

## Application of Production Function Apparatus in Regional Forecasting-Analytical Calculations of Economic Growth and Social Development

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**Abstract:** The paper presents an analysis of the empirical and theoretical literature, considering fundamental concepts of the theory of production functions. A comparative analysis of economic and mathematical models applied in regional socio-economic researches based on different types of production functions is carried out. The choice of functional form is an important aspect of constructing production functions taking into account the limitations of their use. A kind of modified production function with the autonomous progress pace is suggested for the purposes of forecast-analytical calculations of economic growth and social development at the regional level. The progress parameter is differentiated with respect to the components which characterize the level of social development of the region, Technical Park of productive sectors of regional economic complex, the level of the supportability of the region with modern means of communication. On the basis of modeling the degree of the influence of both technical and social and information factors on the gross output is identified and differentiated in this work. *The* values of the parameters of production functions for both Kursk region as a whole and each sector of the regional economy has been determined and developed by the authors. The necessity of applying the developed models for the purposes of regional forecasting and analytical researches has been proved.

**Key words:** Region • Sustainable development of the region • Social and economic growth • Factors of the development • Regional development models • Production functions and their parameters • Targets • Analysis and forecasts of regional development

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### INTRODUCTION

Modern Russian economy is characterized by retention of rather flat trends of deterioration of development growth factors, a high level of social and economic differentiation of the regions. In this regard, the study of the nature of dynamic processes taking place in the economy, the search for the ways of creating a sustainable basis for economic growth, the development of the arsenal of methods and techniques for the formation and application of state economic policy aimed at maintaining sustainable growth and sustainable socio-economic development of the country, including the regional level, are now becoming ever more urgent. Activization of scientific searching in terms of the study of the dialectics of views on economic growth and

carrying out a more profound analysis of the provisions established in various theories of economic growth is of particular importance from the theoretical and methodological point of view. These aspects are scientifically proved in [1, 2]. In the applied aspect there are special requirements for regional economic policy: on the one hand, it should have a system-wide orientation and, on the other hand, it should take into account the specific nature of a particular area. These aspects are highlighted, for example, in [3].

### MATERIALS AND METHODS

Nowadays economic-mathematical models and tools are increasingly used in regional analytics and prognostics. Some of them were developed previously

and some are being produced now by national and foreign scholars and experts in the field of territorial management and economics. First of all, it refers to the econometric models of the formation, functioning and development of socio-economic systems and their component subsystems of various levels [4].

Applied use of forecasting and analytical calculations based on advanced scientific achievements is a necessary and important component of the system of management of all socio-economic processes, both for Russia as a whole and for its each region (subordinate entity of the Federation). So, in econometric studies of various ranks production functions (PF) apparatus is widely used. In this apparatus the dependent variables usually take on the values of the amount of output and the independent variables take on the values of the volumes used and spent on the production resources or production factors.

Cobb-Douglas production function (CDPF) is traditionally used in economic forecasting and analytical calculations, both for a country as a whole and for its constituent regions, (see e.g. [5, 8]). It was developed as a practical application of the ideas of Jean-Baptiste Say in 1929 by American scientists and economists Charles Cobb and Paul Douglas. In this functional dependence the volume value  $Y$ , as an absolute factor of output (including performed work, delivered services, etc.) is determined by the existing assets of production factors which are labour and capital and defined by the factors of marginal productivity the effectiveness of their use in the accounting period:

$$Y = A \times K^\alpha \times L^{1-\alpha} \quad (1)$$

where  $A$  is the scale parameter;  $K$  is the capital input;  $\alpha$  is the coefficient of output elasticity of capital;  $L$  is the labour input;  $1-\alpha$  is the coefficient of output elasticity of labour.

The revealed relationship between economic variables in CDPF in relative ratios (or rates of addition, which are usually described in percent) is quite simple and logical:

$$y = k\alpha + l(1-\alpha), \quad (2)$$

where  $y$  is the average annual rate of addition of the amount of gross of output;  $k$  is the average annual rate of addition of capital factor;  $l$  is the average annual rate of addition of the labour factor.

The experience of the implementation of CDPF in forecasting and analytical researches is limited by its main drawback which is the tough relationship between the indicators of the efficiency of production factors, to be more precise, their sum is equal to one. This fact significantly reduces the possibility of its application in identifying and describing functional linkages in economy, because it implies that the growth in output amount is precisely predetermined by the growth of production factors input. The possible direction to overcome this limitation is to modify CDPF introducing some amendments abolishing the initial assumption about the neutrality of scientific and technological progress (STP) impact on economic development [6].

An attempt to modify CDPF and eliminate the limitation for the value of the степенных indicators for the case of production factors was undertaken by Jan Tinbergen (described in detail in [7]). The PF proposed by him took the following form:

$$Y = A \times K^\alpha \times L^\beta \quad (3)$$

where  $\alpha$  is the coefficient of output elasticity of capital;  $\beta$  is the output elasticity of volume of labour.

The relationship between result and factor economic indicators of Tinbergen PF with respect to the rates of addition is as follows:

$$y = k\alpha + l\beta, \quad (4)$$

Such modification allowed eliminating the major limitation of the classical variant of CDPF, taking into account, though implicitly, the influence of STP; however, the STP is not included directly into the number of its factors.

Another CDPF modification belongs to Robert Merton Solow, who proposed to consider the influence of STP on economic development by introducing an independent variable, namely the inclusion into accumulation factors the natural logarithm base of figure of one raised to the power of  $\alpha$  [10]. With regard to the above said Solow PF has the following form:

$$Y = A \times K^\alpha \times L^{1-\alpha} \times e^{\alpha} \quad (5)$$

where  $\alpha$  is the coefficient characterizing addition to the production results under the influence of STP.

With respect to the rates of addition the relationship between result and factor economic indicators of this functional dependence can be represented as follows:

$$y = k\alpha + l(1-\alpha) + \beta, \tag{6}$$

Such a modification is a significant improvement of the given above two-factor model of CDPF, since it eliminates its main weakness - it does not take into account the technological level of production explicitly.

Beginning of Russian modifications of CDPF is associated with the name of the Soviet Professor Aleksandr Anchishkin who differentiated the factor of scientific and technological progress into two components i.e. an independent NTP  $\alpha$  and NTP connected with the character of produced product distribution  $\beta$  [7, p. 71]. As a result, Anchishkin PF got the following form:

$$Y = A \times K^\alpha \times L^\beta \times e^{\gamma t} \tag{7}$$

where  $\alpha$  is the coefficient characterizing addition to the production results under the influence of the independent STP;  $\beta$  is the coefficient representing addition to the production results under the influence of STP depending on the character of produced product distribution, connected directly with the product costs for research, development and introduction into production.

It should be noted that the value of the coefficient ( $\beta$ ) indicates the STP impact on the economic development. With respect to the rates of addition for this functional dependence the relationship between result and factor economic indicators is as follows:

$$y = k\alpha + l(1-\alpha) + \beta, \tag{8}$$

Generally, Russian scientists have made repeated attempts to expand the number of PF factor indicators to get a more complete and adequate explanation of the dynamics of production results. For example, B. Mikhalevskiy made calculations of three-factor PF, adding natural resources costs to labour and capital input (this is described, in particular, in [8]). Undoubtedly, the proposal made by S. Vischnev deserves focused attention; it consists of a significant expansion of the explanatory factors including qualifications of employees, science and research costs, age composition of fixed capital etc. in their number [7].

The next stage in the development of PF apparatus is the construction of a dynamic production function, in which the amount of production output will be determined taking into account the autonomous pace of progress. In case of creating this kind of PF the STP

influence is represented by the introduction of a new multiplier  $e^{\gamma t}$ , where the parameter  $\gamma$  determines the growth rate of production output directly dependent on the impact of STP [6]. In this case, CDPF with independent rate of technological progress takes the following form:

$$Q = a \times L^\alpha \times K^\beta \times e^{\gamma t}, \tag{9}$$

where  $t$  is the time;  $\gamma$  is the STP parameter.

In the applied socio-economic forecasting and analytical studies in case of the development of such dynamic CDPF for the country as a whole or its constituent region, an aggregate output (or aggregate income) of the country or region in comparable prices is taken as a value of the annual (quarterly or monthly) output; and labour input (the number of labour input units for the period under consideration), fixed capital input (the amount of fixed capital used for the period under consideration) and STP are taken as production factors (or resources) [9-11].

Such approach to building CDPF with the autonomous pace of technical progress eliminates most of its disadvantages typical to all previously discussed PF modifications. However, this PF also does not take into account such important components of progress as social factor (i.e. the development of the very productive power), information factor (i.e., the development of means of communication and the formation of the information space); in the XXI century the composition of the factors of economic growth cannot be considered complete without taking into account these factors.

For the purposes of forecasting and analytical calculations of economic growth and social development at the regional level (for a constituent of the Russian Federation), we propose a modified version of the production function with the autonomous pace of progress (MPF<sub>APP</sub>). In the MPF<sub>APP</sub> the parameter of progress  $\gamma$  from the functions (9) is differentiated into components, characterizing: 1) the level of social development of the region; 2) the technical park of productive sectors of regional economic complex; 3) the level of supportability of the region with modern means of communication. Taking into account the above said, MPF<sub>APP</sub> has the following form:

$$Q = a_0 \times L^\alpha \times K^\beta \times e^{a_1 z_1 t} \times e^{a_2 z_2 t} \times e^{a_3 z_3 t} \tag{10}$$

$$\text{or } Q = a_0 \times L^\alpha \times K^\beta \times e^{(a_1 z_1 + a_2 z_2 + a_3 z_3) t}, \tag{11}$$

where  $Q$  is the output (including the performance of work and services) in the region for the period (a year);  $a_0$  is the scale parameter;  $a_1, a_2, a_3$  are the numerical coefficients;  $L$  is the labour production factor (the volume value of labour resources);  $K$  is the capital production factor (the volume value of fixed assets or capital input in production);  $\alpha, \beta$  is the production factors parameters of elasticity;  $z_1$  is the coefficient characterizing the level of social development of the region;  $z_2$  is the indicator characterizing the technical park of the sectors of regional economy (the level of renewal of fixed production assets);  $z_3$  is the parameter characterizing the supportability with modern means of communication;  $t$  is the time (years).

In the functions such as (10) and (11) the amount of annual gross regional product and the volume of production assets are measured in cost units and labour input is measured in the average annual number of employees; the levels of social development, renewal of fixed assets and the supportability with communications are measured in fractions of a unit.

**RESULTS AND DISCUSSION**

A new approach to econometric modeling of regional growth allows identifying and differentiating the degree of the influence of both technical, social and information factors on the gross output. Indeed, taking the logarithm of the resulting MPF<sub>APP</sub>:

$$\ln Q = \ln a_0 + \alpha \ln L + \beta \ln K + a_1 z_1 t + a_2 z_2 t + a_3 z_3 t, \quad (12)$$

and then differentiating it with respect to  $t$ , we obtain a functional relationship between the rate of addition of gross output ( $P_Q$ ) and the rates of addition of all production factors (including three components of progress that is technical, social and informational):

$$P_Q = \alpha P_L + \beta P_K + a_1 z_1 + a_2 z_2 + a_3 z_3. \quad (13)$$

As a result of transformations of the derived MPF<sub>APP</sub> the value of the rate of addition of gross output in the regional economic complex is equal to the weighted total of the rates of addition to the of the main production factors and the levels of social, technical and informational progress.

The coefficients (parameters) of the proposed MPF<sub>APP</sub> for the economy of Kursk region and for its industrial and agricultural sectors were determined on the basis of the evidence concerning the socio-economic development of Kursk region for the period of 25 years (gross regional product, the number of employees, the volume of fixed assets, the level of social development, the level of renewal of the funds, the level of information and communication supportability; all the indicators were considered in total for the regional economy and separately for its industrial and agricultural sectors) (Table 1).

Verification of the reliability of the MPF<sub>APP</sub> functions executed using the coefficient of determination ( $R^2=0,98 \square 0,99$ ), Fisher's ratio test ( $F=19,6 \square 39,6 > F_{crit.}=19,3$  at 5% significance level) and the Durbin-Watson statistics

Table 1: Proposed production functions (MPF<sub>APP</sub>) for Kursk region

Modeling objects	Proposed production functions
MPF <sub>APP</sub> general form	$Q = a_0 \times L^\alpha \times K^\beta \times e^{(a_1 z_1 + a_2 z_2 + a_3 z_3)t}$
Regional economy	$Q = 0,317.L0,77.K0,585.e^{0,01z_1t+0,47z_2t-0,236z_3t}$
Industrial sector	$Q = 0,25.L1,241.K0,476.e^{0,123z_1t+0,089z_2t-0,162z_3t}$
Agricultural sector	$Q = 0,002764.L2,327.K0,295.e^{0,071z_1t-0,965z_2t+0,039z_3t}$

Table 2: Evaluated production functions (MPF<sub>APP</sub>) with respect to the rates of addition

Modeling objects	Relationship between the rates of addition
MPF <sub>APP</sub> with respect to the rates of addition	$P_Q = \alpha P_L + \beta P_K + a_1 z_1 + a_2 z_2 + a_3 z_3.$
Regional economy	$P_Q = 0,77P_L + 0,585P_K + 0,01z_1 + 0,47z_2 - 0,236z_3$
Industrial sector	$P_Q = 1,241P_L + 0,476P_K + 0,123z_1 + 0,089z_2 - 0,162z_3$
Agricultural sector	$P_Q = 2,327P_L + 0,295P_K + 0,071z_1 - 0,965z_2 + 0,039z_3$

( $DW=1,54 \square 1,67$ ) confirmed that the proposed modified version of the production function with the autonomous pace of progress in the form (10) or (11) adequately reflects the relationship in the regional economy, industry and agriculture of Kursk region and can be used for solving the tasks of analysis, forecasting and programming the economic growth and social development of the region. Table 2 shows  $MPF_{APP}$  constructed with respect to the rates of addition.

### CONCLUSION

In conclusion, it is worth noting that the comparative analysis of the relative indicators of the dynamics of production factors and results, as well as of the values of elasticity coefficients allows conducting a very interesting study of functioning of the regional socio-economic system and determining the possibility of its growth with the use of various factors and reserves.

As a criterion of forecasting capabilities of the evaluated  $MPF_{APP}$  representing regression multiplicative models, we take the ratio of the standard regression error  $S$  to the mean value of the dependent variable. For three functions represented in Table 1, this ratio is 0.64%, 1.37% and 0.43%, respectively. Therefore, since the ratio of the standard regression error to the mean value of the dependent variable for all three  $MPF_{APP}$  is small and there is no residual autocorrelation proved using statistics  $DW$ , it fair to say that the developed econometric models are reliable and can be used in regional forecasting and analytical calculations of the economic growth and social development.

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