Middle-East Journal of Scientific Research 22 (7): 1003-1013, 2014
ISSN 1990-9233

DOI: 10.5829/idosi.mejsr.2014.22.07.21959

The Influence of the Stark Shift and the Gravitational Field on the Interaction of a Two-Level Atom with a Single Mode Cavity Field

[1]N.H. Abd El-Wahab and [2]Ahmed Salah

[1]Department of Mathematics, Faculty of Science, Minia University, Minia, Egypt
[2]Department of Mathematics, Faculty of Science, South Valley University, Qena, Egypt

Abstract: In this paper, a model describing the interaction of a two-level atom with one-mode cavity field including the gravitational field and the Stark-shift is discussed. The analytical solution for this model is presented when the atom and the field are initially prepared in its excited state and coherent state, respectively. The obtained results are employed to examine the dynamical behaviors of the atomic inversion and the field entropy. We observed that increasing of the gravitational field parameter decreases the amplitudes of the oscillations and increases the collapse time as well as the collapse period is elongated and the fluctuations amplitudes decrease in the presence of the Stark shift.

Key words: Jayness-Cummings model (JCM) · The gravitational field · The Stark-shift · Entanglement

INTRODUCTION

The interaction between electromagnetic fields and matter (atoms) lies at the heart of quantum optics. After more than three decades the Jaynes-Cummings model (JCM) [1] is still the best model to represent this concept. This model is one of the exactly solvable models describing the interaction between a two-level atom and a single mode cavity field. Also, this model has been realized experimentally [2]. It gives arise to many quantum phenomena that cannot be explained in classical terms, such as the collapses and revivals of the atomic population inversion [3] and squeezing of the field [4]. Recent experiments with Rydberg atoms and microwave photons in a superconducting cavity have turned the JCM from a theoretical curiosity to a useful and testable enterprise [5]. Furthermore, the quantum property of a system containing an atom interacting with an optical field in a cavity is one of the main contents of quantum optics. Especially the time evolution of the field (or atomic) entropy is attracting many researchers due to its potential applications in the field of quantum information. Phoenix and Knight [6] have shown that entropy is a useful operational measure of the purity of a quantum state.

Recently, atomic beams with very low velocities are generated experimentally in laser cooling and atomic interferometry [7]. It is obvious that for atoms moving with a velocity of few millimeters or centimeters per second for a time period of several milliseconds or more, the influence of earth's acceleration becomes important and cannot be neglected [8]. For this reason, it is interest to study the temporal evolution of a moving atom simultaneously exposed to the classical homogenous gravitational field and a single-mode traveling wave field. Since any quantum optical experiment in the laboratory is actually made in a non-inertial frame, it is important to estimate the influence of earth's acceleration on the outcome of the experiment. Recently, a semiclassical description of a two-level atom interacting with a running laser wave in a gravitational field is investigated [9,10]. Furthermore, a complementary scheme based on an SU(2) dynamical algebraic structures to investigate the influence of the gravity on the quantum-nondemolition (QND) measurement of atomic momentum in the dispersive JCM is demonstrated [11].

In this paper, we study a model describing the interaction of a two-level atom with one-mode cavity field including the classical homogenous gravitational field and the Stark-shift. the dynamical behaviors of the atomic

Corresponding Author: N.H. Abd El-Wahab, Department of Mathematics, Faculty of Science, Minia University, Minia, Egypt. E-mail: dr_math1111@yahoo.com.

inversion and the field entropy is investigated. The paper is organized as follows: in Section 2 we give a description of the system and formulas. Section 3 is devoted to the discussion of the evolution of the atomic inversion. In Section 4, we investigate the field entropy. Some conclusions are presented in Section 5.

Description of the Model: The model we consider here describes the non-resonant and multi-photons case of a two-level atom interacting with a single mode cavity field. The considered model includes the classical homogenous gravitational field and the Stark-shift terms. The total Hamiltonian of the system under consideration and in the RWA can be written as ($\hbar = c = 1$):

$$\hat{H} = \hat{H}_0 + \hat{H}_I, \tag{1}$$

where $\hat{H}_0$ ($\hat{H}_I$) is the free (interaction) part of Hamiltonian. The free part $\hat{H}_0$ is defined as:

$$\hat{H}_0 = \frac{\vec{P}^2}{2M} - M\vec{g}\cdot\vec{x} + \sum_{j=1}^{2}\omega_j\hat{\sigma}_{jj} + \Omega\hat{a}^\dagger\hat{a}, \tag{2}$$

where $\vec{P}$ is the momentum operator, M is the mass of atom, $\vec{g}$ is the earth's gravitational acceleration, $\vec{x}$ is the position vector in x-direction, ω_j is the energy of level $|j\rangle$, $\hat{\sigma}_{ij} = |i\rangle\langle j|$, $(i, j = 1,2)$ are the lowering and raising operators between levels i and j when $i \neq j$ and are the population operators for $i = j$ and $\hat{a}(\hat{a}^\dagger)$ is the annihilation (creation) operator of the field of frequency Ω. The interaction part $\hat{H}_I$ in (1) is defined as:

$$\hat{H}_I = (\beta_1\sigma_{11} + \beta_2\sigma_{22})\hat{a}^\dagger\hat{a} + \lambda\left[\hat{a}^m e^{im\vec{k}\cdot\vec{x}}\sigma_{12} + \hat{a}^{\dagger m} e^{-im\vec{k}\cdot\vec{x}}\sigma_{21}\right]. \tag{3}$$

A Stark shift is caused by the intermediate level $|\ell\rangle$ and the corresponding parameter β_1 and β_2 are $\beta_1 = \lambda_1^2/\Delta$, $\beta_2 = \lambda_2^2/\Delta$ and $\lambda = \lambda_1\lambda_2/\Delta$ where λ_1 and λ_2 are the coupling strengths of the intermediate level with the lower and upper levels of the two-level atom, respectively, λ is the usual coupling constant between the field and the atom, $\vec{k}$ is the propagation vector. We would like to point out that: to produce the Stark-shift, one has to make the adiabatic elimination of the third virtual energy level which in fact can be achieved for $m \geq 2$ where m is the multiplicity for the photon.

It is important to mention that the stander JCM is given when one puts $\vec{P} = \vec{x} = 0$, $\beta_1 = \beta_2 = 0$ and $m = 1$. Also, the effective two-level *m-photon* Hamiltonian by eliminating m-1 intermediate non-resonantly coupled level is obtained where $\vec{P} = \vec{x} = 0$. Moreover, the model [3] is given when the earth gravitational acceleration is not taken into account.

In what follows, we shall present some interesting properties of the atom (field) operators of the considered model. The operators $\hat{\sigma}_{ij}$ are the generators of the unitary group satisfying the following commutation relations:

$$\left[\hat{\sigma}_{ij}, \hat{\sigma}_{k\ell}\right] = \hat{\sigma}_{i\ell}\delta_{jk} - \hat{\sigma}_{kj}\delta_{\ell i}, \left[\hat{a}^m, \hat{\sigma}_{k\ell}\right] = \left[\hat{a}^{\dagger m}, \hat{\sigma}_{k\ell}\right] = 0, \tag{4}$$

where δ_{ij} is the Kroneker symbol and $\sigma_{ij}|j\rangle = |i\rangle$.

Also, the operators $\hat{a}$ and $\hat{a}^\dagger$ satisfy the canonical commutation relation $\left[\hat{a}, \hat{a}^\dagger\right] = 1$ while $[\hat{a}, \hat{a}] = \left[\hat{a}^\dagger, \hat{a}^\dagger\right] = 0$. In the general form it is easy to show that:

$$\left[\hat{a}, \hat{a}^{\dagger m}\right] = m\hat{a}^{\dagger(m-1)}, \left[\hat{a}^\dagger, \hat{a}^m\right] = -m\hat{a}^{(m-1)}. \tag{5}$$

Moreover, the field operators satisfy the following relations:

$$\begin{aligned} \hat{a}^m|n\rangle &= \sqrt{\frac{n!}{(n-m)!}}|n-m\rangle \quad n > m, \\ \hat{a}^{\dagger m}|n\rangle &= \sqrt{\frac{(n+m)!}{n!}}|n+m\rangle, \end{aligned} \tag{6}$$

As shown previously, the gravitational influence of atoms in an interferometer and in a running laser wave can not be neglected [9,10]. Also, we consider that the atom moving in the Earth's gravitational field is equivalent to a free atom moving in a uniformly accelerated reference frame. Under these conditions we have:

$$\begin{aligned} &\vec{P} = \vec{P}_0 + Mgt, e^{\pm i\vec{k}\cdot\vec{x}}\left|\vec{P}_0\right\rangle = \left|\vec{P}_0 \pm \vec{k}\right\rangle, \\ &\vec{x}\left|\vec{P}_0\right\rangle = \frac{\vec{P}_0 t}{M}\left|\vec{P}_0\right\rangle. \end{aligned} \tag{7}$$

Now, we turn our attention to find the wave function of the system under consideration.

The Wave Function: Here, we consider that at time $t > 0$ the atom is in the excited state $|1\rangle$ and the cavity mode prepared in the coherent state

$$|\alpha\rangle = \sum_n q_n|n\rangle, \tag{8}$$

where q_n describes the amplitude of the state $|n\rangle$ which is defined as $q_n = \exp(-\bar{n}/2)\bar{n}^{n/2}/\sqrt{n!}$ and $\bar{n}$ is the initial mean photon number.

Also, assuming that at time $t = 0$ the field and the atom are decoupled, the initial state vector of system can be written as:

$$\left|\psi(0)\right\rangle = \left|\alpha\right\rangle \otimes \left|1\right\rangle = \sum_{n=0}^{\infty} q_n \left|n,1\right\rangle \quad (9)$$

Moreover, let $|P_0\rangle$ be the eigen-function for the operator $\hat{P}$ with an eigen value $\vec{P}_0$, $|n\rangle$ for $\hat{n} = \hat{a}^{\dagger}\hat{a}$ with n and $|j\rangle$ for j^{th} atomic state, where $\left|\vec{P}_0, j, n\right\rangle = \left|\vec{P}_0\right\rangle \otimes |j\rangle \otimes |n\rangle$. According to these assumption the coupled atom-field wave function of the system at an arbitrary time t can be expressed as :

$$\left|\psi(t)\right\rangle = \sum_{n=0}^{\infty} q_n \begin{bmatrix} A(t)e^{-i\gamma_1 t}\left|\vec{P}_0,1,n\right\rangle + B(t)e^{-i\gamma_2 t} \\ \left|\vec{P}_0 - m\vec{k}, 2, n+m\right\rangle \end{bmatrix}, \quad (10)$$

With

$$\gamma_1 = \frac{\vec{P}_0^2}{2M} + \omega_1 + \Omega n - \frac{1}{2}\vec{P}_0 \cdot \vec{g}\,t + \beta_1 n,$$
$$\gamma_2 = \frac{\left(\vec{P}_0 - m\vec{k}\right)^2}{2M} + \omega_2 + \Omega(n+m) - \frac{1}{2}\left(\vec{P}_0 - m\vec{k}\right)\cdot \vec{g}\,t + \beta_2(n+m). \quad (11)$$

where the expressions $A(t)$ and $B(t)$ are the probability amplitudes which determine the initial state $|\psi(0)\rangle$. Using the time-dependent Schrödinger equation $i(d/dt)|\psi(t)\rangle = \hat{H}_I|\psi(t)\rangle$ and the actions of the atomic and field operators on the wave function $|\psi(t)\rangle$ we have the following system of ordinary differential equations for the probability amplitudes:

$$i\frac{dA(t)}{dt} = fB(t)e^{i\Delta(t)t}, \quad i\frac{dB(t)}{dt} = fA(t)e^{-i\Delta(t)t} \quad (12)$$

with

$$f = \lambda\sqrt{\frac{(n+m)!}{n!}} \qquad \Delta(t) = \Re - \frac{m}{2}\vec{k}\cdot\vec{g}t,$$
$$\Re = \Delta + \nu_1 - \nu_2. \qquad \nu_1 = \beta_1 n, \quad (13)$$
$$\nu_2 = \beta_2(\mathrm{n+m}), \quad \Delta = \omega_1 - \omega_2 - m\Omega - \frac{m^2k^2}{2M} + \frac{m\vec{k}\cdot\vec{P}_0}{M}.$$

Now, let us first start with the solution of (12) in the off-resonance case, when the Stark-shift is taken into account and the classical homogenous gravitational field, is absent. The probability amplitudes $A(t)$ and $B(t)$ are given as:

$$A(t) = \exp\left[\left(\frac{\Delta}{2} + \frac{\nu_1 - \nu_2}{2}\right)it\right]\left\{\cos(\mu t) - i\left(\frac{\Delta}{2} + \frac{\nu_1-\nu_2}{2}\right)\frac{\sin(\mu t)}{\mu}\right\},$$
$$B(t) = -if\exp\left[-\left(\frac{\Delta}{2} + \frac{\nu_1-\nu_2}{2}\right)it\right]\frac{\sin(\mu t)}{\mu}, \quad (14)$$

where

$$\mu^2 = \left(\frac{\Delta}{2} + \frac{\nu_1 - \nu_2}{2}\right)^2 + f^2, \quad (15)$$

Furthermore, the probability amplitudes in the resonant case, in the absence of the Stark-shift and the classical homogenous gravitational field is taken in to account are given by:

$$A(t) = e^{\frac{i\vec{k}\cdot\vec{g}}{2}t^2}\,{}_1F_1\left[\frac{1+\xi}{2}, \frac{1}{2}, \frac{i\vec{k}\cdot\vec{g}}{2}t^2\right]$$
$$B(t) = \varepsilon\left(-2^{\xi}\Gamma\left[\frac{\xi+1}{2}\right]H\left[-\xi,(1+i)\varepsilon t\right] + \sqrt{\pi}\,{}_1F_1\left[\frac{\xi}{2}, \frac{1}{2}, \frac{i\vec{k}\cdot\vec{g}}{2}t^2\right]\right) \quad (16)$$

where

$$\xi = \frac{if^2}{m\vec{k}\cdot\vec{g}}, \quad \varepsilon = -\frac{(1-i)\sqrt{m\vec{k}\cdot\vec{g}}\,\Gamma\left[\frac{1}{2}(\xi+2)\right]}{f\sqrt{\pi}\,\Gamma\left[\frac{1}{2}(\xi+1)\right]} \quad (17)$$

where ${}_1F_1$, H and Γ are the confluent hypergeometric, Hermite and Gamma functions, respectively.

Moreover, when both Stark-shift and the classical homogenous gravitational field are taken into account, the probability amplitudes are given by:

$$A(t) = e^{-i\nu_1 t}e^{i\Delta(t)t}$$
$$\left\{C_1 H\left[-(\xi+1),\eta(t)\right] + C_2\,{}_1F_1\left[(\xi+1)/2, 1/2; \eta^2(t)\right]\right\}, \quad (17)$$
$$B(t) = e^{-i\nu_2 t}\left\{C_1 H\left[-\xi,\eta(t)\right] + C_2\,{}_1F_1\left[\xi/2, 1/2; \eta^2(t)\right]\right\}$$

Where

$$C_1 = \frac{{}_1F_1\left[\frac{\xi}{2}, \frac{1}{2}; \eta^2(0)\right]}{H\left[-(\xi+1),\eta(0)\right]{}_1F_1\left[\frac{\xi}{2},\frac{1}{2};\eta^2(0)\right] - H\left[-\xi,\eta(0)\right]{}_1F_1\left[\frac{(\xi+1)}{2},\frac{1}{2};\eta^2(0)\right]},$$
$$C_2 = \frac{-H\left[-\xi,\eta(0)\right]}{H\left[-(\xi+1),\eta(0)\right]{}_1F_1\left[\frac{\xi}{2},\frac{1}{2};\eta^2(0)\right] - H\left[-\xi,\eta(0)\right]{}_1F_1\left[\frac{(\xi+1)}{2},\frac{1}{2};\eta^2(0)\right]}, \quad (18)$$

With

$$\eta(t) = \frac{(1+i)\left(m\vec{k}\cdot\vec{g}\,t - \Re\right)}{2\sqrt{m\vec{k}\cdot\vec{g}}}, \tag{19}$$

The detuning parameter $\Delta(t)$ depends on the recoil energy $m^2k^2/2M$, the classical homogenous gravitational field parameter $\vec{k}\cdot\vec{g}$ and the Doppler shift $m\vec{P}_O\cdot\vec{k}/M$. Getting the explicit form of the wave function $|\psi(t)\rangle$ for the system under consideration, therefore we are in a position to discuss any phenomena related to this system. In the following subsections, we shall employ the obtained results to discuss the effect of the Stark shift and the classical homogenous gravitational field on the time evolution of both the atomic inversion and the field entropy.

The Atomic Population Inversion: The atomic population inversion of the atom is one of the important atomic dynamical variables of the atomic systems. It usually gives us information about the behavior of the atom during the interaction period. The atomic inversion is defined as the difference between the probability of finding the atom in the excited state and in the ground state. From the wave function (10), we can evaluate the time evolution of the atomic inversion as follows:

$$W(t) = \langle\sigma_{11}\rangle - \langle\sigma_{22}\rangle = \sum_{n=0}^{\infty}|q_n|^2\left\{|A(t)|^2 - |B(t)|^2\right\}. \tag{20}$$

Since the resulting series cannot be analytically summed in a closed form, we will evaluate them numerically. To discuss the atomic inversion, we have plotted several figures versus the scaled time λt for different values of the given parameters.

For example in Figs. (1a-1d), we consider the case in which the values of the parameter $\vec{k}\cdot\vec{g}=0$, 0.3, 0.6 and 0.9 taking into account fixed values for other parameters. We set the detuning parameter Δ = 0, the mean photon number $\bar{n}=20$, the Stark shift parameters $\beta_1=\beta_2=0$ and m = 1. In this case, we notice that for longer time the atomic inversion shows small oscillations around zero. Also, Fig. (1a) shows that the atomic inversion evolves periodically and the oscillations increase whereas the amplitudes decrease as the scaled time increases. In the presence of classical homogenous gravitational field parameter $\vec{k}\cdot\vec{g}$, we can see in Figs. (1b-1d) the amplitude of the fluctuations in this case is less than the absence gravitational field case, however the revival period is elongated. Also, we notice that increasing of the gravitational field parameter decreases the amplitudes of the oscillations and increases the collapse time.

Also, we plot in Fig. 2 the atomic inversion in the absence of the Stark-shift parameters $\beta_1=\beta_2=0$ taking into consideration the same values of the other parameters except Δ = 5. In Fig. 2a, we can see that the function is shifted upward and fluctuates around 0.1 for the case $\vec{k}\cdot\vec{g}=0$. On the other hand Figs. (2b-2d), show that the function is shifted downwards by increasing the gravitational field. This is due to the modification in the detuning parameter which affected by the time where $\Delta = \Delta(t)$. Thus, we can say that, the effect of the gravity parameter leads to stronger interaction between the atom and the field where the atomic system in this case would store more energy.

In order to examine the effect of the Stark shift into consideration we have to consider 2-photon process, m = 2. For instance in Fig. (3), we put β_1 = 0.5, β_2 = 2 and Δ = 0. We observed that the period of revivals becomes smaller, since for longer time, the atomic inversion shows small periodic oscillations around positive values. Furthermore, it is obvious that the atomic inversion shows rapid oscillations around 0.6 and it oscillates around the positive value which means that this deformation exceeds the energy stored in the atomic subsystem. On the other hand, when $\vec{k}\cdot\vec{g}=0.3,$ 0.6 and 0.9, the function is shifted to higher values while the fluctuations amplitude increased. However, there is slight elongation in the period of collapses between revivals compared to the standard JCM. This means that the Stark-shift has strongly affects on the system behavior. Also, we can see a decrease in the fluctuations amplitudes as shown in Figs. (3a-3d).

In Figs. (4a-4d), we put the Stark shift parameters $\beta_1 = \beta_2 = 1$, one observes that the atomic inversion shows small oscillations around zero for all values of $\vec{k}\cdot\vec{g}$. Moreover, the collapse period is elongated and the fluctuations amplitudes decrease as seen from this figure.

Finally, when the detuning parameter is taken place in the interaction Δ = 5, we notice that the function changes its behavior just for the cases $\vec{k}\cdot\vec{g}\neq 0$. It is obvious that the function is shifted just above zero, the collapse period is elongated and the fluctuations amplitudes are decreased. Also, It is shown that as gravitational field parameter increases the collapse is increasing and the oscillations go back around zero again as shown in Figs. (5a-5d). Thus we may conclude that the effect of the classical homogonous gravitational field on the atomic inversion is strong enough to make essential changes in its behavior and it is pronounced in most cases. In the next section we are going to discuss the entanglement degree due to the quantum field entropy for the system under consideration.

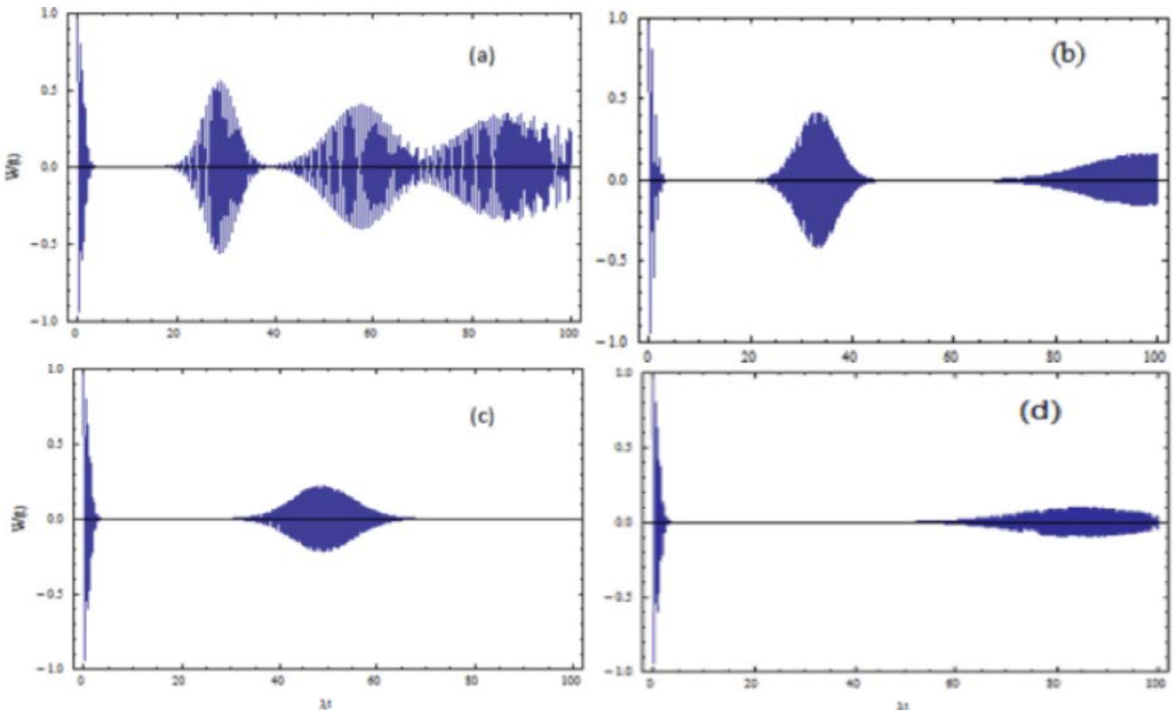

Fig. 1: The time evolution of the atomic inversion $W(t)$ against the scaled time λt with $\Delta = 0$, $m = 1$, $\bar{n} = 20$, $\beta_1 = \beta_2 = 0$ and different values of parameter $\vec{k} \cdot \vec{g}$ (a) $\vec{k} \cdot \vec{g} = 0$ (b) $\vec{k} \cdot \vec{g} = 0.3$ © $\vec{k} \cdot \vec{g} = 0.6$ and (d) $\vec{k} \cdot \vec{g} = 0.9$.

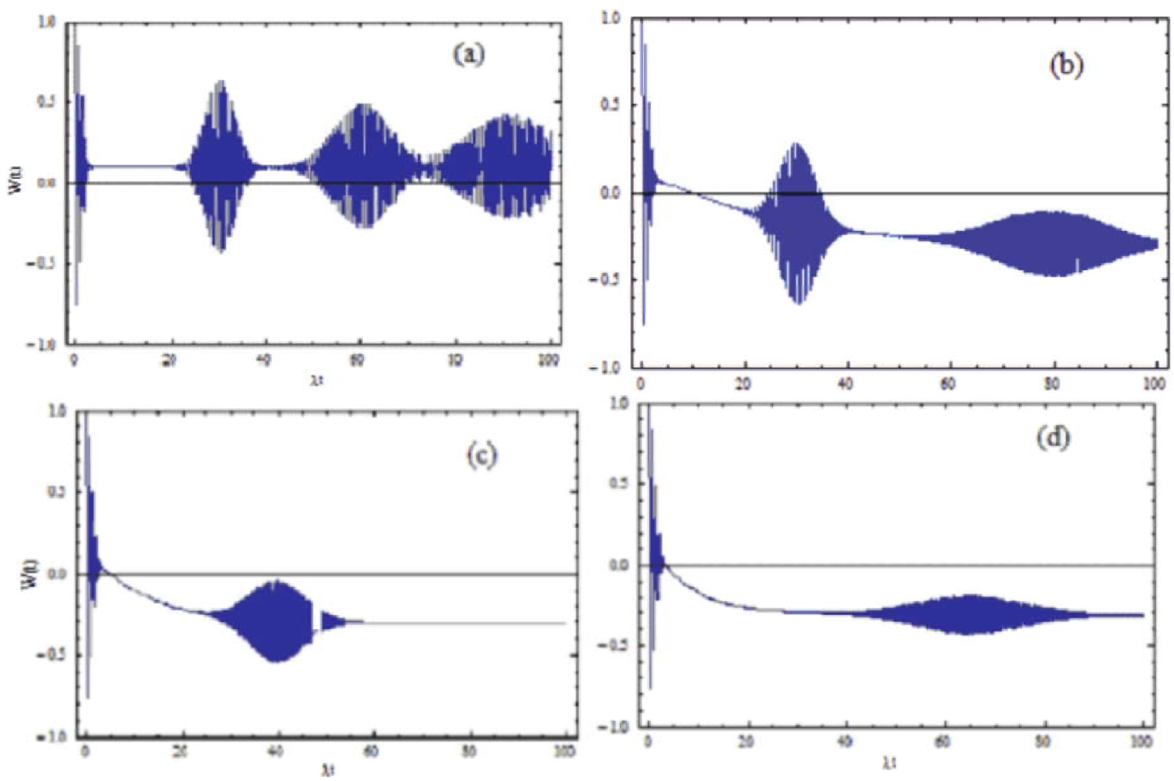

Fig. 2: The same as in Fig. 1 but for $\Delta = 5$.

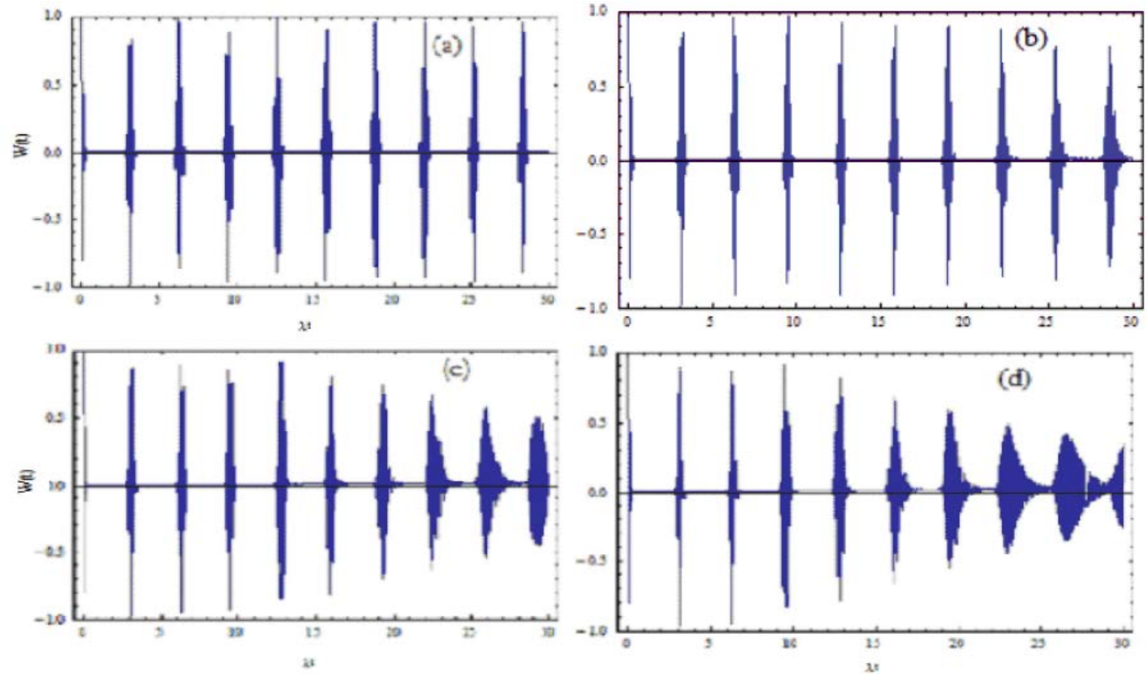

Fig. 3: The same as in Fig. 1 but for $m = 2$, $\beta_1 = 0.5$ and $\beta_2 = 0.5$.

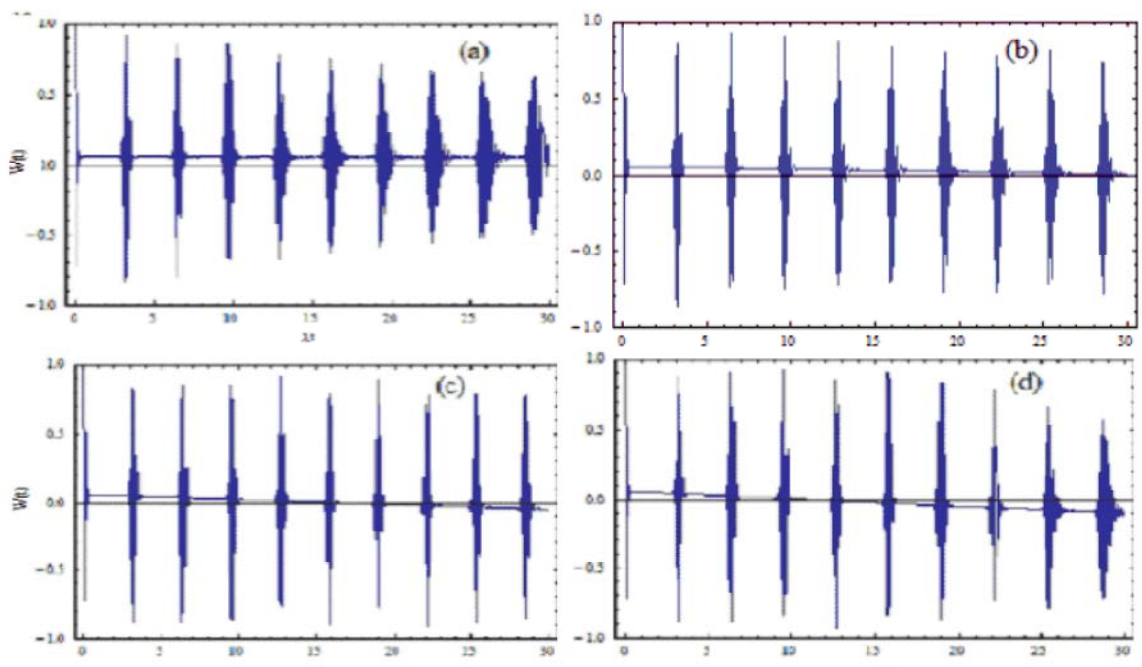

Fig. 4: The same as in Fig. (1) but for $m = 2$, $\beta_1 = \beta_2 = 0.5$.

The Entropy of the Cavity Field: The entanglement is an important parameter in quantum optics, where it is the corner-stone of the quantum information theory. In fact, much attention has been focused on the properties of the entanglement between field and atom, in particular the entropy of the system. The entropy is a very useful operational measure of the purity of the quantum state. The time evolution of the field entropy reflects the time evolution of the degree of entanglement between the atom and the field. Thus, the field entropy is used as a measurement of the entanglement degree for atomic system. Quantum mechanically, the field entropy is defined by:

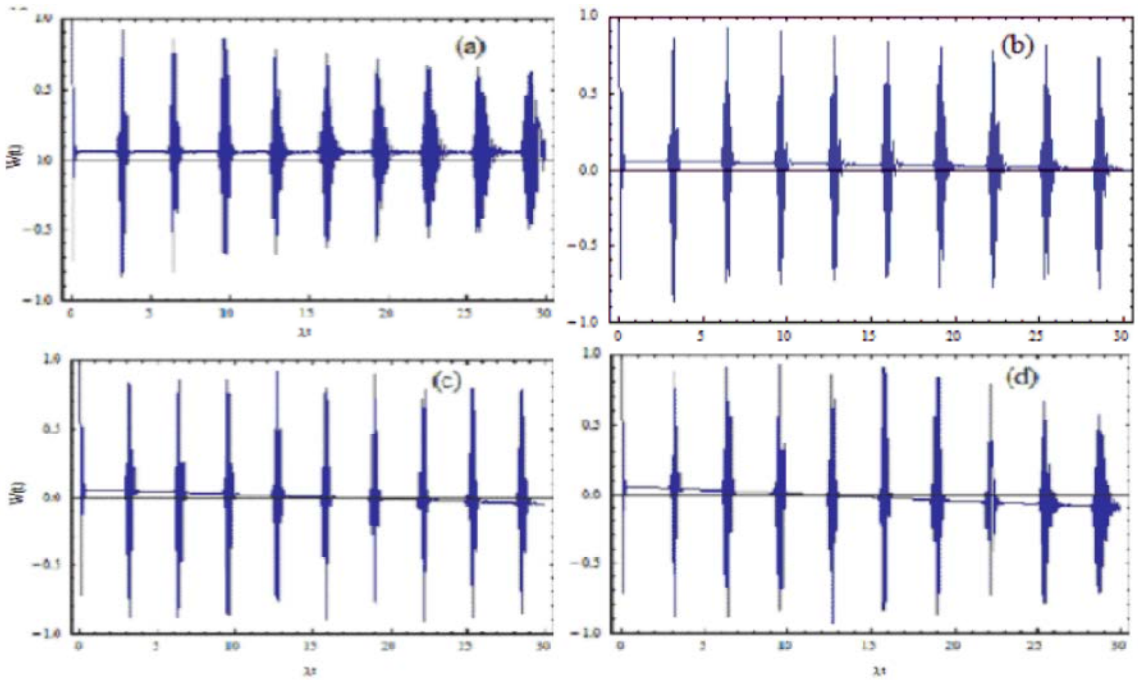

Fig. 5: The same as in Fig. (1) but for Δ = 5.

$$S = -Tr\{\rho \ln \rho\}, \tag{21}$$

where ρ is the density operator for a given quantum system with Boltzmann's constant equal to 1. Consider the field and the atom interacts with each other. Quantum entropies are generally difficult to compute, because they involve the digitalization of large (and, in many cases, infinite dimensional) density matrices. If ρ describes a pure state, then $S = 0$ and if ρ describes a mixed state, then $S \neq 0$. Let S_A and S_F denote the entropies of two interacting systems and let S denotes the entropy of the composite system. Araki and Lieb showed that the entropy satisfies the triangle inequalities:

$$|S_A - S_F| \le S \le S_A + S_F. \tag{22}$$

A nice illustration of these inequalities in the context of the JCM has been given (Knight and Phoenix, 1991). The entropies of the atom and the field, when treated as separated systems, are defined through the corresponding reduced density operators as:

$$S_{A(F)} = Tr_{A(F)}\left(\rho_{A(F)} \ln \rho_{A(F)}\right). \tag{23}$$

If we assume that the system starts from a pure state, then $S = 0$ and if ρ describes a mixed state, then $S \neq 0$. The density operator for a given quantum system can be written in the form:

$$\rho_f(\mathrm{t}) = Tr_{atom}\{|\psi(t)\rangle\langle\psi(t)|\} = \begin{pmatrix} \langle A|A\rangle & \langle A|B\rangle \\ \langle B|A\rangle & \langle B|B\rangle \end{pmatrix}, \tag{24}$$

where

$$|A\rangle = \sum_{n=0}^{\infty} q_n A(t)|n\rangle,\ |B\rangle = \sum_{n=0}^{\infty} q_n B(t)|n+m\rangle, \tag{25}$$

We can go to a basis in which the density matrix of the field is diagonal and we can express the field entropy in terms of the eigenvalues $\lambda_f^{\pm}(t)$ of the reduced field density operator, as follows:

$$S_f = -\left[\lambda_f^{+}(t)\ln\lambda_f^{+}(t) + \lambda_f^{-}(t)\ln\lambda_f^{-}(t)\right], \tag{26}$$

where $\lambda_f^{\pm}(t)$ is an eigenvalue of the density operator $\rho_f(\mathrm{t})$. Then, the eigenvalues and the eigenstates of the density matrix of the field are given by:

$$\begin{aligned} \lambda_f^{\pm} &= \langle A|A\rangle \pm \exp(\pm\theta)|\langle A|B\rangle|, \\ &= \langle B|B\rangle \mp \exp(\mp\theta)|\langle A|B\rangle|, \end{aligned} \tag{27}$$

where

$$\theta = \sinh^{-1}\left(\frac{1}{2|\langle A|B\rangle|}\left(\langle A|A\rangle - \langle B|B\rangle\right)\right). \tag{28}$$

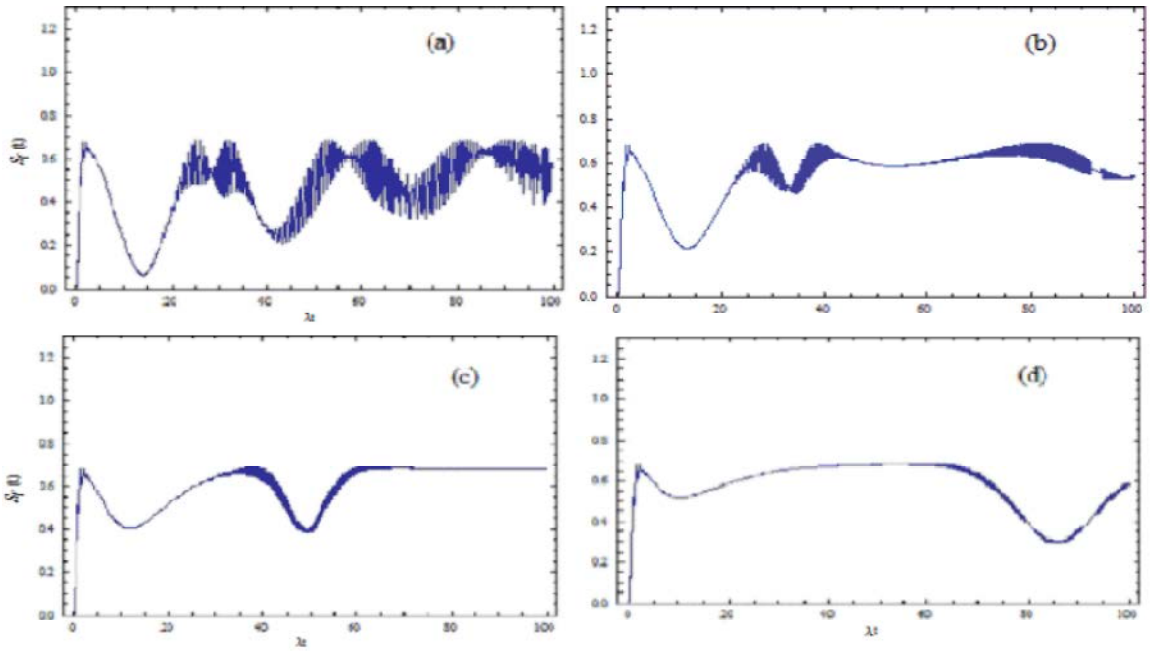

Fig. 6: The same as in Fig. 1 but for the field entropy $S_f(t)$.

where $\langle A|A\rangle$, $\langle B|B\rangle$ and $\langle A|B\rangle$ can be calculated from (25). Now, we turn to examine the temporal evolutions of the entropy field related to the present model. This will be done on the basis of the analytical solution obtained in the previous section. For this reason, we plot several figures to display the behavior of the field entropy against the scaled time for different values of the given parameters. For example in Fig. 6 we set the same data as in Fig. 1. Also, the value of the initial mean photon number to be $\bar{n}=20$. We notice that the maximum value of the entropy is approximately 0.7 and this value is affected by the gravity-parameter. Furthermore, it is observable that, the first maximum of the field entropy at $t > 0$ is achieved in the collapse time, while at one-half of the revival time the entropy reaches its local minimum. Also, the entropy shows fluctuations after a certain period of time. As the gravity-parameter increases value of maximum is decreased as shown in Figs. (6b-6d), in addition fluctuations start after a longer period time by comparison with the case $\Delta = 0$ as in Fig. 6a. Also, we observe that the time delaying more fluctuations with increasing of the parameter $\vec{k}\cdot\vec{g}$.

In Fig. 7, we investigate the effect of the detuning parameter where we put $\Delta = 5$ with different values of the gravity-parameter on the field entropy. We see that the entropy affected by the gravity-parameter where the maximum value of the entropy is approximately 0.4. Also the entropy reaches its local minimum compared with the case of absence the detuning Fig. 6a. However, when $\vec{k}\cdot\vec{g}=0.6$ the fluctuations of the entropy and its period time decrease; but when $\vec{k}\cdot\vec{g}=0.9$ the fluctuations occur for a short time and its period time also decrease as shown in Fig. 7d.

To examine the effect of the Stark shift, we consider the case in which $\Delta = 5$, $m = 2$, $\beta_1 = 0.5$ and $\beta_2 = 2$. In this case a drastic change occurs in the entropy function behavior, while for the case $\vec{k}\cdot\vec{g}=0$ an increase in the fluctuations number and consequently more period of squeezing at different intervals is observed. Also, the value of the maximum field entropy obviously decreases (Fig. 8a). On the other hand, when the gravity-parameter takes into account, the values of the field entropy decreases and the fluctuations increase as shown in Figs. (8a-8d).

In Fig. 9, we plot the entropy when the same values in Fig. 8 are taken with $\beta_1=\beta_2=1$. In this case, we can see different behavior from the previous case $\beta_1 = 0.5$ and $\beta_1 = 2$.. It is remarkable that the field entropy evolves periodically and shows of the disentangled between the field and the atom. Furthermore, we notice that the amplitude is decreasing by increasing the time and the gravity parameter.

Finally, when the detuning parameter takes place $\Delta = 5$, while all other parameters have the same values as the previous case. The minimum value of the entropy function is reduced and the behavior is similar to the previous case except there is more interference between the fluctuations pattern as shown in Fig. 10.

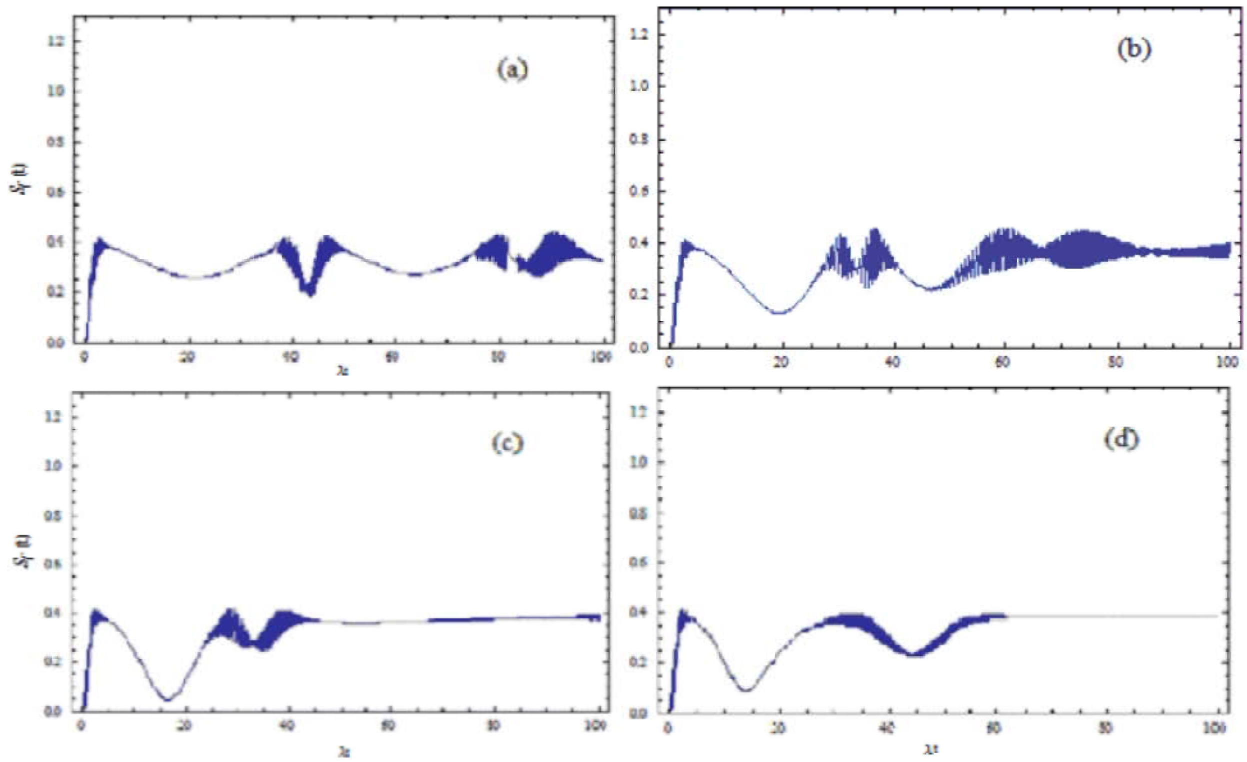

Fig. 7: The same as in Fig. 2 but for the field entropy $S_f(t)$.

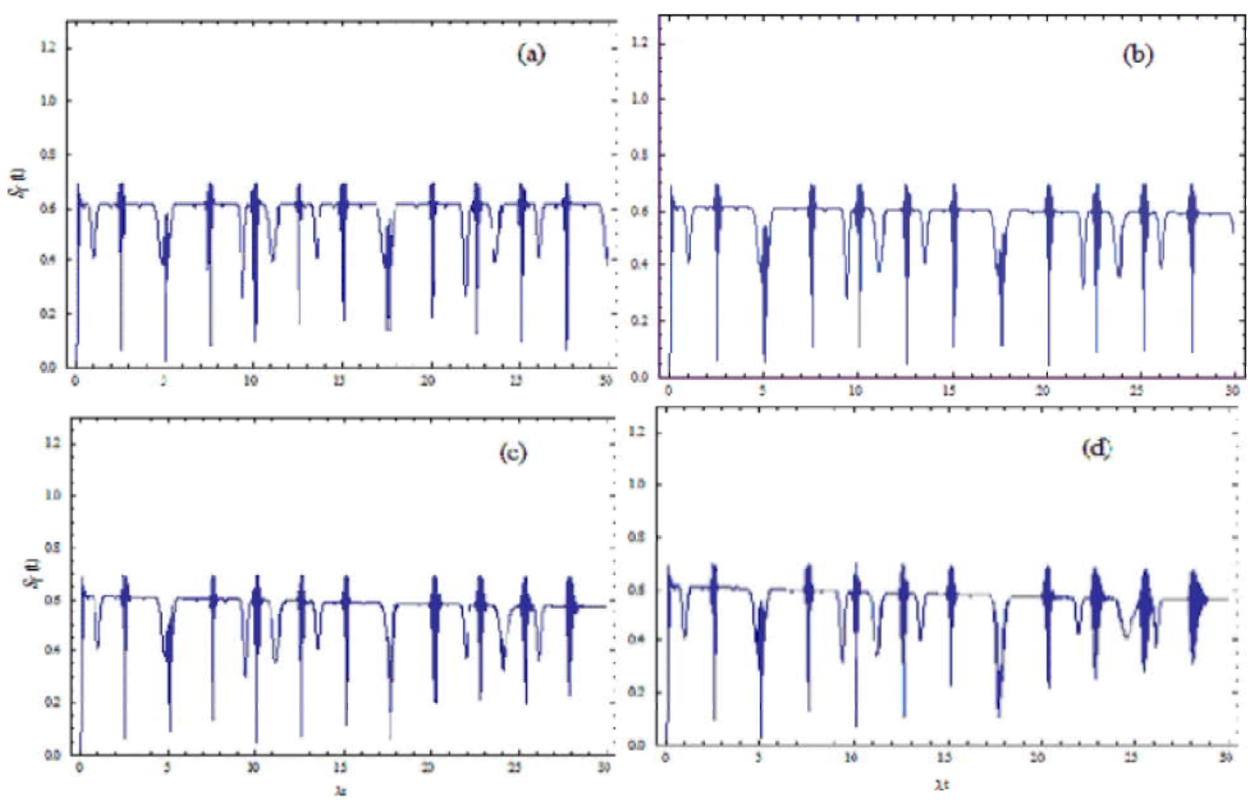

Fig. 8: The same as in Fig. 3 but for the field entropy $S_f(t)$.

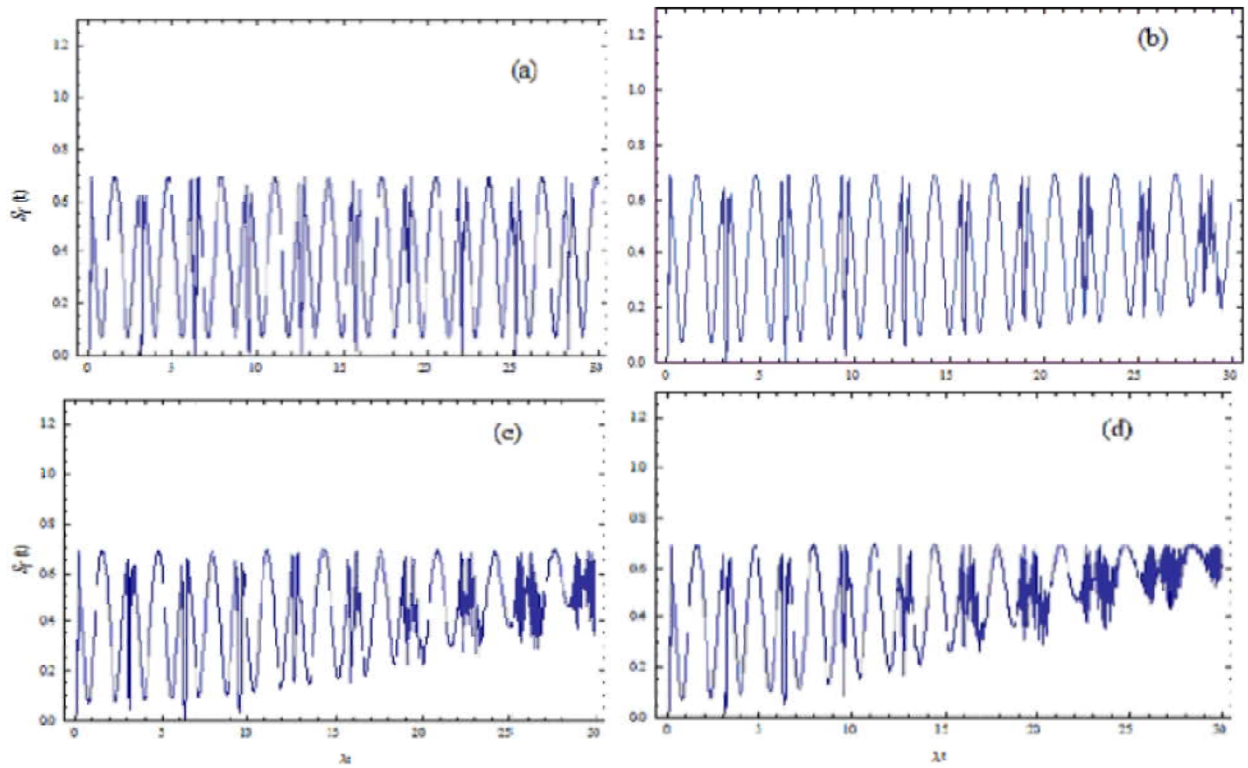

Fig. 9: The same as in Fig. 4 but for the field entropy $S_f(t)$.

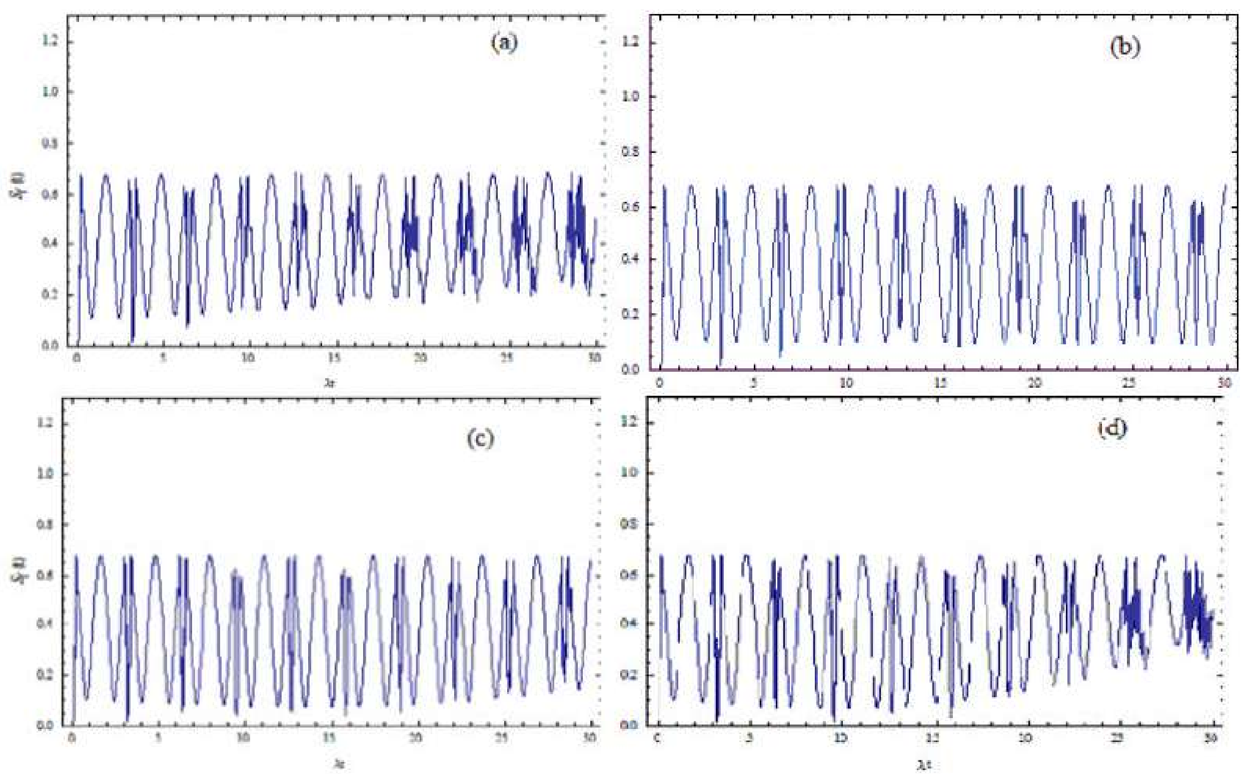

Fig. 10: The same as in Fig. 5 but for the field entropy $S_f(t)$.

CONCLUSION

We studied the interaction between one-mode cavity field and two-level atom in a pure momentum eigenstate in the presence of both the Stark shift and the classical homogenous gravitational field. We obtained the wave function of this atomic system and calculated some aspects related of this system such as the atomic population inversion and the field entropy. The effect of the detuning, the Stark shift and the classical homogenous gravitational field parameters on the atomic inversion and the field entropy is investigated. It is important to say that the classical homogenous gravitational field, the Stark shift and the detuning parameters have an obvious effect on the properties of the atomic population inversion and on the field entropy.

REFERENCES

1. Jaynes, E.T. and F.W. Cummings, 1963. Proc. IEEE, 51: 89.
2. Rempe, G., H. Walther and N. Klien, 1987. Phys. Rev. Lett., 57: 353.
3. Eberly, J.H., N.B. Narozhny and Sanchez-Mondragon, 1980. J. J. Phys. Rev. Lett., 44: 1323.
4. Yoo, H.I. and J. H Eberly, 1985. Phys. Rep., 118: 239.
5. Raimond, J.M., M. Brune and S. Haroche, 2001. Rev. Mod. Phys., 73: 565.
6. Phoenix, S.J.D. and P.L. Knight, 1991. Ann. Phys. (N.Y.) 186, 3811988; Phys. Rev. A, 44: 6023.
7. Adamas, C., M. Sigel and J. Mlynek, 1994. Phys. Rep., 240: 143.
8. Kastberg, A., W.D. Philips, S.L. Rolston, R.J.C. Spreeuw and P. S.Jessen, 1995. Phys. Rev. Lett., 74: 1542.
9. Lammerzahl, C. and C.J. Borde, 1995. Phys. Lett. A, 203: 59.
10. Marzlin, K.P. and J. Audertsch, 1995. Phys. Rev. A, 53: 1004.
11. Mohammadi, M., M.H. Naderi and M. Soltanolkotabi, 2006. J. Phys. A: Math. Gen. 39: 11065.