Middle-East Journal of Scientific Research 21 (12): 2247-2249, 2014
ISSN 1990-9233

DOI: 10.5829/idosi.mejsr.2014.21.12.570

Normalized Thermal Energy and Resistivity with Anisotropic Pressure

Muhammad Asif

Department of Physics, COMSATS Institute of Information Technology,
Lahore 54000 Pakistan

Abstract: The relationships between Normalized Thermal Energy [β_p,β_t] and resistivity, with anisotropic pressure and no particle sources in cylindrical tokamak geometry are derived. The relationships, $\beta_P = \left(\frac{\eta_{II}}{\eta_\perp}\right)$ *and* $\beta_t = \left(\frac{\eta_{II}}{\eta_\perp}\right)^2$ are obtained. These relationships, are a generalization of the usual Bennett relation and it is of importance for tokamak confinement and heating.

Key words: PACC: 52.35. Py, 52.55. Fa

INTRODUCTION

It is well known that the conventional neoclassical theory, NCT [1], is not in agreement with experiments since the electron thermal transport in tokamaks is much greater than that predicted, although the electrical conductivity, apparently, is not anomalous. For this reason two modifications of the theory have been proposed, namely the extended neoclassical theory, ENCT [2] and the pseudoneoclassical theory, PNCT [3]. It was previously shown [4-6] that a wide range of tokamak experiments are consistent with the extended neoclassical theory [7, 8]. Furthermore it was found [6], using a numerical analysis, that from the poloidal and toroidal components of Ohm's law for a tokamak there follows a relation between the normalized thermal energy content, β and the normalized plasma current, 1/q, which only depends on the plasma refuelling and not on thermal-energy transport [9]. Previously, the relation was examined only numerically and it is the purpose of [9, 10] to provide an approximate analytical expression for this relation valid for tokamaks, showing explicitly the dependence on the fuelling parameters. The relation, which can be considered as a generalization of the Bennett [11] relation, implies a constraint on the thermal energy content which cannot be violated, whatever the energy transport present and the plasma heating used. It should be noted that a different generalized Bennett relation, analogous to that given above, can be obtained by following the same procedure but using Ohm's law written for the NCT. The main focus of this paper is to show the relation between Normalized Thermal Energy and Resistivity, with anisotropic pressure and no particle sources in cylindrical tokamak geometry. These relationships, are a generalization of the usual Bennett relation and it is of importance for tokamak confinement and heating. The consideration of the generalized Bennett relation is of importance for the problem of plasma confinement and heating, since it provides a direct constraint on the thermal-energy content which cannot in any case be violated, whatever the energy transport present.

The Extended Neoclassical Theory: We consider the problem of determining the steady state in cylindrical tokamak geometry with anisotropic pressure and no particle sources. It is well known that the conventional neoclassical theory is unable to reproduce experimental results. However, it has recently been shown [4-6] that the extended neoclassical theory, ENCT [7-8], where the electron-electron collision rate is assumed to be anomalously high while the electron-ion collision rate is

Corresponding Author: Muhammad Asif, Department of Physics,
COMSATS Institute of Information Technology, Lahore 54000 Pakistan.

Coulombian, is in good agreement with tokamak experiments. We will therefore refer to this theory and first of all we recall the expressions for the toroidal and poloidal components of Ohm's law, which can be put in the following form (see ref. [8]), with no particle sources in the limit of large aspect ratio and circular magnetic surfaces [9, 10]:

$$j_\phi = E_\phi / \eta_{II} \quad (1)$$

$$\frac{dp}{dr} = -E_\phi B_\theta / \eta_\perp \quad (2)$$

Here both anisotropic models and MHD model [12] with, $p = \left(\frac{p_{II} + p_\perp}{2}\right)$ approximation and $\beta = \frac{(\beta_{p_{II}} + \beta_{p\perp})}{2}$ (ideal MHD) are used. where all quantities are surface-averaged and depend on the minor radius r, j_ϕ is the toroidal component of the current density, E_ϕ is the electric field, η_{II} and $\eta_\perp$ are the parallel and perpendicular resistivities, B_ϕ and B_θ are the toroidal and poloidal components of the magnetic field. The safety factor is $q = rB\phi / RB\theta$ and R is the major radius.

The minor radius of the last closed magnetic surface will be indicated by a and B_θ and j_ϕ are related by Ampere's law [9, 10]:

$$j_\phi = \frac{1}{\mu_0 r}\frac{d(rB_\theta)}{dr} \quad (3)$$

Substituting eq. (1) into (2), one obtains the following equation:

$$\frac{dp}{dr} = -\frac{\eta_{II} j_\phi B_\theta}{\eta_\perp} \quad (4)$$

It should be stressed that, in comparing tokamak experiments with theory, one can certainly use the cylindrical approximation where all quantities depend only on minor radius r; however, one must then consider the relations between surface-averaged quantities. In this case the perpendicular component of Ohm's law contains the toroidal corrections [Pfirsch-Schluter and viscosity or trapping terms] which are proportional to the square of the safety factor and which are absent for the exactly cylindrical case. In this paper these terms were not considered.

The Normalized Thermal-Energy Content, β_p and Resistivity: Here both anisotropic models and MHD model [12] with, $p = \left(\frac{p_{II} + p_\perp}{2}\right)$ approximation and $\beta = \frac{(\beta_{p_{II}} + \beta_{p\perp})}{2}$ (ideal MHD) are used. The normalized thermal-energy content, β_p, can be defined by [9, 10]:

$$\beta_p = \frac{2\mu_0}{B_\theta^2(a)}\langle p\rangle \quad (5)$$

where $\langle ...\rangle$ indicates the volume average and, in the cylindrical approximation,

$$\langle p\rangle = \frac{2}{a^2}\int_0^a p(r) r dr \quad (6)$$

so that, if $p(a) = 0$, we also have, by integrating by parts,

$$\langle p\rangle = -\frac{1}{a^2}\int_0^a \frac{dp}{dr} r^2 dr \quad (7)$$

Substituting eq. (7) into (5), one obtains the following equation:

$$\beta_p = -\frac{2\mu_0}{a^2 B_\theta^2(a)}\int_0^a \frac{dp}{dr} r^2 dr \quad (8)$$

Substituting eq. (3) and eq. (4) into eq.(8), one obtains the following equation:

$$\beta_p = \left(\frac{\eta_{II}}{\eta_\perp}\right) \quad (9)$$

The Normalized Thermal-energy Content, β_t and Resistivity: Here both anisotropic models and MHD model [12] with, $p = \left(\frac{p_{II} + p_\perp}{2}\right)$ approximation and $\beta = \frac{(\beta_{p_{II}} + \beta_{p\perp})}{2}$ (ideal MHD) are used. The normalized thermal-energy content, β_t, can be defined by [9, 10]:

$$\beta_t = \frac{2\mu_0}{B_\phi^2(a)}\langle p\rangle \quad (10)$$

where $\langle ...\rangle$ indicates the volume average and, in the cylindrical approximation, Substituting eq. (7) into (10), one obtains the following equation:

$$\beta_t = -\frac{2\mu_0}{a^2 B_\phi^2(a)} \int_0^a \frac{dp}{dr} r^2 dr \qquad (11)$$

Substituting eq. (3) and eq. (4) into eq.(11), one obtains the following equation:

$$\beta_t = \frac{\eta_{II}}{\eta_\perp}\left(\frac{B_\theta}{B_\phi}\right)^2 \qquad (12)$$

Haas and Thyagaraja [13] assumed the phenomenological form $\left(\frac{\eta_{II}}{\eta_\perp}\right) = \left(\frac{B_\theta}{B_\phi}\right)^2$. where β_ϕ and B_θ are the toroidal and poloidal components of the magnetic field. With these assumptions eq.(12) can be written in the form

$$\beta_t = \left(\frac{\eta_{II}}{\eta_\perp}\right)^2 \qquad (13)$$

These relationships, are a generalization of the usual Bennett relation and are of importance for tokamak confinement and heating. It should be noted that a different generalized Bennett relation, analogous to that given above, can be obtained by following the same procedure but using Ohm's law written for the NCT. The pseudoneoclassical theory [3], has been proposed, where the enhanced collisionality not only modifies the tokamak collisionality regime but it is also allowed to affect the electron distribution function. Thus both the parallel and the perpendicular resistivities become equal to the Spitzer perpendicular resistivity. When this theory is used, the corresponding generalized relation is quantitatively very close to that resulting from the ENCT. In this paper we have considered the classical system which obey the Ohm's law. Quantum effect has not been consider because the Ohm's law destroyed. The relations (9, 13) are simple models valid only for classical system.

CONCLUSION

We have obtained relations between Normalized Thermal Energy [β_p, β_t] and resistivity, with no particle sources in cylindrical tokamak geometry. The relationships, $\beta_p = \left(\frac{\eta_{II}}{\eta_\perp}\right)$ and $\beta_t = \left(\frac{\eta_{II}}{\eta_\perp}\right)^2$ are obtained. These relationship, are a generalization of the usual Bennett relation and it is of importance for tokamak confinement and heating. The consideration of the generalized Bennett relation is of importance for the problem of plasma confinement and heating, since it provides a direct constraint on the thermal-energy content which cannot in any case be violated, whatever the energy transport present and the heating used.

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