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The Evaluation of Investment Projects by Using the Bayesian Criterion and Reducing the Degree of Subjectivity

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Abstract: In general, investment decision is an evaluation of the proposed alternatives for the investor using a set of indicators. It seems to be appropriate to use a method of the potential distribution of probabilities when investors know only the data of relevant characteristics of the investment projects. The application of the method is presented and it is shown that the quantitative estimates calculated by this method are relative and strongly depend on the choice of the base project.

Key words: Generalized indicator · Bayesian criterion · Shannon entropy · Subjectivity

INTRODUCTION

Investment decision is generally an evaluation of the alternatives proposed for the investor on the basis of the indicators and the selection of the projects according to the existing conditions (constraints). If possible, the multi-criteria problem usually reduces to a one-criterion issue by introducing a generalized criterion to simplify the problem [1]. In our case, this criterion could be the generalized index of the investment project attractiveness.

For the convolution of partial indicators related to a particular investment project, it seems reasonable to use the method of the potential distribution of probability. An information situation exploiting this method is characterized by the fact that investors know only the data on the corresponding private characteristics of investment projects. In this case, it seems appropriate to put forward a hypothesis of a linear convolution of some partial dimensionless parameters [1].

There is a sufficient number of different methods for determining the weights of such convolutions. They are all based on a particular behavior model of the social and economic systems, which is usually postulated informally. Meanwhile, a greater objectivity is typical of the models built using the principle of maximum uncertainty. One possible approach to evaluate these weights, which is based on this principle, is the method the potential distribution of probability. The content of this situation may be represented by the following scheme.

MATERIALS AND METHODS

Let consider n investment projects which, in their purpose and contents, are competitors in terms of investing funds. Each of these projects is associated with a set of characteristics that define its investment attractiveness.

Let such characteristics be m. Define $x_{ij}\,(i=\overline{1,n},\ j=\overline{1,m})$ as particular indicators of comparable projects. Initial data in this case are conveniently situated in a matrix

$$X=\begin{bmatrix} x_{11} & x_{21} & x_{m1} \\ x_{12} & x_{22} & x_{m2} \\ x_{1n} & x_{2n} & x_{mn} \end{bmatrix}.$$

Weight of the j–th characteristic in the distribution of funds to achieve the desired level of investment project efficiency is generally unknown. It is required to assess the weight of each characteristic in the distribution of resources taking into account the objectively existing uncertainties.

The principle of a potential distribution postulates an application of the Bayesian criterion as a comprehensive indicator for measuring the attractiveness of the project. It has the following form

$$b_j=\sum_{j=1}^{m} r_j p_{ij}, \tag{1}$$

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Table 1: Characteristics of alternative investment projects

	Projects				
Projects characteristics	1	2	3	4	5
1. Net Present Value (NPV), mln. rub.	1	1.3	0.7	2.6	1.1
2. Profitability Index (PI)	1.2	1.5	1.3	1.7	1
3. Internal Rate of Return (IRR),%	15.5	14.2	17.5	13	17
4. Return on investment (ROI),%	45	30	65	35	50
5. Payback period, years	3	4	5	3	6

where r_{ij}- dimensionless parameters, $r_{ij} = x_{ij}/x_{эj}$, if an increase in x_{ij} leads to growth of b and $r_{ij} = x_{эj}/x_{ij}$; if the increase in x_{ij} leads to the reduction of b; $x_{эj}$ - characteristics of the standard, which is considered as one of the projects.

Then the weighting factors $p_j = (j = \overline{1,m})$, reflecting a pattern of environment behavior are found by maximizing the Shannon entropy [2, 3]

$$H = -\sum_{j=1}^{m} p_j \ln p_j \to \max \tag{2}$$

under the constraints

$$\sum_{j=1}^{m} p_j = 1, \ \prod_{j=1}^{m} \frac{-p\,j}{r\,j} = const. \tag{3}$$

It can be shown that the expression for estimating weights in this case has the form

$$p_j = \left(\sum_{i=1}^{n} rij \right)^{-1} \left[\sum_{j=1}^{m} \left(\sum_{i=1}^{n} rij \right)^{-1} \right]^{-1} \tag{4}$$

Constraints (3) postulate the normalization and constancy of the geometric mean. Physically, this means that the relative increase in the weight of the j–th characteristic is in proportion to the relative increment of the level of the same characteristic among the totality of the considered projects and the proportionality coefficient depends on the level achieved.

Thus, by calculating with expression (4) the significance coefficients, it is possible not only to rank the private indicators on their contribution, but also to choose the most attractive project from the offered alternatives. The efficiency of the method is demonstrated in the following example. Initial data for five specific indicators of five alternative projects are shown in Table 1.

Table 2: Importance (significance) of the characteristics

Projects characteristics	Coefficients
1. Net Present Value (NPV), mln. rub.	0,16
2. Profitability Index (PI)	0,18
3. Internal Rate of Return (IRR),%	0,20
4. Return on investment (ROI),%	0,20
5. Payback period, years	0,26

Reduced matrix of initial data, calculated by expressions (2), where the standard accepted is project 1, is as follows:

$$R = \begin{bmatrix} 1 & 1,3 & 0,7 & 2,6 & 1,1 \\ 1 & 1,25 & 1,08 & 1,42 & 0,83 \\ 1 & 0,92 & 1,13 & 0,84 & 1,1 \\ 1 & 0,67 & 1,44 & 0,78 & 1,1 \\ 1 & 0,75 & 0,6 & 1 & 0,5 \end{bmatrix} \tag{5}$$

Then the matrix of calculated by expressions (1-4) integrated indicators of investment attractiveness of alternative projects equals

$$B = [1 \quad 0,94 \quad 0,98 \quad 1,24 \quad 0,9]. \tag{6}$$

The weighting coefficients for particular projects characteristics calculated by the expression (4) are summarized in Table 2.

Analyzing the results of the calculations, we can conclude that the most attractive for the investor is project 4, because it has the highest generalized index.

Emphasis on the Subjective Evaluations of the Importance of Project Characteristics: Another conclusion that can be drawn on the basis of the initial data and the calculations is that the payback period is the defining characteristic of these projects is and has the highest weighting factor. However, it makes sense to take into account the opinions and experience of qualified experts in the evaluation of the project characteristics importance. For this purpose, it is advisable to take into account the subjective opinion of experts in the formation of the matrix (5).

Table 3: Subjective priority of characteristics

Projects characteristics	Priority	Coefficients
1. Net Present Value (NPV), mln. rub.	4	0,13
2. Profitability Index (PI)	3	0,20
3. Internal Rate of Return (IRR),%	5	0,07
4. Return on investment (ROI),%	1	0,33
5. Payback period, years	2	0,27

Table 4: Generalized evaluation of the characteristics importance

Projects characteristics	Coefficients
1. Net Present Value (NPV), mln. rub.	0,14
2. Profitability Index (PI)	0,19
3. Internal Rate of Return (IRR),%	0,13
4. Return on investment (ROI),%	0,27
5. Payback period, years	0,27

Table 5: Expert opinion in the evaluation of projects preference

	Projects				
Parameters	1	2	3	4	5
Project priority	2	1	4	5	3
Assessment of the "weight" of preferences	0,27	0,33	0,13	0,07	0,2
"Potential assessment" (6)	1	0,94	0,98	1,24	0,9
Generalized assessment of investment attractiveness	0,64	0,64	0,56	0,66	0,55

Typically, these problems are solved by estimates formation (usually in points) for all characteristics and then assigned weighting coefficients for characteristics in order to convolute them further into a generalized index. However, in this case, the problem, which is shown on the stage of grading, is to formalize the intuitive approach. The method based on the minimization of participation of experts' opinion should be recognized as a more objective method. This approach requires the expert to place a number of preferences for project characteristics and weights are calculated using the principle of maximum uncertainty. It can be shown, that under these conditions, the most objective scale is Fishburn estimates [3, 4].

$$\hat{p}_j = \frac{2(m-l+1)}{m(m+1)}, \; j=\overline{1,m} \tag{7}$$

where n - number of estimated characteristics; j - rank in the scale of priorities for the j-th characteristic.

In other words, it suffice to place the data in order of importance (significance, impact, etc.) and to determine the weights by the expression (7). Then the results in Table 2 should be recalculated according to the subjective factor of the first order (the importance of the project characteristics). Continuing the example, we can assume that, in the opinion of experts, the prioritization of the relevant characteristics of the projects and the weights look like as shown in Table 3.

Then weighting factors for characteristics of the projects taking into consideration a subjective factor can be calculated by the expression

$$Q_j = \gamma P_{\text{э}j} + (1-\gamma)P_{Пj}, \; j=\overline{1,m} \tag{8}$$

where γ - the degree of trust to experts; $P_{\text{э}j}$ - expert (subjective) assessment of the j-th weighting factor; $P_{Пj}$ - potential (objective) assessment of the j-th weighting factor; n- number of estimated characteristics.

The results of this recalculation with a 50% level of confidence in expert opinions are summarized in Table 4. The analysis of the results indicates the sensitivity of the method to both an objective and a subjective factor (Tables 2 and 4).

Changing γ from no-confidence level (0%) to absolute confidence level (100%), we see the convergence of the results to the limits either for the purely objective or for the purely subjective assessment.

Emphasis on the Experts' Opinions in the Evaluation of Alternative Investment Projects: So far we have considered a problem of the subjective opinions of experts in assessing the significance of the projects characteristics. The second scale of the original Table 1 includes a list of projects. So, expert opinion must be formalized by taking into account the preferences among investment projects. According to the experts, projects are ranked in the order of preferences and then with an expression similar to (7), weights reflecting the quantitative measure of preference are estimated (taking into account the subjective factor of the second order). With the problem being solved, let us assume that the evaluation by experts allowed to place the projects in the order of preferences, presented in Table 5. From the calculation results, summarized in Table 5, it is seen that the subjective evaluation given by the experts does not agree with the more objective and potential estimates. Thus, the generalized evaluation of investment attractiveness, calculated by the expression similar to (8), takes into account both of these factors.

Thus, the most preferred investment project is project 4.

Investigation of the Effect of Projects Choice: We have shown above that formalizing information situation of potential distribution of probability involves the formation of Bayesian criterion (1) to assess the weights of which we introduce the dimensionless parameters r_{ij}. It uses the

Table 6: Initial data for investigation

	Projects *(i)*				
Characteristics *(j)*	1	2	3	4	5
1	11	34	24	67	76
2	23	23	54	46	34
3	21	12	34	45	56
4	23	32	23	32	23
5	43	56	12	11	44

Table 7: Input data (project 1 – basic one)

	Projects *(i)*				
Characteristics *(j)*	1	2	3	4	5
1	1	3,09	2,18	6,09	6,90
2	1	1,00	2,34	2,00	1,47
3	1	0,57	1,61	2,14	2,66
4	1	1,39	1,00	1,39	1,00
5	1	1,30	0,27	0,25	1,02

concept of a "standard" and each of the projects can be considered as such. In fact, it is necessary to consider the following feature of this method.

Let us apply the abstract matrix X that contains m specific indicators (characteristics) of some n comparable projects in Table 6.

To go to the dimensionless matrix of indicators, we use the expression $r_{ij} = x_{ij}/x_{Bj}$ in formula (1).

The following Table 7 presents input data in case project 1 is selected as a standard (basic project).

The use of the expression (4) when selecting project 1 as the basic one (standard) gives the following values of weights

$$P^{(1)} = [0,07 \quad 0,17 \quad 0,17 \quad 0,23 \quad 0,36]^T$$

Similarly, the weights are calculated when selecting project 2, 3, ...: as a standard.

$$P^{(2)} = [0,17 \quad 0,14 \quad 0,08 \quad 0,25 \quad 0,36]^T;$$

$$P^{(3)} = [0,13 \quad 0,35 \quad 0,24 \quad 0,20 \quad 0,08]^T;$$

$$P^{(4)} = [0,28 \quad 0,22 \quad 0,23 \quad 0,21 \quad 0,06]^T;$$

$$P^{(5)} = [0,27 \quad 0,14 \quad 0,25 \quad 0,13 \quad 0,21]^T;$$

CONCLUSION

Analyzing the results, it must be admitted that the choice of the project as the base one affects the weighting factors of their characteristics. In other words, the weight of the private indicator in the complex characteristic of the project is highly dependent on the choice of the base object for comparison. Since the weighting factors are only for internal operations, their use for other purposes ignoring this method is incorrect.

Let us consider the effect of the base project selection on a generalized indicator (1). To do this, using the above-mentioned weight P_j, we calculate the value of the indicator (1) for the different cases of base project selection:

$$b^{(1)} = [1,00 \quad 1,27 \quad 1,17 \quad 1,56 \quad 1,79];$$

$$b^{(2)} = [0,79 \quad 1,00 \quad 0,92 \quad 1,22 \quad 1,41];$$

$$b^{(3)} = [0,86 \quad 1,09 \quad 1,00 \quad 1,33 \quad 1,53];$$

$$b^{(4)} = [0,64 \quad 0,82 \quad 0,75 \quad 1,00 \quad 1,15];$$

$$b^{(5)} = [0,56 \quad 0,71 \quad 0,65 \quad 0,87 \quad 1,00];$$

The comparison $b^{(i)}$, $i=\overline{1,n}$, shows that the selection of the base project also strongly affects the absolute values of the generalized indicator. Therefore, values can be used only for comparison on a «better or worse» principle in the formation of a number of preferences for the projects under consideration. Thus, it is easy to see that, in all cases, when selecting the basic project, a number of preferences remains identical: 5, 4, 2, 3, 1, despite the fact that the absolute values vary significantly in case the basic project changes.

Thus, the potential distribution of probability can be successfully used for the qualitative comparison of a number of projects in the form of preferences. The quantitative evaluation of both weights and generalized indicators calculated by this method is relative and strongly depends on the choice of the base project.

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