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On a Type of Para Kenmotsu Manifold

[1]T. Satyanarayana and [2]K.L. Sai Prasad

[1]Department of Mathematics,
Pragathi Engineering College, Kakinada, India
[2]Department of Mathematics,
GVP College of Engineering For Women, Visakhapatnam, India

Abstract: The object of this paper is to study a class of almost para contact metric manifold namely para Kenmotsu (briefly p-Kenmotsu) manifold in which R(X, Y).C = 0 where C is the conformal curvature tensor of the manifold and R is the Riemannian curvature and R(X, Y) is considered as a derivation of the tensor algebra at each point of the manifold for tangent vectors X and Y.

Key words: Kenmotsu Manifold · Curvature Tensor · Ricci Tensor · Tangent Vector

INTRODUCTION

Sato [1] defined the notions of an almost para contact Riemannian manifold. After that, T. Adati and K. Matsumoto [2] defined and studied para-Sasakian and SP-Sasakian manifolds which are regarded as a special kind of an almost contact Riemannian manifolds. Before Sato, Kenmotsu [3] defined a class of almost contact Riemannian manifolds. In 1995, B. B. Sinha and K. L. Sai Prasad [4] have defined a class of almost para contact metric manifolds namely para Kenmotsu (briefly p-Kenmotsu) and SP-Kenmotsu manifolds. They also have studied the curvature properties of p-Kenmotsu manifold and the curvature properties of semi-symmetric metric connection of SP-Kenmotsu manifold.

Let M_n be an n-dimensional differentiable manifold equipped with structure tensors (ϕ, ξ, η) where ϕ is a tensor of type (1,1), ξ is a vector field, η is a 1-form such that

$$\eta(\xi) = 1 \tag{1.1}$$

$$\phi^2(X) = X - \eta(X)\xi; \quad \bar{X} = \phi X \tag{1.2}$$

Then M_n is called an almost para contact manifold.w

Let 'g' be the Riemannian metric g satisfying such that, for all vector fields X and Y on M,

$$g(X, \xi) = \eta(X) \tag{1.3}$$

$$\phi\xi = 0, \ \eta(\phi X) = 0, \ \text{rank } \phi = n - 1 \tag{1.4}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\,\eta(Y) \tag{1.5}$$

Then the manifold M_n [1] is said to admit a almost para contact Riemannian structure (ϕ, ξ, η, g).

A manifold of dimension 'n' with Riemannian metric 'g' admitting a tensor field 'ϕ' of type (1,1), a vector field 'ξ' and a 1-form 'η ' satisfying (1.1), (1.3) along with

$$(\nabla_X \eta)Y - (\nabla_Y \eta)X = 0 \tag{1.6}$$

$$(\nabla_X \nabla_Y \eta)Z = [-g(X,Z) + \eta(X)\,\eta(Z)]\,\eta(Y) + [-g(X,Y) + \eta(X)\,\eta(Y)]\eta(Z) \tag{1.7}$$

$$\nabla_X \xi = \phi^2 X = X - \eta(X)\xi \tag{1.8}$$

is called a para Kenmotsu manifold or briefly P-Kenmotsu manifold [4]. This paper deals with type of p-Kenmotsu manifold in which

$$R(X, Y).C = 0 \tag{1.9}$$

where C is the conformal curvature tensor of the manifold and R is the Riemannian curvature.

Let (M^n, g) be an n-dimensional Riemannian manifold admitting a tensor field 'ϕ' of type (1,1), a vector field 'ξ' and a 1-form 'η' satisfying

Corresponding Author: K.L. Sai Prasad, Department of Mathematics, GVP College of Engineering For Women, Visakhapatnam, India.

$(\nabla_X \eta)Y = g(X,Y) - \eta(X)\,\eta(Y)$ (1.10)

$g(X, \xi) = \eta(X)$ and $(\nabla_X \eta)Y = \varphi(\bar{X}, Y)$ where φ is an associate of ϕ

(1.11)

is called a special P-Kenmotsu manifold or briefly SP-Kenmotsu manifold [4]. In this paper it is proved that if in a P-Kenmotsu manifold (M^n, g) $(n > 3)$ the relation (1.9) holds then the manifold is conformally flat and hence is an SP-Kenmotsu manifold. Also it is shown that a conformally symmetric P-Kenmotsu manifold (M^n, g) is an SP-Kenmotsu manifold for $n > 3$. (since it is known that C = 0 when n = 3, it has taken that n > 3).

It is known that [4] in a P-Kenmotsu manifold the following relations hold:

$S(X, \xi) = -(n-1)\,\eta(X)$ (1.12)

$g[R(X, Y)Z, \xi] = \eta[R(X, Y, Z)] = g(X, Z)\,\eta(Y) - g(Y, Z)\,\eta(X)$

(1.13)

$R(X, \xi) = -1$ (1.14)

$R(X, \xi, \xi) = -X + \eta(X)\xi$ (1.15)

$R(X, \xi, X) = \xi$ (1.16)

$R(\xi, X, \xi) = X$ (1.17)

$R(X, Y, \xi) = \eta(X)Y - \eta(Y)X$; when X is orthogonal to ξ.

(1.18)

where S is the Ricci tensor and R is the Riemannian curvature. Moreover, it is also known that a P-Kenmotsu manifold cannot be flat and a P-Kenmotsu manifold satisfying R(X, Y).W = 0 i.e., a projectively flat P-Kenmotsu manifold is said to be Einstein manifold with the constant curvature $-n(n-1)$.

The above results will be used in the next section.

P-kenmotsu Manifold Satisfying R(X, Y).C = 0:

We have

$$C(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}\,[g(X,Z)QX - g(X,Z)QY + S(Y,Z)X - S(X,Z)Y] + \frac{r}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y] \quad (2.1)$$

where 'r' is the scalar curvature and 'Q' is the symmetric endomorphism of the tangent space at each point corresponding to the Ricci tensor 'S' [5] i.e.,

$$g(QX, Y) = S(X, Y). \quad (2.2)$$

Then

$$\eta(C(X,Y)Z) = g(C(X,Y)Z, \xi) = \frac{1}{n-2}\left[\left(\frac{r}{n-1}+1\right)(g(Y,Z)\eta(X) - g(X,Z)\eta(Y)) - (S(Y,Z)\eta(X) - S(X,Z)\eta(Y))\right] \quad (2.3)$$

Putting $Z = \xi$ in (2. 3), we get

$$\eta(C(X,Y)Z) = 0 \quad (2.4)$$

Again putting $X = \xi$ in (2. 3), we get

$$\eta(C(\xi, Y)Z) = \frac{1}{n-2}\left[\left(\frac{r}{n-1}+1\right)g(X, Z) - S(Y, Z) - \left(\frac{r}{n-1}+1\right)\eta(Y)\eta(Z)\right]. \tag{2. 5}$$

Now

$$(R(X, Y).C)(U, V)W = R(X, Y)C(U, V)W - C(R(X, Y)U, V)W - C(U, R(X, Y)V)W - C(U, V)R(X, Y)W.$$

In virtue of (1.9) we get

$$R(X, Y)C(U, V)W - C(R(X, Y)U, V)W - C(U, R(X, Y)V)W - C(U, V)R(X, Y)W = 0. \tag{2. 6}$$

Therefore

$$g(R(\xi, Y)C(U, V)W, \xi) - g(C(R(\xi, Y)U, V)W, \xi) - g(C(U, R(\xi, Y)V)W, \xi) - g(C(U, V)R(\xi, Y)W, \xi) = 0. \tag{2. 7}$$

From this it follows that

$$'C(U, V, W, Y) - \eta(Y)\eta(C(U, V)W) + \eta(U)\eta(C(Y, V)W) + \eta(V)\eta(C(U, Y)W) + \eta(W)\eta(C(U, V)Y) - g(Y, U)\eta(C(\xi, V)W) - g(Y, V)\eta(C(U, \xi)W) - g(Y, W)\eta(C(U, V)\xi) = 0 \tag{2. 8}$$

where

$'C(U, V, W, Y) = g(C(U, V)W, Y)$.

Putting Y = U in (2.8) we get

$$C(U, V, W, U) - \eta(U)\eta(C(U, V)W) + \eta(U)\eta(C(U, V)W) + \eta(V)\eta(C(U, U)W) + \eta(W)\eta(C(U, V)U) - g(U, U)\eta(C(\xi, V)W) - g(U, V)\eta(C(U, \xi)W) - g(U, W)\eta(C(U, V)\xi) = 0 \tag{2. 9}$$

Let $\{e_i\}$, i = 1, 2, . . ., n be an orthonormal basis of the tangent space at any point. Then the sum 1 = i = n of the relation (2. 9) for $U = e_i$ gives

$$\eta(C(\xi, V)W) = 0 \tag{2.10}$$

By using (2. 4), we have from (2. 8)

$$'C(U, V, W, Y) - \eta(Y)\eta(C(U, V)W) + \eta(U)\eta(C(Y, V)W) + \eta(V)\eta(C(U, Y)W) + \eta(W)\eta(C(U, V)Y) - g(Y, U)\eta(C(\xi, V)W) - g(Y, V)\eta(C(U, \xi)W) = 0 \tag{2. 11}$$

In virtue of (2. 5) and (2. 10) we have

$$S(V, W) = \left(\frac{r}{n-1}+1\right)g(V, W) - \left(\frac{r}{n-1}+n\right)\eta(V)\eta(W). \tag{2. 12}$$

Using (2.3), (2.4) and (2.12) the relation (2.11) reduces to

'C (U, V, W, Y) = 0. (2. 13)

From (2.13) it follows that

C(U, V) W = 0. (2. 14)

Thus we can state the following theorem:

Theorem 1: A P-Kenmotsu manifold (M^n, g) (n > 3) satisfying the relation R(X, Y).C = 0 is conformally flat and hence is an SP-Kenmotsu manifold.

For a conformally symmetric Riemannian manifold, we have ∇ C = 0 [6] and hence for such a manifold R(X, Y).C = 0 holds. Thus we have the following corollary of the above theorem:

Corollary 1: A conformally symmetric P-Kenmotsu manifold (M^n, g) (n > 3) is an SP-Kenmotsu manifold.

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