i\gamma_2 t} \times |\vec{P}_0 - m\vec{k},2,n+m\rangle + C(n,t)e^{-i\gamma_3 t}|\vec{P}_0 - m\vec{k},3,n+m\rangle \Big\} \qquad (7)$$

where

$$\gamma_1 = \frac{\vec{P}_0^2}{2M} + \omega_1 + \Omega n$$

$$\gamma_{\ell+1} = \frac{(\vec{P}_0 - m\vec{k})^2}{2M} + \omega_{\ell+1} + \Omega(n+m) \qquad (8)$$

The quantities A(n,t), B(n,t) and C(n,t) are the probability amplitudes and $q_n$ is defined as

$$q_n = \exp(-\bar{n}/2)\,\bar{n}^{n/2}/\sqrt{n!} \qquad (9)$$

where $\bar{n}$ is the initial mean photon number. Applying Schrödinger equation, we obtain the following system of coupled ordinary differential equations:

$$i\dot{A}(t) = V_1 A(t) + \tilde{f}_1 B(t)\cos(\delta_1 t)e^{i\tilde{\Delta}_1 t} + \tilde{f}_2 C(t)\cos(\delta_2 t)e^{i\tilde{\Delta}_2 t}$$

$$i\dot{B}(t) = V_2 B(t) + \tilde{f}_1 A(t)\cos(\delta_1 t)e^{-i\tilde{\Delta}_1 t}$$

$$i\dot{C}(t) = V_2 C(t) + \tilde{f}_2 A(t)\cos(\delta_2 t)e^{-i\tilde{\Delta}_2 t} \qquad (10)$$

and

$$V_1 = \chi n(n-1)$$

$$V_2 = \chi(n+m)(n+m-1)$$

$$\tilde{f}_\ell = \varepsilon_\ell \sqrt{\frac{(n+m)!}{n!}}$$

$$\tilde{\Delta}_\ell = \omega_1 - \omega_{\ell+1} + m\Omega - \frac{m\vec{k}\cdot\vec{P}_0}{M} + \frac{m^2k^2}{2M} \qquad (11)$$

It should be noted that when $\delta_\ell = 0$, then the coupling parameter between the atom and the field is constant as shown in [17]. We employ an approximation where the rapid oscillating terms can be ignored. This can be achieved if one can adjust the detuning so that the difference between $\Delta_\ell$ and the parameter $\delta_\ell$ becomes too small (slowly oscillating term), the coupled differential equation (10) becomes

$$i\dot{A}(t) = V_1 A(t) + f_1 B(t)\left(e^{i(\tilde{\Delta}_1+\delta_1)t} + e^{i(\tilde{\Delta}_1-\delta_1)t}\right) + f_2 C(t)\left(e^{i(\tilde{\Delta}_2+\delta_2)t} + e^{i(\tilde{\Delta}_2-\delta_2)t}\right)$$

$$i\dot{B}(t) = V_2 B(t) + f_1 A(t)\left(e^{-i(\tilde{\Delta}_1-\delta_1)t} + e^{-i(\tilde{\Delta}_1+\delta_1)t}\right)$$

$$i\dot{C}(t) = V_2 C(t) + f_2 A(t)\left(e^{-i(\tilde{\Delta}_2-\delta_2)t} + e^{-i(\tilde{\Delta}_2+\delta_2)t}\right) \qquad (12)$$

where

$$f_\ell = \frac{1}{2}\tilde{f}_\ell$$

As one can see there are two exponential terms in each equation: one is rapidly oscillating terms $\exp[i(\delta_\ell + \tilde{\Delta}_\ell)t]$ and the other is slowly varying terms $\exp[i(\delta_\ell - \tilde{\Delta}_\ell)t]$. In this case if we neglect the rapidly varying term compared with the slowly varying term, then (12) reduces to:

$$i\dot{A}(t) = V_1 A(t) + f_1 B(t)e^{i\Delta_1 t} + f_2 C(t)e^{i\Delta_2 t}$$

$$i\dot{B}(t) = V_2 B(t) + f_1 A(t)e^{-i\Delta_1 t}$$

$$i\dot{C}(t) = V_2 C(t) + f_2 A(t)e^{-i\Delta_2 t} \qquad (13)$$

where

$$\Delta_\ell = \tilde{\Delta}_\ell - \delta_\ell$$

This approximation is physically acceptable and it may be compared with the well known RWA. It is interesting to point out that such approximation has been used extensively to derive several physical models, for instance the frequency converter and parametric amplifier models. To solve the coupled system (13) we consider that the atom and the field are initially prepared in superposition state and coherent state, respectively. In this case the initial wave function can be written as;

$$|\psi(t=0)\rangle = \sum_{n=0}^{\infty} q_n\left[\cos(\theta)|1,n\rangle + \sin(\theta)e^{i\Psi}|3,n+m\rangle\right] \qquad (14)$$

where $\Psi$ is the relative phase of the two levels. It is obvious that for $\theta = 0$ and $(\pi/2)$, the atom is initially in the upper (lower) state $|1\rangle$ ($|3\rangle$), respectively.

Firstly, we consider that the Kerr medium is absent and the system in the off-resonance case ($\Delta_\ell = \Delta$). Hence, the probability amplitudes under these initial conditions have the form;

$$A(t) = e^{\frac{i\Delta t}{2}}\left(\cos\theta\cos\gamma t - i\left\{\Delta\cos\theta + 2f_2 e^{i\Psi}\sin\theta\right\}\frac{\sin\gamma t}{2\gamma}\right)$$

$$B(t) = \frac{f_1 e^{\frac{-i\Delta t}{2}}}{f_1^2+f_2^2}\Big[\left(2f_2 e^{i\Psi}\sin\theta + \Delta\cos\theta\right)\left(2\gamma\cos\gamma t + i\Delta\sin\gamma t\right) + 2\cos\theta\left(2i\gamma\sin\gamma t + \Delta\cos\gamma t\right) - 2\gamma\left(2f_2 e^{i\Psi}\sin\theta + \Delta\cos\theta\right) - 2\Delta\cos\theta\Big]$$

$$C(t) = \frac{f_2 e^{\frac{-i\Delta t}{2}}}{f_1^2+f_2^2}\Big[\left(2f_2 e^{i\Psi}\sin(\theta) + \Delta\cos(\theta)\right)\left(2\gamma\cos(\gamma t) + i\Delta\sin(\gamma t)\right) + 2\cos(\theta)\left(2i\gamma\sin(\gamma t) + \Delta\cos(\gamma t)\right) - 2\gamma\left(2f_2 e^{i\Psi}\sin(\theta) + \Delta\cos(\theta)\right) - 2\Delta\cos(\theta) - e^{i\Psi}\sin(\theta)\Big] \quad (15)$$

where

$$\gamma = \sqrt{\left(\frac{\Delta}{2}\right)^2 + f_1^2 + f_2^2}$$

Now, let us consider that the more general case when detuning and nonlinearity take place in the interaction. In this case, the probability amplitudes are given by

$$A = -e^{i(\Delta_2-\delta)t}\sum_{j=1}^{3} C_j(\mu_j + V_2)e^{i\mu_j t}$$

$$B = e^{i(\Delta_2-\Delta_1)t}\sum_{j=1}^{3}\frac{C_j}{f_2 f_1}\left[\left(\mu_j + V_2\right)\left(\mu_j + \Delta_2 + V_1\right) - f_2^2\right]e^{i\mu_j t}$$

$$C = \sum_{j=1}^{3} C_j e^{i\mu_j t} \quad (16)$$

where

$$C_j = \frac{\Re_1 + \Re_2}{\mu_{jk}\mu_{ji}} \quad (17)$$

With

$$\Re_1 = e^{i\Psi}\sin(\theta)\left(f_2^2 + V_2^2 + V_2(\mu_k + \mu_i) + \mu_k\mu_i\right)$$

$$\Re_2 = f_2\cos(\theta)\left(V_1 + V_2 + \Delta_2 + \mu_k + \mu_i\right) \quad (18)$$

where $\mu_{jk} = \mu_j - \mu_k \quad i \neq j \neq k = 1,2,3$ and $\mu$ satisfies the third-order equation

$$\mu^3 + x_1\mu^2 + x_2\mu + x_3 = 0 \quad (19)$$

The general expressions for these roots are given by

$$\mu_j = -\frac{1}{3}x_1 + \frac{2}{3}\sqrt{x_1^2 - 3x_2}\cos\left(\xi + \frac{2}{3}(j-1)\pi\right) \quad (20)$$

with

$$\xi = \frac{1}{3}\cos^{-1}\left(\frac{9x_1x_2 - 2x_1^3 - 27x_3}{2\left(x_1^2 - 3x_2\right)^{3/2}}\right) \quad (21)$$

where

$$x_1 = V_1 + 2V_2 - \Delta_1 + 2\Delta_2$$

$$x_2 = \Delta_2(V_1 + 3V_2 - \Delta_1 + \Delta_2) + V_2(2V_1 + V_2) - \Delta_1(V_1 + V_2) - f_1^2 - f_2^2$$

$$x_2 = \begin{bmatrix}\Delta_2(V_1 + 3V_2 - \Delta_1 + \Delta_2) \\ +V_1(V_2 - \Delta_1) - f_1^2 - f_2^2\end{bmatrix}V_2 - f_2^2(\Delta_2 - \Delta_1) \quad (22)$$

In the next section, we shall obtain some statistical aspects of the formulated model such as the momentum increment and the field entropy.

## THE MOMENTUM INCREMENT

Let us first use the well known definition of the expectation value of any dynamical operator

$$\langle\hat{O}(t)\rangle = \langle\psi(t)|\hat{O}|\psi(t)\rangle$$

and find for the generators $\sigma_{ii}$ are

$$\langle\sigma_{11}\rangle = \langle A|A\rangle,\ \langle\sigma_{22}\rangle = \langle B|B\rangle,\ \langle\sigma_{33}\rangle = \langle C|C\rangle \quad (23)$$

with

$$|A\rangle = \sum_{n=0}^{\infty} q_n A(t)|n\rangle$$

$$|B\rangle = \sum_{n=0}^{\infty} q_n B(t)|n+m\rangle$$

$$|C\rangle = \sum_{n=0}^{\infty} q_n C(t)|n+m\rangle \quad (24)$$

Also, the expectation value of the atomic momentum increment is given by $\langle\Delta\vec{P}\rangle = \langle\vec{P}\rangle - \vec{P}_0$, it can be written as

$$\langle\Delta\vec{P}\rangle = -m\vec{k}\left[\langle\hat{\sigma}_{22}\rangle + \langle\hat{\sigma}_{33}\rangle\right] \quad (25)$$

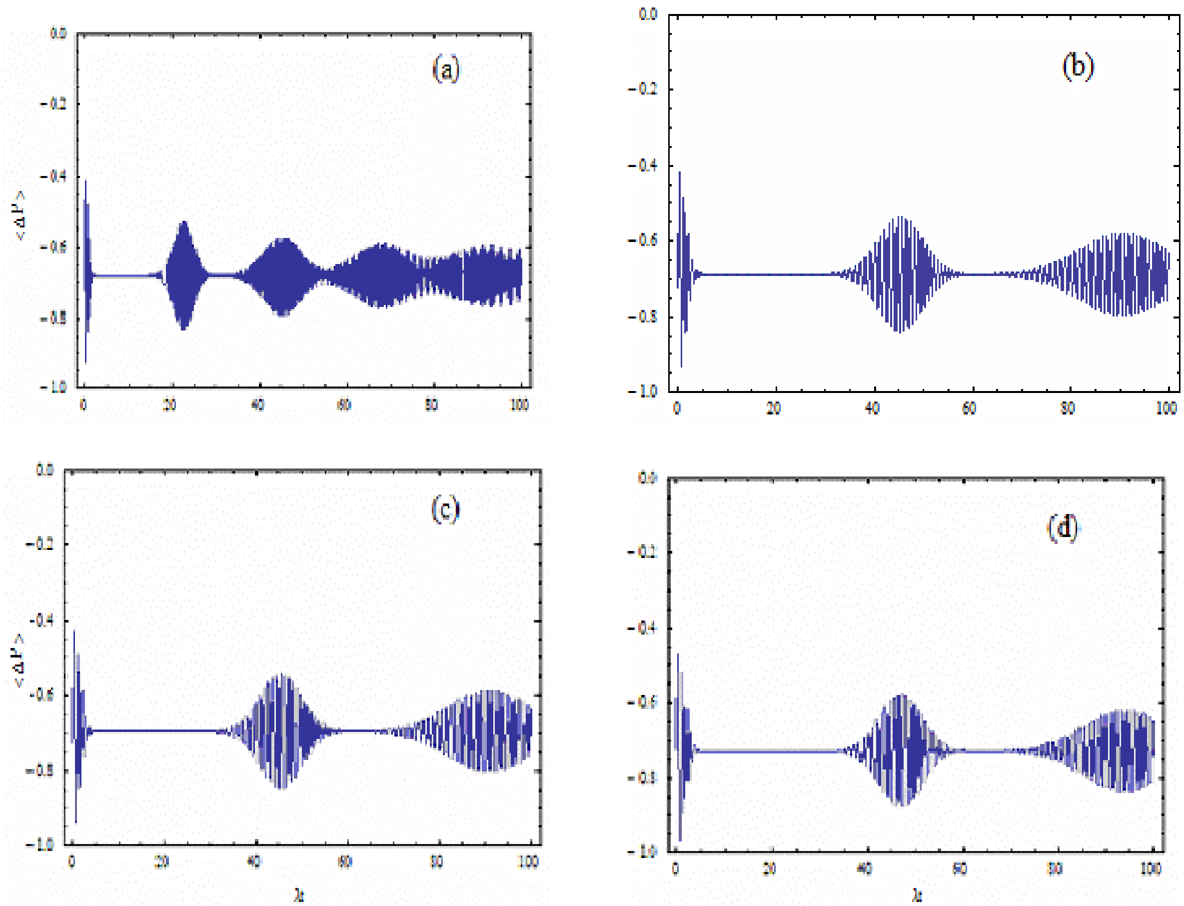


Fig. 1: The time evolution of the momentum increment $\langle \Delta\vec{P} \rangle$ versus the scaled time $\lambda t$ for different values of $\delta$ with $m = 1$, $\bar{n} = 25$ and $\Delta_1 = \Delta_1 = \chi = 0$ (a) $\delta = 0$, (b) $\delta = 0.1$ (c) $\delta = 0.3$ and (d) $\delta = 0.9$

To discuss the momentum increment, we plot several figures for different values of the given parameters. In all figures, we shall concentrate on the case $\theta = \pi/4$, i.e. when the atom is initially in superposition state and take the relative phase $\Psi = \pi/4$, in addition to $m = 1$, the mean photon number $\bar{n} = 25$ and $\lambda_\ell = \lambda$ .We plot the momentum increment $\langle \Delta\vec{P} \rangle / k = -(\sigma_{22} + \sigma_{33})$ versus the scaled time $\lambda t$. Figure 1(a-d) correspond to the momentum increment in the absence of both the detunings ($\Delta_1 = \Delta_2 = 0$) and the nonlinear interaction of the Kerr-like medium ($\chi = 0$). Whereas curves (a), (b), (c) and (d) corresponding to $\delta = 0$, $\delta = 0.3$, $\delta = 0.6$ and $\delta = 0.9$, respectively. From this Figure, we see that the momentum increment evolves periodically and the oscillations increase whereas the amplitudes decrease as the scaled time increases. This delaying time is in fact almost twice the usual time for the case of standard three-level atom via the modifications which have been introduced to the detuning parameters as a result of neglecting the fast oscillating term.

In the presence of the detuning parameter in off-resonant case ($\Delta_1 = \Delta_2 = 10$) with the absence of Kerr medium, Fig. 2, shows that the momentum increment is shifted upward and fluctuates around-0.59. The amplitude of the fluctuations in this case is less than the exact resonance case, however the revival period is observed to be elongated. We can say that the effect of time dependent coupling parameter leads to stronger interaction between the atom and the field where the atomic system in this case would store more energy. This is observed from the appearance of $\delta$ in the modified of the detuning parameters.

In Fig. 3(a-d), we consider the nonresonant case $\Delta_1 \neq \Delta_2$. We plot the momentum increment for $\Delta_1 = 5$ and $\Delta_2 = 3$, in the absence of the Kerr-like medium. We observed that the momentum increment is shifted upwards and fluctuates around-0.53 for the case $\delta = 0$ and around-0.49 for the case $\delta \neq 0$. Furthermore, we noticed that from these curves the occurrence of collapse and revival depends upon the detuning parameters. However, the revival period is elongated and the collapse time increases $|\Delta_1 - \Delta_2|$ increases.

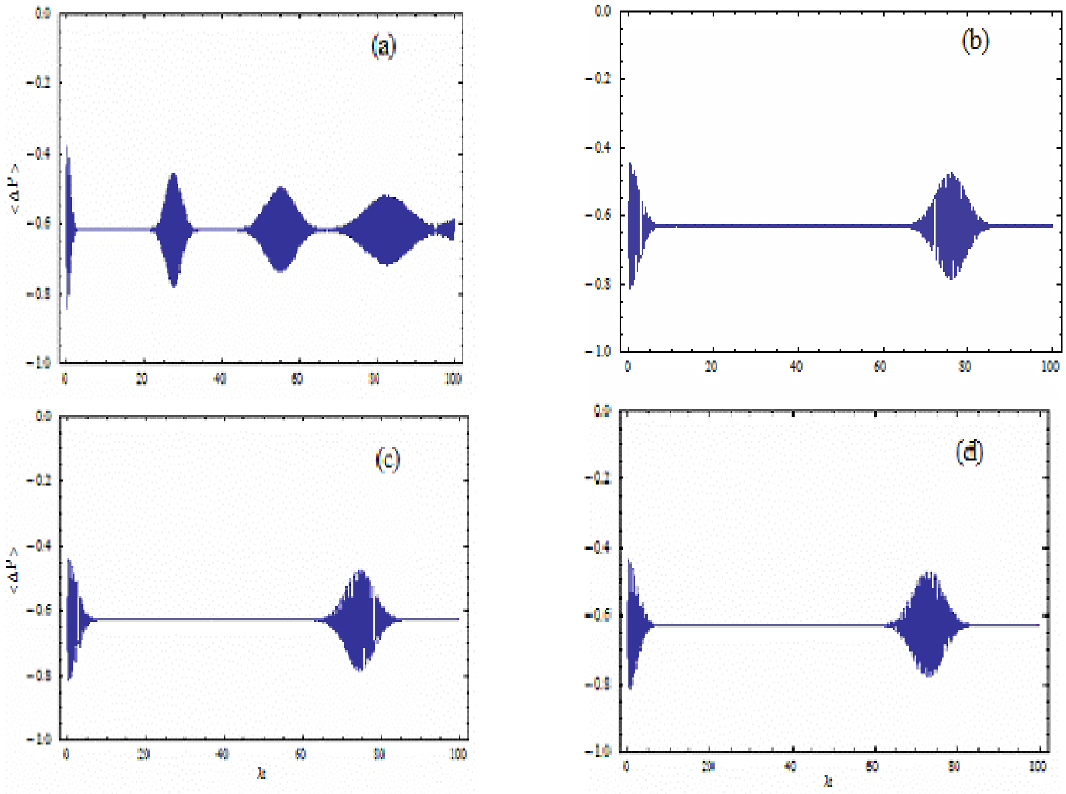


Fig. 2: The same as in Fig. 1 but for $\Delta_1 = \Delta_2 = 10$

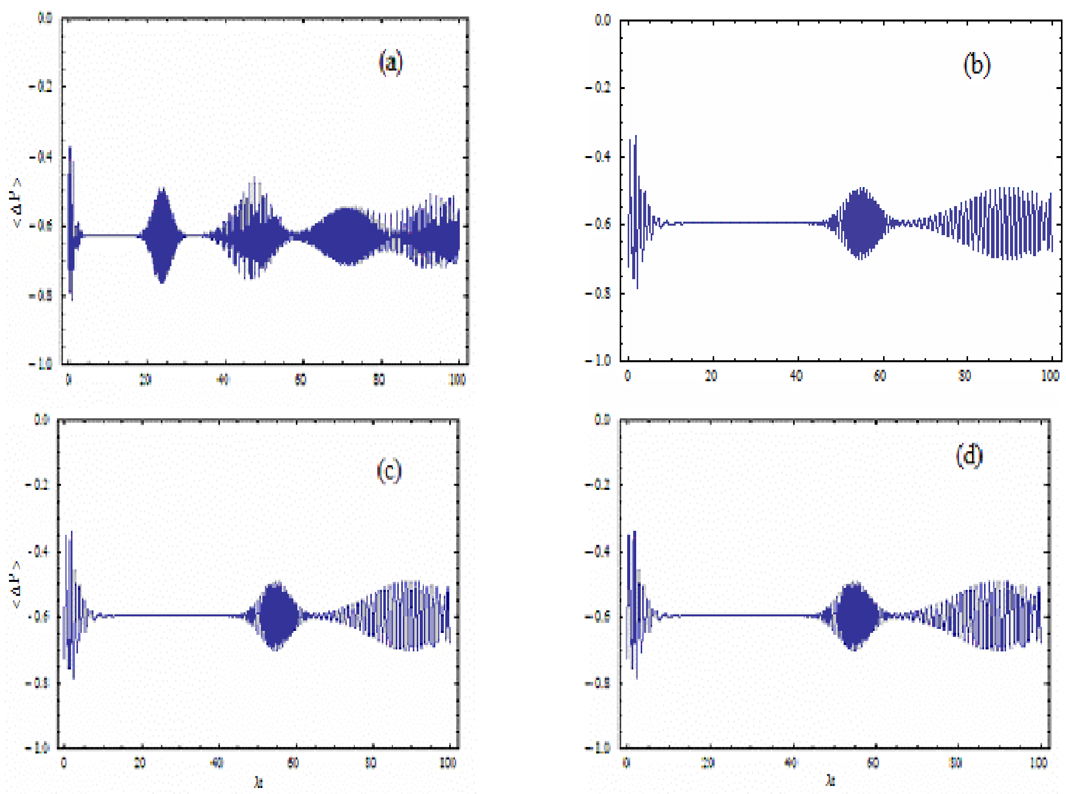


Fig. 3: The same as in Fig. 1 but for $\Delta_1 = 5$ and $\Delta_2 = 3$

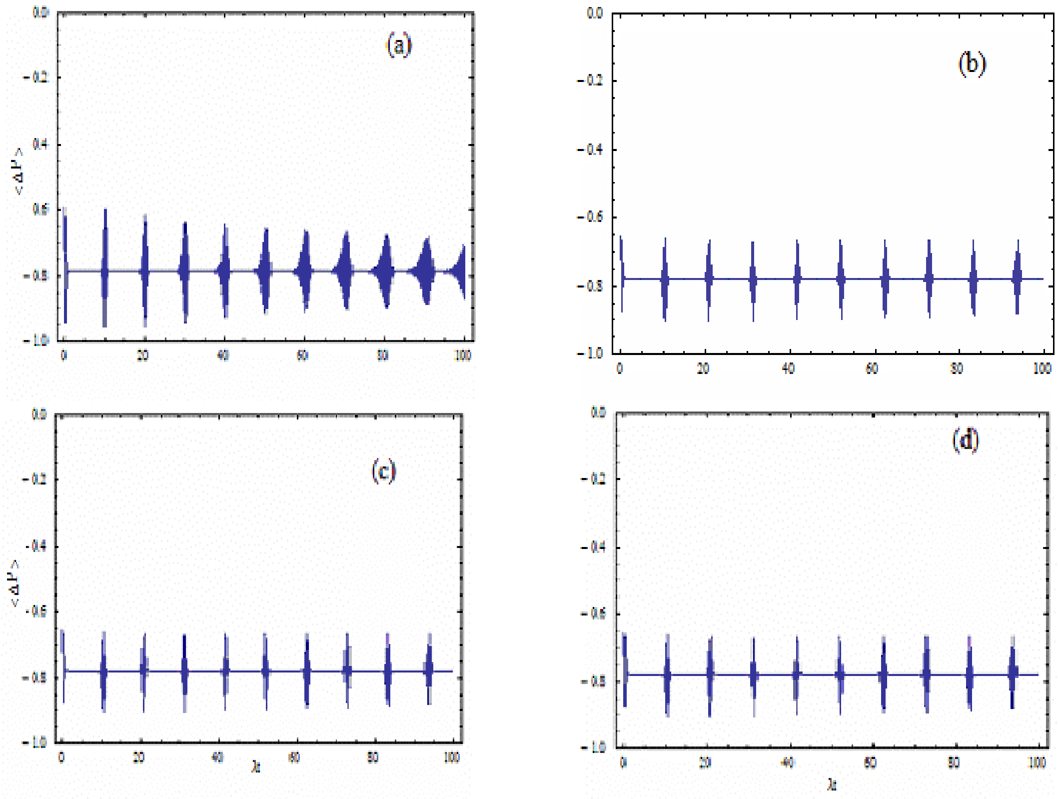


Fig. 4: The same as in Fig. 1 but for $\chi = 0.3$

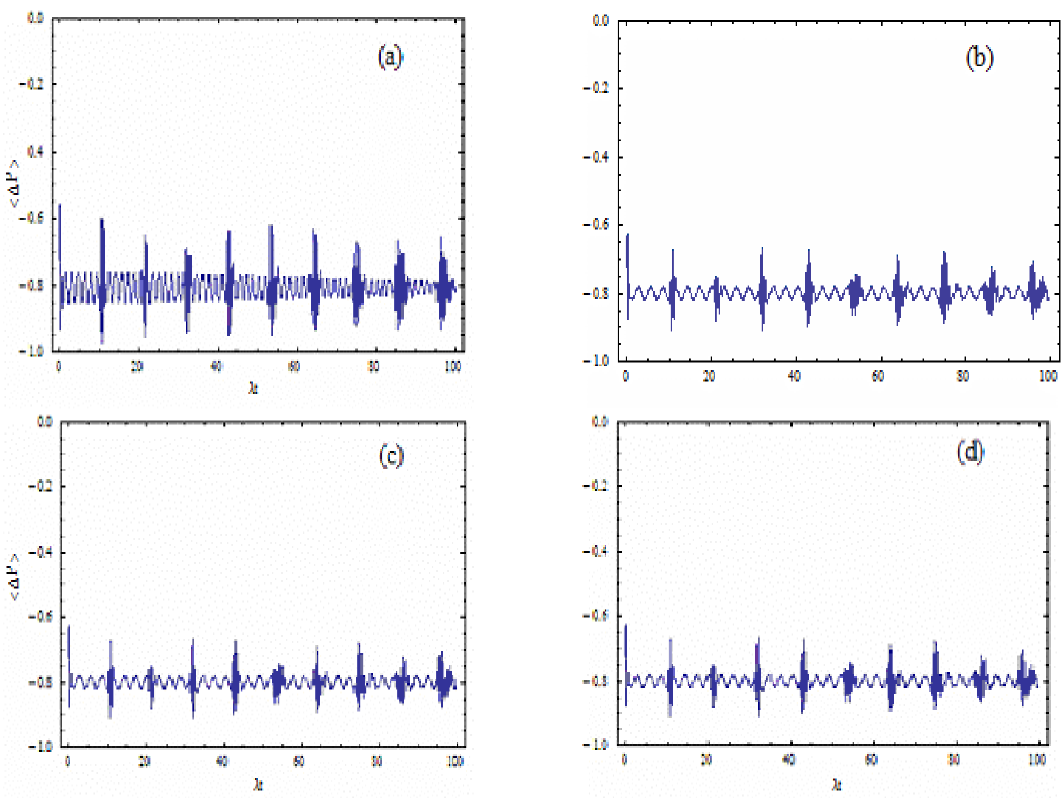


Fig. 5: The same as in Fig. 1 but for $\Delta_1 = 3$, $\Delta_2 = 5$ and $\chi = 0.3$

On the other hand, the amplitude of the oscillations decreases as $|\Delta_1+\Delta_2|$ increases. Moreover, the amplitude of the fluctuations in this case is less than the exact resonance case. To examine the effect of the Kerr-like medium on the present system, we plot the momentum increment in Fig. 4(a-d) in the absence of the detuning parameter, taking into consideration the same values of the other parameters except $\chi = 0.3$. In this case, we realize that the standard three level atom behavior ($\delta = 0$) where the momentum increment function fluctuates around -0.56, showing collapse and revivals and the Kerr-like medium acts like the detuning. As soon as the effect of the parameter $\delta$ leads to a slight decreasing in the revival period which means slight offset under the effect of the Kerr-like medium. The influence of the detuning parameters on the momentum increment in the presence of the Kerr nonlinearity can be seen in Fig. 5 where $\chi = 0.3$ and $\Delta_1 = 5$, $\Delta_2 = 10$. We observed from this figure the collapse and revival occur in the longest time as $|\Delta_1-\Delta_2|$ increases as well as the amplitude of the oscillations decreases. We conclude that the detuning parameters and the Kerr-like medium changes the properties of the momentum increment.

## THE FIELD ENTROPY

In this section, we use the field entropy as a measurement of the degree of entanglement between the field and the atom of the system under consideration. Knight and co-workers [50] have developed a general method to calculate the various field eigenstates in a simple way. Using this method, we obtain the eigenvalues of the reduced density operator. Since the trace is invariant under a similarity transformation, we can go to a basis in which the density matrix of the field is diagonal and we can express the field entropy $S_F(t)$ in terms of the eigenvalue $\lambda_F^i$, i = 1,2,3 of the reduced field density operator as

$$S_F(t) = -\sum_{i=1}^{3} \lambda_F^i(t) \ln \lambda_F^i(t) \qquad (26)$$

where

$$\lambda_F^i = -\frac{y_1}{3} + \frac{2}{3}\sqrt{y_1^2 - 3y_2}\cos\left(\eta + \frac{2}{3}(i-1)\pi\right) \qquad (27)$$

and

$$\eta = \frac{1}{3}\cos^{-1}\left(\frac{9y_1y_2 - 2y_1^3 - 27y_3}{2\left(y_1^2 - 3y_2\right)^{3/2}}\right) \qquad (28)$$

with

$$y_1 = -\langle A|A\rangle - \langle B|B\rangle - \langle C|C\rangle$$

$$y_2 = \langle A|A\rangle\langle B|B\rangle + \langle B|B\rangle\langle C|C\rangle + \langle C|C\rangle\langle A|A\rangle - |\langle A|B\rangle|^2 - |\langle B|C\rangle|^2 - |\langle C|A\rangle|^2$$

$$y_3 = \langle A|A\rangle|\langle B|C\rangle|^2 + \langle B|B\rangle|\langle C|A\rangle|^2 + \langle C|C\rangle|\langle A|B\rangle|^2 - \langle A|A\rangle\langle B|B\rangle\langle C|C\rangle - \langle A|B\rangle\langle B|C\rangle\langle C|A\rangle - \langle A|C\rangle\langle C|B\rangle\langle B|A\rangle \qquad (29)$$

Now, we shall investigate numerically the dynamic of the field entropy in Fig. (6-10) with the same initial parameters of Fig. (1-5), respectively. In Fig. 6 correspond to the field entropy in both the absence of the detuning and the Kerr-like medium. It is observed that the maximum and minimum values of the field entropy are achieved during the state time evolution. Also, we noticed that the field entropy evolves periodically and the oscillations increase whereas the amplitudes decrease as the scaled time increases. At one-half of the revival time the entropy attains its minimum. As soon as the value of the parameter $\delta$ increases more fluctuations occur. The influence of the detuning parameter in off-resonant case ($\Delta_1 = \Delta_2 = 10$) on the field entropy in the absence of Kerr-medium can be seen in Fig. 7. It is observed that the first maximum value of the field entropy decreases, the period of revivals becomes longer and the time area of vibration of the entropy is compressed. In Fig. 8, we consider the nonresonant case $\Delta_1 \neq \Delta_2$, we plot the entropy field S(t) with $\Delta_1 = 5$ and $\Delta_2 = 3$ in the absence of the Kerr-like medium. We noticed that the field entropy has minimum value and the collapse time of the entropy becomes longer as $|\Delta_1-\Delta_2|$ increases. On the other hand, the amplitude of the oscillations decreases as $|\Delta_1-\Delta_2|$ increases. Also, for $\delta = 0$, 0.3, 0.6 and 0.9, the field entropy reduces its maximum value (this phenomenon is more pronounced when $\delta = 0$). On the other hand, we observed that the situation is changed at $\delta \neq 0$, the fluctuations in the function are seen with interference between the patterns at the half period of the considered time. To visualize the effect of the Kerr-like medium in the absence of the detuning, we plot S(t) as shown in Fig. 9 for different values of the parameter $\delta$ and $\chi = 0.3$. The entropy function S(t) fluctuated and it is rapidly after half period of the considered time. Also, the Kerr-like medium implies to increase the minimum values of the field entropy. Moreover, we noticed that the Kerr-like medium increases the amplitudes of the field entropy is decrease. In the case of $\delta \neq 0$, this phenomenon is more pronounced and occurs once at the onset of the interaction. The influence of the detunings on the field entropy in the presence of the Kerr medium ($\chi = 0.3$) is visualized in Fig. 10. We noticed that the

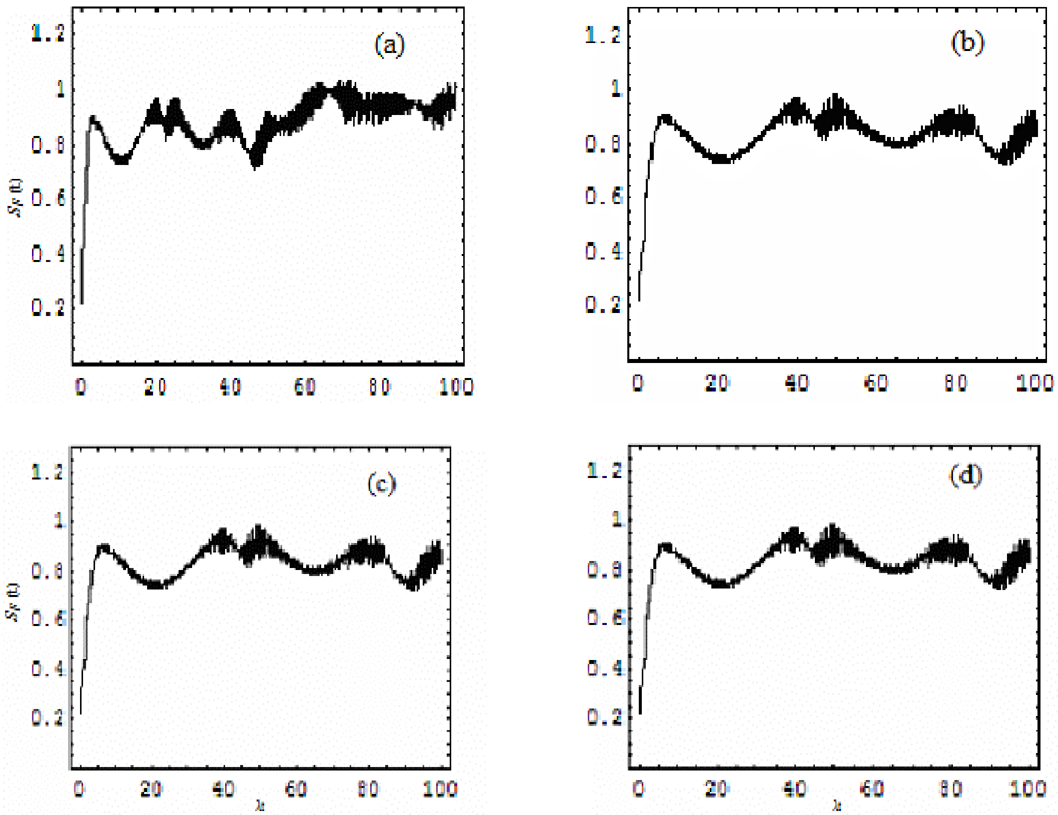


Fig. 6: The same as in Fig. 1 but for the field entropy $S_F(t)$

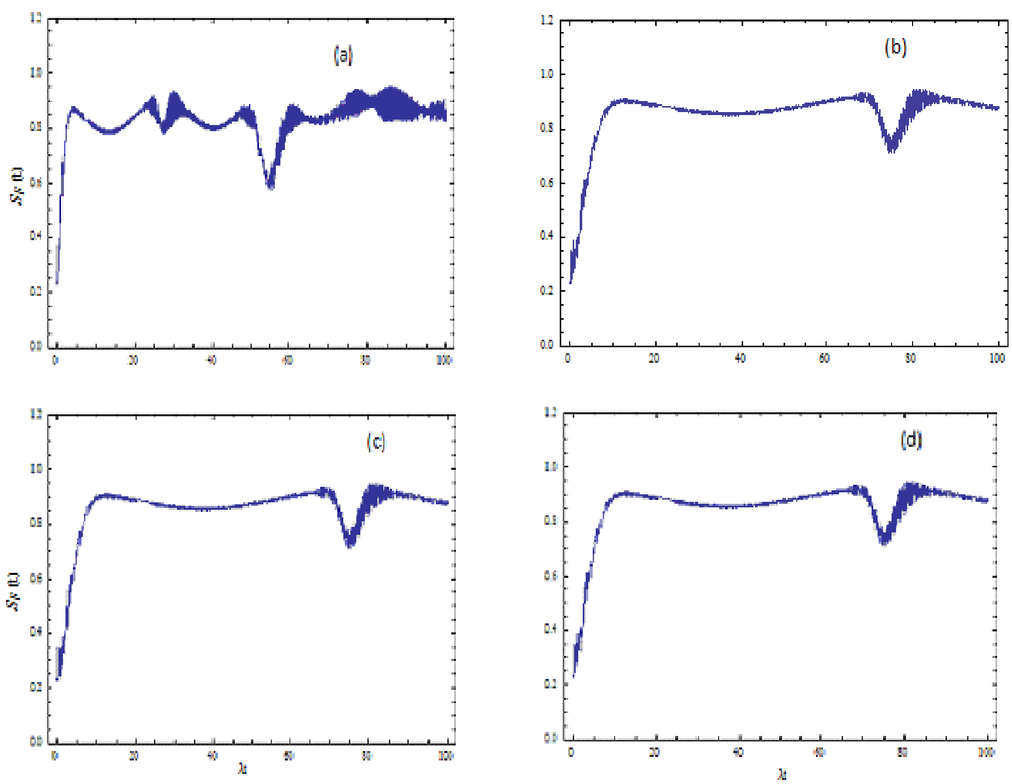


Fig. 7: The same as in Fig. 2 but for the field entropy $S_F(t)$

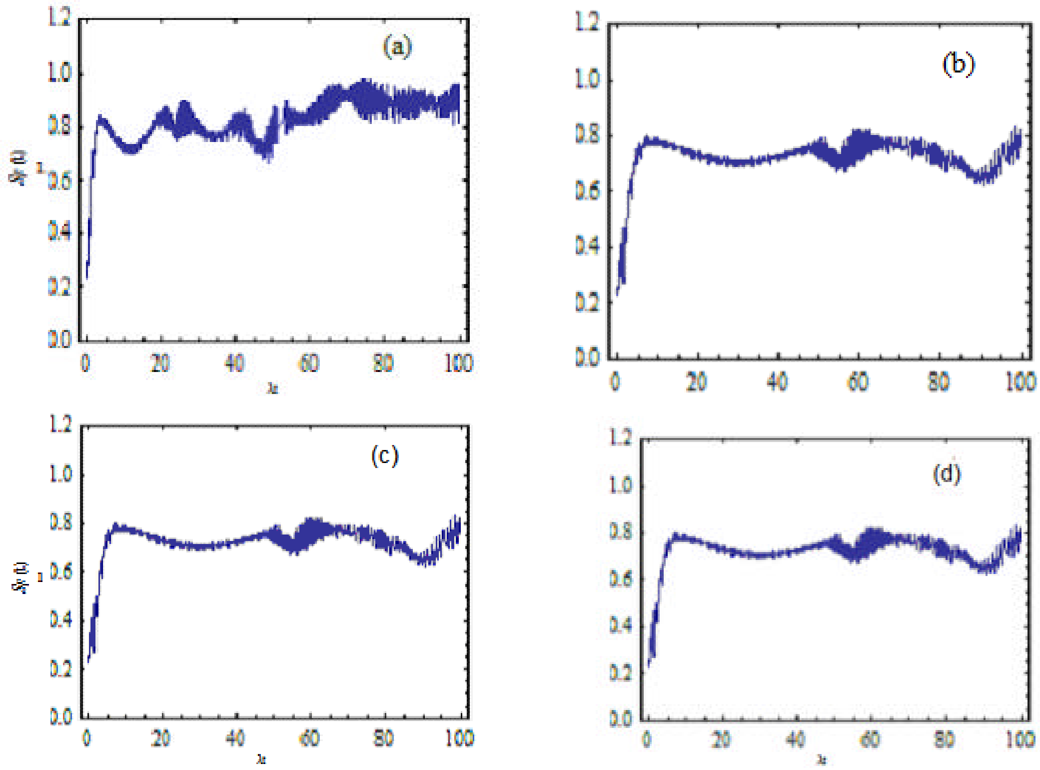


Fig. 8: The same as in Fig. 3 but for the field entropy $S_F(t)$

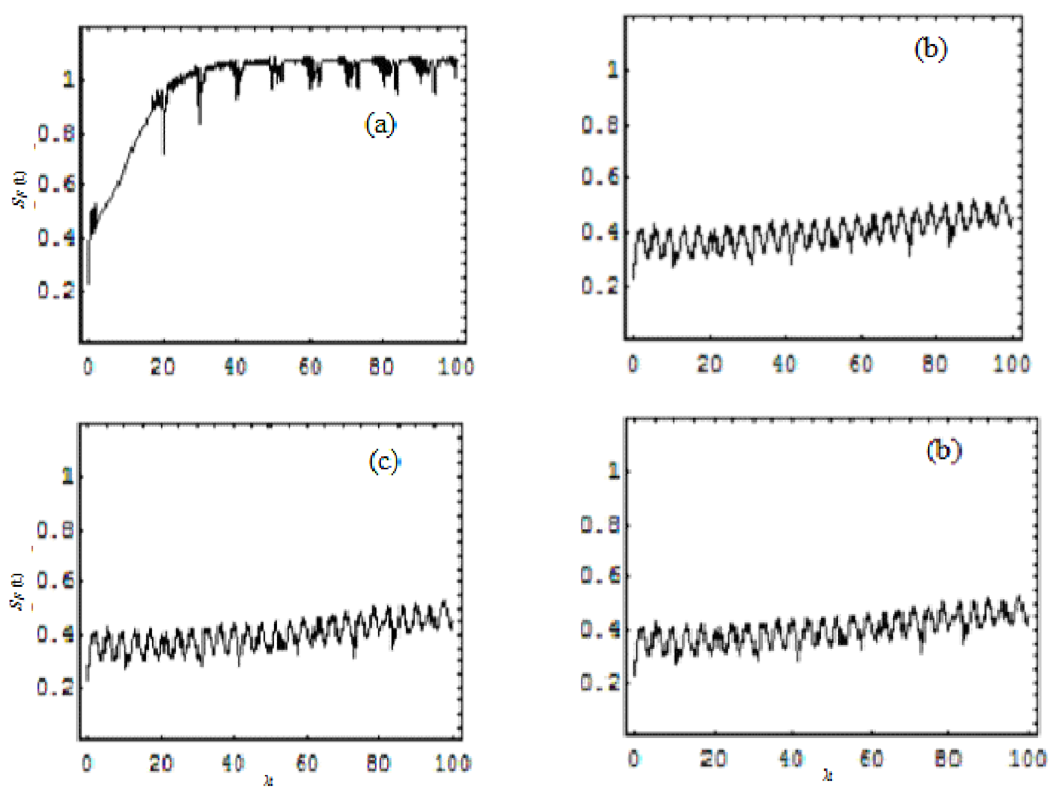


Fig. 9: The same as in Fig. 4 but for the field entropy $S_F(t)$

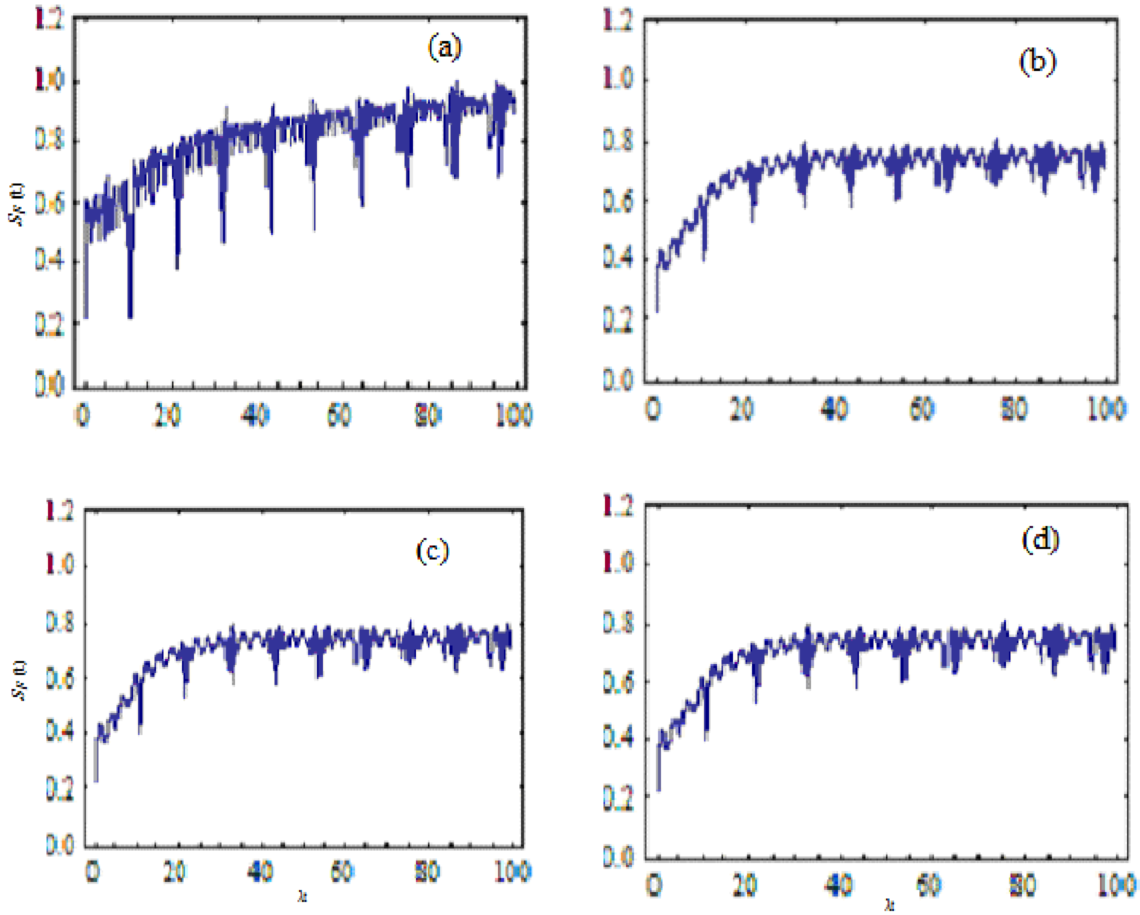


Fig. 10: The same as in Fig. 5 but for the field entropy $S_F(t)$

Kerr-like medium leads to increase the maximum values of the field entropy. In this case, the field and the atom almost retain in strong entanglement.

## CONCLUSIONS

The system of a three-level atom with a momentum eigenstate interacting one-mode cavity field in the presence of a nonlinear Kerr-like medium is studied. The coupling parameter between the atom and the field is modulated to be time-dependent. Under certain approximation similar of the RWA with $\delta \approx \Delta_i$, an exact solution is given. The momentum increment behaviors as well as the entropy field are investigated. The influence of the detuning parameters, the Kerr-like medium and the coupling parameter on the evolution of the momentum increment and the field entropy is analyzed. We conclude that there is no difference between the field entropy behavior in the usual stander three level atom model compared to the present case, except some delay in the time related to the collapses and revivals phenomena when $\delta \neq 0$.

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