Submission of Kalman Filter in Sensorless Momentum and Pose Evaluation

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Abstract: In this paper, the application of the extended Kalman filter (KF) to estimate the states of a stepper motor is described. A dynamic model, describing the operation of a two-phase stepper motor, was derived and used for sensorless speed and position measurement. The simulation results of full state estimation, using full order extended Kalman filter, is presented. The effects of noise and the sensitivity of the algorithm to motor parameters are thoroughly investigated.

Key words: Stepper motor • Kalman filter • Sensorless • Speed • Position

INTRODUCTION

Over the past few years, the use of stepper motors has increased. The reasons for this include: better reliability due to the elimination of mechanical brushes, better heat dissipation as the windings are located on the stator and not on the rotor, higher torque-to-inertia ratio due to a lighter rotor and lower prices.

Stepper motors were originally designed to be used in open loop control. Their inherent stepping ability allows for accurate positioning without feedback [1]. Closed-loop control of stepper motors has been used increasingly in the last two decades to achieve faster response times and higher resolution capabilities. Adaptive control was successfully demonstrated in [2] to achieve higher precision by canceling torque-ripple effects. The stepper motor can also be operated at higher speeds, by taking nonlinear effects into consideration [3].

Kalman filter is one of the greatest inventions in the history of statistical estimation theory [4]. Its most immediate applications have been for the control of complex dynamic systems such as manufacturing processes, aircraft, ships, or spacecraft. In the present paper KF is applied to the estimation of speed and position of a two-phase stepper motor. The paper presents a complete dynamic model of a two phase stepper motor. Simulation results of KF application to the problem in hand and their discussion is also provided.

Stepper Motor Drive System: A typical drive system for a stepping motor is represented by the block diagram in Fig. 1, the number of phases being four in this example.

Here the step command pulses are given from an external source and it is expected that the stepping motor is able to follow every pulse. This type of operation is referred to as the ‘open–loop’ drive. The open–loop drive is attractive and widely accepted in applications of speed and position control [6]-[9]. However, the performance of a stepping motor is limited under the open-loop mode. The drive may fail to follow a pulse command when the frequency of the pulse train is too high or inertial load is too heavy. Moreover the motor motion tends to be oscillatory in open–loop drives.

The performance of stepping motor can be improved to a great extent by employing position feedback and/or speed feedback to determine the proper phase(s) to be switched at proper timings. This type of control is termed ‘closed–loop’ drive. The closed–loop control is advantageous over the open-loop control not only in that step failure never occurs but also that the motion is much

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quicker and smoother [5]. In closed-loop control, a position sensor is needed for detecting the rotor position. As a typical sensor, nowadays, an optical encoder is used and it is usually coupled to the motor shaft. Position and speed sensors have some disadvantages:

- They are usually expensive.
- The sensor and the corresponding wires will take up space.
- In defective and aggressive environments, the sensor might be the weakest part of the system. Especially, the last item degrades the system’s reliability and reduces the advantage of a closed-loop drive system.

In the more advanced systems, instead of an additional mechanical sensor, rotor position and/or speed is sensed by observation of waveforms of the currents in the motor windings.

On the other hand, avoiding sensor means use of additional algorithms and added computational complexity that requires high-speed processors for real time application. As digital signal processors have become cheaper and their performance greater, it has become possible to use them for controlling electrical drives as a cost effective solution. Some relatively new fully digitized methods, used for sensorless speed/position estimation, utilize this enhanced processing capacity [6].

Usually sensorless control is defined as a control scheme where no mechanical parameters like, speed and torque, are measured. In recent years, nonlinear observers are used to estimate parameters and states of electrical machines. Model-based Sensorless schemes, generally, rely on accurate system modeling and accurate model parameters values [10].

**Dynamic Modelling of a Stepper Motor:** We consider a two-phase permanent magnet (PM) stepper motor. A simplified schematic of a motor with one pole-pair is shown in Fig. 2. Commanded voltages \( v_a \) and \( v_b \) control the two-phase currents \( i_a \) and \( i_b \) [11][12]. The magnetic field, due to the stator current, interacts with the permanent magnet on the rotor to create a torque, so that the rotor will tend to align itself with the magnetic field produced by the currents. Applying a sequence of voltage to each phase in succession will cause the rotor to step. The size of each step is 90° in the case of Fig. 1.

In general, it is determined by \( n \), the number of rotor teeth. A sinusoidal characteristic of the magnetic field in the air gap is assumed [13].

![Fig. 2: Diagram of a two-phase stepper motor [7].](image)

The dynamic equations, derived in [7], are cast in state-space form as follows:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

where \( x \) is the output vector, \( R \) is the resistance of the coils, \( L \) is the inductance of the coils, \( \lambda \) is the motor constant depending on the design of the rotor, \( J \) is the inertia of the rotor and the load, \( \theta (t) \) is the actual rotor position, \( \theta_a \) is the location of the coil \( a \) in the stator, \( \omega \) is the rotational velocity of the rotor and \( i_a (t) \) is the current in the coil \( a \) as function of time.

The model described above conforms to the general state-space model given by

\[
\begin{align*}
x_{k+1} &= F_k x_k + B_k u_k + w_k \\
z_k &= H_k x_k + v_k
\end{align*}
\]

\( x, u \) and \( y \) are called the state, input and output vectors, respectively.

**States Estimation Using Kalman Filter:** A linear state-space model is given by:

\[
\begin{align*}
x_{k+1} &= F_k x_k + B_k u_k + w_k \\
z_k &= H_k x_k + v_k
\end{align*}
\]

The covariance matrices of the noises are defined as:

\[
\begin{align*}
\text{cov} (w_k) &= E\{w_k w_k^T\} = Q \\
\text{cov} (v_k) &= E\{v_k v_k^T\} = R
\end{align*}
\]

where \( E\{.\} \) denotes the expected value, \( F, B \) and \( H \) are matrices. The Kalman filter is decomposed into two steps: predict and update, as shown in Table 1.
where $K$ is the Kalman gain matrix, used in the update observer and $P$ is the covariance matrix for the state estimation, containing information about the accuracy of the estimate.

**Speed and Position Estimation Using the EKF:** In the present work, the EKF is applied to estimate the speed and position, of a two phase stepper motor drive, using current measurements [14]. The sensorless speed and position estimator was implemented, in Matlab, using the EKF algorithm given by (9) through (13). For our simulation we selected three G series 34 Frame stepper motors manufactured, by SmartDrive [8], whose parameters are given in Table 2.

Simulation is used to estimate motor speed and position using the EKF algorithm described in the previous section. The simulation provides the computed (true) and the estimated winding currents, $i_b$ and $i_a$, together with speed, $\omega$ and position, $\theta$. The state estimation covariance, $P$, is also computed. This is an indicator of the goodness of the estimate. Effects of measurement noise and sensitivity to changes in motor parameters, $R$ and $L$, are also investigated. The noise covariances $R$ and $Q$ used throughout the simulation are given:

Measurement noise covariance:

\[
R = 0.8^2 \tag{14}
\]

Process noise covariance:

\[
Q = \begin{bmatrix}
0 & 0.01^2 & 0 & 0 \\
0 & 0 & (0.5)^2 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The values used in both matrices are obtained using trial and error approach. Exact values are extremely difficult to find. This requires a lengthy statistical analysis that is beyond the scope of this paper. Due to page number limitation, for this paper, only results relevant to the motor SMR 341 are presented. This motor has the following parameters: $R=0.4$, $L=1.75$ mH, $J=1.6*10^{-4}$ mH, $\lambda=0.1$ Nm/A, $B=0.001$ Nms/rad.

Figure 3 (a) shows that the estimated value of rotor speed converges to the true value within 0.27 seconds. The tracking of rotor position is shown in Fig. 3 (b). It can be seen that there is a sharp increase in the estimated value before the algorithm is able to lock on the true value. It is clear that good estimation is possible in less than 0.4 seconds.

**Effects of Measurement Noise:** Considering the SM343 and increasing the process noise level in the covariance $Q$ leads to deterioration of the algorithm capability in tracking the system states. This yields meaningless
results as shown in Fig. 4 (a) and (b)[15]. These figures show clearly the strong dependency of the algorithm on the process noise. Measurement noise, R, was found to be less significant in estimating the states. Therefore, it is advisable to take a great care in choosing the elements of the matrix Q.

CONCLUSION

The focus of this work was the sensorless speed and position measurement of a two-phase stepper motor. A dynamic mathematical model of the stepper motor was derived and formulated into state space model for easy analysis and manipulation. The EKF was, then, applied to the developed motor model. Three motors of different sizes, whose parameters were provided in the manufacturer’s data sheet, were used to test the effectiveness of the developed EKF-based sensorless algorithm. Simulation results have shown good agreement between true and estimated states. Effects of process noise were also investigated. This clearly indicated that the application of the EKF requires good evaluation of measurement noise linked with thorough understanding of the system under investigation. The simulations carried out, in this work, show the great potential of the investigated technique.

REFERENCES

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