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Modelling Air Flows in Premises Using Conformal Mapping

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Abstract: At the moment the methods based on mathematical modelling are increasingly used for more accurate aerodynamic calculations of ventilation systems and air conditioning of premises of various purposes. The method involving solving a system of equations with particular derivatives is a most developed one and is vulnerable to criticism as to how accurate the resulting calculations are. In this paper an analytical method for determining air flows in ventilation of premises of complex configuration based on conformal mapping and almost free from natural computer calculation errors. The obtained solution is analytical and thus can prove to be more accurate compared to solving a system of differential equations. The method of designing current lines in premises with partitions can well allow for the geometric characteristics of the premises in designing displacement ventilation which typically has a low vortex formation.

Key words: Modelling · Conformal mapping · Conditioning · Function of a complex variable · Ventilation

INTRODUCTION

Mathematical modelling of speed, concentration and temperature fields is the most promising method of studying the existing laws of change of the microclimate parameters that affect health and safety requirements and technological requirements for the indoor air of premises. This allows for hands-on accurate solutions of multi-dimensional tasks as well as taking into account the way variable thermophysical characteristics and boundary conditions influence them.

The methods applied in mathematical and any other type of modelling are based on studying the characteristics of different objects by investigating their analogues [1]. It should be noted that a mathematical mdoel is an ideal actual object and it should be a clear reflection of the most essential characteristics of the phenomena with less important features being left out. Commonly, in order to make practical calculations more simple, simplified models and dependencies might be used without compromising the accuracy. As a result, a model becomes more simple and available for research.

In modelling temperature and velocity fields, Russian scientists have been long developing a method based on that of local heat balances where premises was divided into areas, their parameters considered equal. The major issue is difficulty justifying this division and identifying the heat exchange coefficient between them.

Numerical methods of modelling hydrogasdynamics [2-4] have recently been used for designing air flow velocity which are commonly based on the Navier Stock's system of equations. Studying this system involves a great deal of computer calculations and errors can undermine the accuracy of the final result. Therefore other approaches need to be employed which would be as much similar as analytical solutions.

Conformal mapping [7] is becoming increasingly used in mathematical physics for aerodynamics of small velocities which basically rely on the properties of a first order elliptic integral and its reverse function which is called an elliptic sinus

$$F(z, \alpha) = \int_{o}[dt/v((1-t^2)(1-\alpha^2t^2))],\ (0<\alpha<1) \qquad (1)$$

In these functions the upper subplane is used as an intermediate region. In this paper using the example of a premises with three partitions, we deal with a principle of designing a mathematical model of air velocity fields which is based on the conformal map theory.

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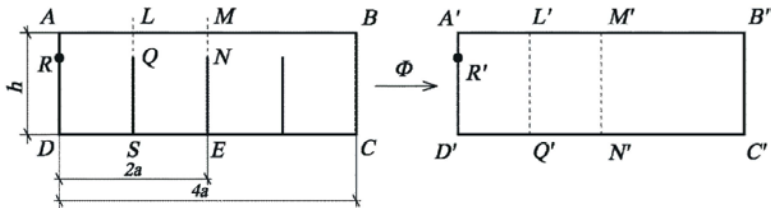

Fig. 1: Conformal map Φ of the original figure

MATERIALS AND METHODS

Designing a air current line: Designing air current lines takes two stages. First, air current line for the current premises are designed. For that, a conformal map Φ of the initial rectangle onto another rectangle is designed (Figure 1).

The original current lines are maps of the horizontal current lines of the simplified figure. In particular, the upper horizontal and lower broken sides of the figure *ABCD* are transformed into the upper and lower horizontal sides of a new rectangle *A'B'C'D'* respectively. According to the known symmetry principle of the theory of functions of a complex variable, it would suffice to design a conformal map Φ_1 of half the original rectangle with a hole *MN* onto some rectangle of unknown size with a line *M'N*' taking up the whole wall. If there is a map Φ_1, the original map Φ coincides with Φ_1 in the left half of the original figure with three partitions and the right half is continued in the same manner.

Designing a conformal map Φ_1 can be reduced to finding an even more simple map using the same symmetry principle. In this case it is necessary to look into a supplementary task which involves designing a conformal map Φ_2 of a rectangle *ALSD* with a line *LQ* onto some rectangle with a line *L'Q'* (Figure 1) and a conformal map Φ_2 is designed as a superposition of three maps

$$\Phi_2 = F(w,\beta) \circ wF^{-1}(z,\alpha), \tag{2}$$

where $F(w, \beta)$ is a map which is designed using a first order elliptic integral with some parameter β; $w = g(\zeta)$ is some linear-functional automorphism (a self-map) of the upper subplane; $\zeta = F^{-1}(z,\alpha)$ is an elliptic sinus with a known parameter α.

According to Formula (2), a linear functional map can be written as follows:

$$w = g(\zeta) = (A\,\zeta+B)/(\zeta+D), \tag{3}$$

where *A*, *B*, *D* are some real coefficients.

A map Φ_1, which is a solution of the original problem, is also designed as a superposition of three maps and the common map is designed in the reverse order as a superposition of simple maps.

Determining the velocity at a random point in space: Secondly, scalar velocities at random points in space are identified. If a map of a complex figure onto ist simplified image is known, tangents to ist current lines (determining the flow velocity) are calculated using a simple complex variable differentiation, Formula 4.

$$dw/dz=v_x+iv_y. \tag{4}$$

where v_x is a velocity projection onto the axis *OX*; $v_ó$ is a velocity projection onto the axis *OY*; *w* is a complex potential of some flow.

Scalar air flow velocity is given by:

$$v=|v_x+iv_y|=v(v^2_x+ v^2_y). \tag{5}$$

Since in designing current lines determining a medium flow velocity in premises with partitions, the above map is used and the air velocity is a product of the variables of each transformation stage

$$v=A_1\bullet A_2\bullet A_3\bullet A_4\bullet A_5\bullet A_6\bullet A_7\bullet A_8, \tag{5}$$

where A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8 are derivatives of the functions at each transformation step.

RESULTS AND DISCUSSION

For the premises shown in Fig. 1 it would suffice to design the lines which would be maps of the horizontal sections of the rectangle *1,622×4,97* affected by the map Φ_2^{-1} in order to visualize the air flows. Following the stretch of the resulting figure, there is another map used for these lines which transforms the figure into the rectangle with a partition. Finally, it would suffice to design a symmetrical map of the current lines to the symmetry axis.

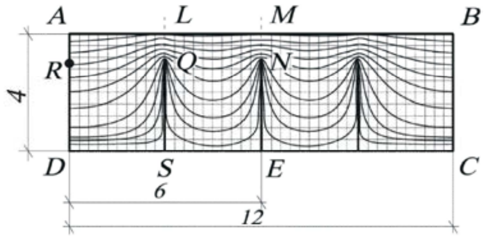

Fig. 2: Scheme ''current lines'' designed using MAPLE package

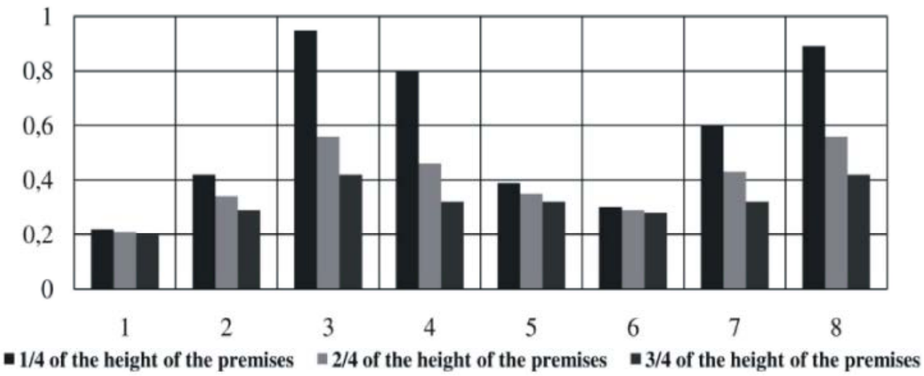

Fig. 3: Distribution of the air flow velocities in the premises with partitions

The final scheme of the air current lines for the current premises with partitions following joining the calculation results will be as shown in Figure 2 [5-7].

Formula 5 suggests that in order to identify the air flow velocities at any point in space, it is necessary that we know the variable at this point at each transformation stage. Therefore a mathematical model which determines the air velocity at any point of the investigated space of the current premises is as follows:

- For the first fourth of the premises:

$$v_I = \frac{11{,}593}{\sqrt{\left|\dfrac{\left(t_1^2-1\right)\left(87{,}8\cdot t_1^2-100\right)\left(t_3^2-1\right)\left(13{,}69\cdot t_3^2-100\right)}{\left(0{,}1\cdot t_1+0{,}3311\right)^4\left(t_2^2-1\right)\left(4{,}84\cdot t_2^2-6{,}25\right)\left(0{,}1\cdot t_3+0{,}919\right)^4\left(t_4^2-1\right)\left(0{,}09\cdot t_4^2-25\right)}\right|}}; \quad (6)$$

- For the second fourth of the premises:

$$v_{II} = \frac{2{,}799}{\sqrt{\left|\dfrac{\left(t_1^2-1\right)\left(87{,}8\cdot t_1^2-100\right)\left(t_3^2-1\right)\left(13{,}69\cdot t_3^2-100\right)}{\left(0{,}1\cdot t_1+0{,}3311\right)^4\left(t_2^2-1\right)\left(4{,}84\cdot t_2^2-6{,}25\right)\left(2\cdot t_3+18{,}71\right)^4\left(t_4^2-1\right)\left(0{,}09\cdot t_4^2-25\right)}\right|}}, \quad (7)$$

where the complex variables t_1, t_2, t_3 and t_4 correspond with the position of the point at certain stages of transformation.

The analysis of the data suggests that the air velocity increases as we are approaching the partition and reaches its maximum at the point where it is reached, which is proved by the calculations carried out based on Navier-Stokes equations detailed in [4]. Besides, as the air flow approaches the upper boundary of the premises, there is a drop in its velocity, which can account for a slowing effect of the surface.

CONCLUSION

The presented mathematical model based on conformal mapping allows for calculations of the fields of air velocity in premises with partions in the whole range of velocities where there is no stagantion zones in the fenced parts of the premises. The model relies on use of the symmetry of the rectangular premises and elliptic integral. The implementation of this function in packages of symbolic mathematics keeps them accurate if the model is numerically studied. An important characteristics of the model is that designing air velocity fields for displacement ventilation can be analytically accurate.

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