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Regression-Cum-Exponential Ratio Type Estimators for the Population Mean

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Abstract: This paper proposes regression-cum-exponential ratio type estimators under simple random sampling without replacement for estimating the finite population mean of study variable by using the information of two auxiliary variables. The mean square error and bias expressions have been derived. The theoretical comparison of proposed estimators has been made with existing estimators. Empirical study is conducted to evaluate the efficiency of proposed estimators.

Key words: Regression estimator . auxiliary variable . exponential estimator . mean square error . bias

INTRODUCTION

The use of auxiliary information has been studied by various authors in various forms to improve the efficiency of their constructed estimator. In the use of auxiliary variables, ratio, product and regression estimators are corner stone's in the estimation of population characteristics.

Auxiliary information has always been seems effective in increasing the precision of estimates in survey sampling. Grant [7] was the first who estimated the population of England based on auxiliary information. The work of Neyman [13] may be referred as an initial work where auxiliary information has been discussed in detail. Cochran [3] suggested the classical ratio type estimator for the population mean and Cochran [4] used auxiliary information in regression estimators.

Robson [14] and Murthy [11] investigated; if the correlation is negative the product method of estimation is quite effective. Mohanty [10] used two auxiliary variables by combining the regression and ratio method. Bahl and Tuteja [2] were the first, who used exponential type estimators for estimating the population mean. Samiuddin and Hanif [15] introduced ratio and regression estimation procedure by using two auxiliary variables and they have provided modification of Mohanty [10] estimator. Hanif *et al*. [9] proposed an estimator which was the modification of Singh and Espejo [17] estimator. The development is continued in the form of exponential estimators for different situations by many authors such as Singh and Vishwakarma [18], Noor ul amin and Hanif [12] and Sanaullah *et al*. [16].

NOTATIONS AND VARIOUS EXISTING ESTIMATORS

$$e_{\bar{y}} = \frac{\bar{y}-\bar{Y}}{\bar{Y}}, \quad e_{\bar{x}} = \frac{\bar{x}-\bar{X}}{\bar{X}}, \quad e_{\bar{z}} = \frac{\bar{z}-\bar{Z}}{\bar{Z}}$$

$$E(e_{\bar{y}}) = E(e_{\bar{x}}) = E(e_{\bar{z}}) = 0,\ E(e_{\bar{y}}^2) = \theta C_y^2, E(e_{\bar{x}}^2) = \theta C_x^2$$

$$E(e_{\bar{x}}e_{\bar{y}}) = \theta\rho_{xy}C_xC_y, E(e_{\bar{y}}e_{\bar{z}}) = \theta\rho_{yz}C_zC_y \quad (2.1)$$

$$\theta = \frac{1}{n}-\frac{1}{N}, C_y = \frac{S_y}{\bar{Y}}, \rho_{xy} = \frac{S_{xy}}{S_xS_y}, H_{ij} = \rho_{ij}\frac{C_i}{C_j}$$

Cochran [3] and Robson [14] suggested the classical ratio and product estimators, respectively, for estimating the population mean as:

$$t_1 = \bar{y}\left[\frac{\bar{X}}{\bar{x}}\right] \quad (2.2)$$

$$t_2 = \bar{y}\left[\frac{\bar{z}}{\bar{Z}}\right] \quad (2.3)$$

The mean square equations (MSE) of the estimators of t_1 and t_2 are

$$MSE(t_1) \approx \bar{Y}^2\theta\left[C_y^2 + C_x^2(1-2H_{yx})\right] \quad (2.4)$$

$$MSE(t_2) \approx \bar{Y}^2\theta\left[C_y^2 + C_z^2(1+2H_{yz})\right] \quad (2.5)$$

Cochran [4] suggested the classical regression estimator given by

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$$t_3 = \bar{y}_r + b_{yx}\left(\bar{X} - \bar{x}\right) \quad (2.6)$$

The mean square error of t_3 is given by

$$MSE\left(t_3\right) = \theta\bar{Y}^2 C_y^2\left(1 - \rho_{xy}^2\right) \quad (2.7)$$

Bahl and Tuteja [2] proposed the following exponential ratio and product type Estimators

$$t_4 = \bar{y}\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \quad (2.8)$$

$$t_5 = \bar{y}\exp\left(\frac{\bar{z} - \bar{Z}}{\bar{z} + \bar{Z}}\right) \quad (2.9)$$

respectively, the mean square error of the estimators t_4 and t_5 are given by

$$MSE\left(t_4\right) \approx \theta\bar{Y}^2\left[C_y^2 + \frac{C_x^2}{4}\left(1 + 4H_{yx}\right)\right] \quad (2.10)$$

$$MSE\left(t_5\right) \approx \theta\bar{Y}^2\left[C_y^2 + \frac{C_z^2}{4}\left(1 - 4H_{yz}\right)\right] \quad (2.11)$$

respectively, the bias expressions of the estimators t_4 and t_5 are given by

$$Bias(t_4) \approx \frac{\bar{Y}\theta C_x^2}{8}\left[3 - 4H_{yx}\right] \quad (2.12)$$

$$Bias(t_5) \approx \frac{\bar{Y}\theta C_x^2}{8}\left[4H_{yx} - 1\right] \quad (2.13)$$

Mohanty [10] suggested the following the Regression-Cum-Ratio Estimator for estimating the finite population mean

$$t_6 = \left[\bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\frac{\bar{Z}}{\bar{z}}\right] \quad (2.14)$$

The mean square error and bias expressions for t_6 are given by

$$MSE\left(t_6\right) = \theta\bar{Y}^2\left[\begin{array}{c} C_y^2\left(1 - \rho_{xy}^2\right) + C_z^2 \\ -2\rho_{yz}C_yC_z + 2\rho_{xy}\rho_{xz}C_yC_z \end{array}\right] \quad (2.15)$$

$$Bias\left(t_6\right) = \theta\bar{Y}C_z^2 - \theta\bar{Y}\rho_{yz}C_yC_z + b_{yx}\bar{X}\rho_{xz}C_xC_z \quad (2.16)$$

respectively.

The modification of Mohanty [10] have given by Samiuddin and Hanif [15] as

$$t_7 = \left[\bar{y} + k_1\left(\bar{X} - \bar{x}\right)\frac{\bar{Z}}{\bar{z}}\right] \quad (2.17)$$

The mean square error and bias expression of t_7 is given by

$$MSE\left(t_7\right) \approx \theta\bar{Y}^2\left[\begin{array}{c} C_y^2\left(1 - \rho_{xy}^2\right) + C_z^2\left(1 - \rho_{xz}^2\right) \\ -2C_yC_x\left(\rho_{yz} - \rho_{xy}\rho_{xz}\right) \end{array}\right] \quad (2.18)$$

$$Bias\left(t_7\right) = \theta\bar{Y}C_z^2 - \theta\bar{Y}\rho_{yz}C_yC_z + b_{yx}\bar{X}\rho_{xz}C_xC_z$$

Samiuddin and Hanif [15] were using the idea of Chand [6] and suggested the estimator given by

$$t_8 = \bar{y}\frac{\bar{X}}{\bar{x}}\frac{\bar{Z}}{\bar{z}} \quad (2.19)$$

The mean square error and bias expression of t_8 is given by

$$MSE\left(t_8\right) \approx \theta\bar{Y}^2\left(\begin{array}{c} C_x^2 + C_y^2 + C_z^2 - 2C_xC_y\rho_{xy} \\ -2C_yC_z\rho_{yz} + 2C_xC_z\rho_{xz} \end{array}\right) \quad (2.20)$$

$$Bias\left(t_8\right) \approx \theta\bar{Y}\left(C_x^2 + C_z^2 + C_xC_z\rho_{xz} - C_xC_y\rho_{xy} - C_yC_z\rho_{yz}\right) \quad (2.21)$$

respectively.

Hanif *et al*. [9] proposed an estimator which was the modification of the Singh and Espejo [17] estimator, given as

$$t_9 = \left[\left\{\bar{Y} + k_1\left(\bar{X} - \bar{x}\right)\right\}\left\{k_2\frac{\bar{Z}}{\bar{z}} + \left(1 - k_2\right)\frac{\bar{z}}{\bar{Z}}\right\}\right] \quad (2.22)$$

The mean square error and bias expression of t_9 is given by

$$MSE\left(t_9\right) = \theta\bar{Y}^2 C_y^2\left(1 - \rho_{y.xz}^2\right) \quad (2.23)$$

$$Bias\left(t_9\right) \approx \theta\bar{Y}C_z^2\left(\frac{1}{4} - \frac{\bar{Z}^2}{\bar{Y}^2}\beta_{yz.x}^2\right) \quad (2.24)$$

where

$$\rho_{y.xz}^2 = \frac{\rho_{xy}^2 + \rho_{yz}^2 - 2\rho_{xy}\rho_{xz}}{1 - \rho_{xz}^2}, \quad \beta_{yz.x} = \frac{S_y\left(\rho_{yz} - \rho_{xy}\rho_{xz}\right)}{S_x\left(1 - \rho_{xz}^2\right)}$$

PROPOSED ESTIMATOR

In this section, an estimator is developed by combining the concept of Bahl and Tuteja [2] exponential type estimator and classical regression estimator. The generalized regression-ratio estimator in the exponential form is given by

$$t_G = \left[\bar{y} + \alpha\left(\bar{Z} - \bar{z}\right)\right]\exp\left[\gamma\frac{\bar{X} - \bar{x}}{\bar{X} + (\beta - 1)\bar{x}}\right] \quad (3.1)$$

where, α, β are real positive constants and γ may take the values-1 and 1.

In order to obtained the bias and mean square error, rewriting (3.1) by using the notations (2.1), we get

$$t_G = \left[\bar{Y}\left(1 + \bar{e}_y\right) + \alpha\left(\bar{Z} - \bar{Z}\left(1 + \bar{e}_z\right)\right)\right] \exp\left[\gamma\frac{\bar{X} - \bar{X}\left(1 + \bar{e}_x\right)}{\bar{X} + (\beta - 1)\bar{X}\left(1 + \bar{e}_x\right)}\right] \quad (3.2)$$

After some simplification, (3.2) is given by

$$t_G = \left[\bar{Y} + \bar{Y}\bar{e}_y - \alpha\bar{Z}\bar{e}_z\right]\exp\left[\gamma\frac{-\bar{e}_x}{\beta}\left(1 + \left(\frac{\beta - 1}{\beta}\right)\bar{e}_x\right)^{-1}\right] \quad (3.3)$$

Expanding the exponential term up to first degree in (3.3), squaring and taking expectations, we may get mean square error as

$$MSE\left(t_G\right) \approx \theta\bar{Y}^2C_y^2 + \theta\alpha^2\bar{Z}^2C_z^2 + \theta\gamma\bar{Y}^2\frac{C_x^2}{\beta^2} - 2\theta\bar{Z}\alpha\bar{Y}\rho_{yz}C_yC_z + 2\theta\gamma\bar{Y}\alpha\bar{Z}\rho_{xz}\frac{C_xC_z}{\beta} - 2\theta\gamma\bar{Y}^2\rho_{xy}\frac{C_xC_y}{\beta} \quad (3.4)$$

In order to get optimum value of a and ß we partially differentiating equation (3.4) with respect to α and ß and equating to zero we get optimum value of α and ß

$$\alpha = \frac{\bar{Y}}{\bar{Z}}\left[H_{yz} - \gamma\frac{H_{xz}}{\beta}\right] \quad (3.5)$$

or

$$\beta = \frac{\gamma\left(1 - \rho_{xz}^2\right)}{H_{yx} - H_{xz}H_{yz}\frac{C_z^2}{C_x^2}} \quad (3.6)$$

Using (3.6) in equation (3.5), we get

$$\alpha = \frac{\bar{Y}}{\bar{Z}}\left[\frac{H_{yz} - H_{yx}H_{xz}}{\gamma\left(1 - \rho_{xz}^2\right)}\right] \quad (3.7)$$

Now putting the value of (3.6) and (3.7) in equation (3.4) and after some simplifications, we get the minimized mean square error as

$$MSE\left(t_G\right)_{min} \approx \frac{\theta\bar{Y}^2C_y^2}{\left(1 - \rho_{xz}^2\right)}\left[\begin{array}{l}1 - \gamma^2\rho_{xz}^2 - \rho_{yz}^2 \\ -\gamma^2\rho_{xy}^2 + 2\gamma^2\rho_{yz}\rho_{xy}\rho_{xz}\end{array}\right] \quad (3.8)$$

In order to derive the bias of (3.1) we again use (3.3) and simplify as

$$t_G \approx \left[\bar{Y} + \bar{Y}\bar{e}_y - \alpha\bar{Z}\bar{e}_z\right]\exp\left[\gamma\frac{-\bar{e}_x}{\beta} + \gamma\left(\frac{\beta - 1}{\beta^2}\right)\bar{e}_x^2\right] \quad (3.9)$$

Expanding the exponential function up to second degree in (3.9) and after some simplification, we get bias as:

$$Bias\left(t_G\right) \approx \alpha\gamma\theta\bar{Z}\rho_{xz}\frac{C_xC_z}{\beta} + \frac{\theta\gamma\bar{Y}C_x^2}{2\beta^2}\left[1 + 2(\beta - 1) - 2\beta H_{yx}\right] \quad (3.10)$$

Special cases of proposed estimators: For the γ = 1 and γ = -1, the following regression-cum-exponential ratio type and regression-cum-exponential product type estimator in generalized form may be obtained from proposed estimator

$$t_{GR} = \left[\bar{y} + \alpha\left(\bar{Z} - \bar{z}\right)\right]\exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + (\beta - 1)\bar{x}}\right]$$

and

$$t_{GP} = \left[\bar{y} + \alpha\left(\bar{Z} - \bar{z}\right)\right]\exp\left[\frac{\bar{x} - \bar{X}}{\bar{X} + (\beta - 1)\bar{x}}\right]$$

respectively.

For the α = b_{yx}, β = 2, γ = 1 and α = b_{yx}, β = 2, γ = -1, the regression-cum-exponential ratio type and regression-cum-exponential product type estimator may be written as

$$t_{GP} = \left[\bar{y} + b_{yx}\left(\bar{Z} - \bar{z}\right)\right]\exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right]$$

and

$$t_{GP} = \left[\bar{y} + b_{yx}\left(\bar{Z} - \bar{z}\right)\right]\exp\left[\frac{\bar{x} - \bar{X}}{\bar{X} + \bar{x}}\right]$$

respectively.

For the α = b_{yx}, β = 2, γ = 1 and α = b_{yx}, β = 2, γ = -1, the regression-cum-exponential ratio type and regression-cum-exponential product type estimator may be written as

$$t_{GP} = \left[\bar{y} + b_{yx}\left(\bar{Z} - \bar{z}\right)\right]\exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right]$$

and

$$t_{GP} = \left[\bar{y} + b_{yx}\left(\bar{Z} - \bar{z}\right)\right]\exp\left[\frac{\bar{x} - \bar{X}}{\bar{X} + \bar{x}}\right]$$

For the $\alpha = b_{yx}$, $\gamma = 0$, the proposed estimator may take the form of classical regression estimator.

For the $\alpha = 0$, $\beta = 2$, $\gamma = 1$, the proposed estimator may take the form of Bhal and Tuteja [1] exponential ratio type estimator.

For the $\alpha = 0$, $\beta = 2$, $\gamma = -1$, the proposed estimator may take the form of Bhal and Tuteja [1] exponential product type estimator.

EFFICIENCY COMPARISONS OF PROPOSED ESTIMATOR OVER OTHER ESTIMATORS

In this section, the theoretical conditions have been derived, when the deduced estimators performs better as compare to some existing estimators. The comparison of t_{GR} and t_{GP} is made with t_3, t_4, t_6, t_7, t_8 and t_3, t_5, t_9, respectively, using the mean square errors.

Comparison of MSE $(t_{GR})_{min}$ with classical regression estimator

$$MSE\left(t_{GR}\right)_{min} < MSE\left(t_3\right)$$

$$\frac{\theta\bar{Y}^2C_y^2}{\left(1-\rho_{xz}^2\right)}\left[1-\rho_{xz}^2-\rho_{yz}^2-\rho_{xy}^2+2\rho_{yz}\rho_{xy}\rho_{xz}\right] < \theta\bar{Y}^2C_y^2(1-\rho_{xy}^2)$$

or

$$\rho_{y.xz}^2 > \rho_{xy}^2 \qquad (4.1)$$

Comparison of MSE $(t_{GR})_{min}$ with Bahl and Tuteja's [2] Estimator

$$MSE\left(t_{GR}\right)_{min} < MSE\left(t_4\right)$$

$$\frac{\theta\bar{Y}^2C_y^2}{\left(1-\rho_{xz}^2\right)}\left[1-\rho_{xz}^2-\rho_{yz}^2-\rho_{xy}^2+2\rho_{yz}\rho_{xy}\rho_{xz}\right]$$

$$< \bar{Y}^2\theta\left[C_y^2 + \frac{C_x^2}{4}\left(1+4H_{yx}\right)\right]$$

or

$$\rho_{y.xz}^2 > \frac{-C_x^2}{4C_y^2}\left(1-4H_{yx}\right) \qquad (4.2)$$

Comparison of MSE $(t_{GR})_{min}$ with Mohanty [10] Estimator

$$MSE\left(t_{GR}\right)_{min} < MSE\left(t_6\right)$$

$$\frac{\theta\bar{Y}^2C_y^2}{\left(1-\rho_{xz}^2\right)}\left[1-\rho_{xz}^2-\rho_{yz}^2-\rho_{xy}^2+2\rho_{yz}\rho_{xy}\rho_{xz}\right]$$

< [illegible]

or

$$\rho_{y.xz}^2 > \rho_{xy}^2 - \frac{C_z^2}{C_y^2} + 2H_{zx} - 2H_{xy}H_{zx} \qquad (4.3)$$

Comparison of MSE $(t_{GR})_{min}$ with Samiuddin and Hanif [15] Estimator

$$MSE\left(t_{GR}\right)_{min} < MSE\left(t_7\right)$$

$$\frac{\theta\bar{Y}^2C_y^2}{\left(1-\rho_{xz}^2\right)}\left[1-\rho_{xz}^2-\rho_{yz}^2-\rho_{xy}^2+2\rho_{yz}\rho_{xy}\rho_{xz}\right]$$

$$< \theta\bar{Y}^2\left[C_y^2\left(1-\rho_{xy}^2\right)+C_z^2\left(1-\rho_{xz}^2\right)-2C_yC_z\left(\rho_{yz}-\rho_{xy}\rho_{xz}\right)\right]$$

or

$$\rho_{y.xz}^2 > \rho_{xy}^2 - \frac{C_z^2}{C_y^2}\left(1-\rho_{xy}^2\right) - 2\frac{C_x}{C_y}\left(\rho_{yz}-\rho_{xy}\rho_{xz}\right) \qquad (4.4)$$

Comparison of MSE $(t_{GR})_{min}$ with modification of Chand [6] Estimator, which was given Samiuddin and Hanif [15]

$$MSE\left(t_{GR}\right)_{min} < MSE\left(t_8\right)$$

$$\frac{\theta\bar{Y}^2C_y^2}{\left(1-\rho_{xz}^2\right)}\left[1-\rho_{xz}^2-\rho_{yz}^2-\rho_{xy}^2+2\rho_{yz}\rho_{xy}\rho_{xz}\right]$$

$$< \theta\bar{Y}^2\left(C_x^2+C_y^2+C_z^2-2C_xC_y\rho_{xy}-2C_yC_z\rho_{yz}+2C_xC_z\rho_{xz}\right)$$

or

$$\rho_{y.xz}^2 > 2H_{zx} + 2H_{xy} - \frac{C_z^2}{C_y^2}\left(1+2H_{xz}C_y\right) - \frac{C_x^2}{C_y^2} \qquad (4.5)$$

Comparison of t_{GP} with Classical Product Estimator

$$MSE\left(t_{GR}\right)_{min} < MSE\left(t_3\right)$$

$$\frac{\theta\bar{Y}^2C_y^2}{\left(1-\rho_{xz}^2\right)}\left[1-\rho_{xz}^2-\rho_{yz}^2-\rho_{xy}^2+2\rho_{yz}\rho_{xy}\rho_{xz}\right]$$

$$< \bar{Y}^2\theta\left[C_y^2 + C_z^2\left(1+2H_{yz}\right)\right]$$

or

$$\rho_{y.xz}^2 > -\frac{C_z^2}{C_y^2}\left(1+2H_{yz}\right) \qquad (4.6)$$

Comparison of t_{GP} with Bahl and Tuteja [2] Estimator

$$\text{MSE}(t_{GR})_{\min} < \text{MSE}(t_5)$$

$$\frac{\theta\bar{Y}^2C_y^2}{\left(1-\rho_{xz}^2\right)}\left[1-\rho_{xz}^2-\rho_{yz}^2-\rho_{xy}^2+2\rho_{yz}\rho_{xy}\rho_{xz}\right]$$

$$< \bar{Y}^2\theta\left[C_y^2+\frac{C_x^2}{4}\left(1+H_{yx}\right)\right]$$

or

$$\rho_{y.xz}^2 > -\frac{C_x^2}{4C_y^2}\left(1+4H_{yx}\right) \qquad (4.7)$$

Comparison of t_{GP} with Hanif *et al*. [9] Estimator

$$\text{MSE}(t_{GR})_{\min} < \text{MSE}(t_9)$$

$$\frac{\theta\bar{Y}^2C_y^2}{\left(1-\rho_{xz}^2\right)}\left[1-\rho_{xz}^2-\rho_{yz}^2-\rho_{xy}^2+2\rho_{yz}\rho_{xy}\rho_{xz}\right]$$

$$< \theta\bar{Y}^2C_y^2\left(1-\rho_{y.xz}^2\right)$$

or

$$\rho_{y.xz}^2 > 0 \qquad (4.8)$$

If the conditions (4.1)-(4.5) are fulfilled, the performance of t_{GP} is better than the classical regression estimator, Bahl and Tuteja [2] ratio, Mohanty [10], Samiuddin and Hanif [15] and modification of Chand [6] estimators, respectively. Similarly, when the conditions (4.6)-(4.8) are satisfied, the estimator, t_{GP}, is more efficient than the classical product, Bahl and Tuteja [2] product, Hanif *et al*. [9] estimators, respectively.

NUMERICAL EXAMPLE

To show the performance of the proposed estimator in comparison of other estimators in single phase sampling, three original data set used by others authors in literature has been considered. The descriptions of the population are given below.

Population 1: Anderson [1]

Y: Head length of second son
X: Head length of first son
Z: Head breadth of first son
$N = 25$, $n_1 = 15$, $\bar{Y} = 183.84$, $\bar{X} = 185.72$, $\bar{Z} = 151.12$,
$C_y = 0.0546$, $C_x = 0.0488$, $C_z = 0.0546$,
$\rho_{yx} = 0.6932$, $\rho_{yz} = 0.7108$, $\rho_{xz} = 0.7346$

Population II: Cochran [4]

Y: Number of ‘’placebo’’ children
X: Number of paralytic polio cases in the ‘’not inoculated’’ group.
Z: Number of paralytic polio cases in the placebo group

Table 1: Percentage relative efficiencies for the estimators

	Populations		
Estimators	1	2	3
$\bar{y}$	100	100	100
t_3	192.5025048	215.8441522	*
t_4	172.37096	208.8930029	*
t_6	94.41334467	73.87076248	*
t_7	170.5311765	132.0008015	*
t_8	72.29145849	43.21049749	*
t_{GR}	231.9046782	235.0896295	*
t_3	*	*	101.935116
t_5	*	*	95.97932128
t_9	*	*	93.87584582
t_{GP}	*	*	122.499378

$N = 34$, $n_1 = 15$, $\bar{Y} = 4.92$, $\bar{X} = 2.59$, $\bar{Z} = 2.91$,
$C_y = 1.01232$, $C_x = 1.23187$, $C_z = 1.053516$,
$\rho_{yx} = 0.7326$, $\rho_{yz} = 0.643$, $\rho_{xz} = 0.6837$

Population III: Gujrati [8]

Y: The number of wild cats drilled.
X: Price at the well head in the previous period.
Z: Domestic output.

$N = 30$, $n_1 = 12$, $\bar{Y} = 10.6374$, $\bar{X} = 4.44968$, $\bar{Z} = 7.5248$,
$C_y = 0.21783$, $C_x = 0.14732$, $C_z = 0.17986$,
$\rho_{yx} = -0.4285052$, $\rho_{yz} = 0.1377817$, $\rho_{xz} = -0.305424195$

The results of percent relative efficiencies have been given in Table 1. The percent relative efficiencies have been computed by using the formula

$$\text{PRE} = \frac{\text{Var}\left(\bar{Y}\right)}{\text{MSE}\left(t_*\right)}\times 100$$

The results from population 1-2 have shown that the generalized exponential regression-cum-exponential ratio type estimator (t_{GR}) is more efficient as compared to classical regression estimator, Mohanty [10] estimator, exponential ratio-type estimator, Samiuddin and Hanif [15] estimator. The results from population 3 have shown that the generalized regression-cum-exponential product type estimator (t_{GR}) is more efficient as compared to, classical regression type estimator, exponential product-type estimator and Hanif *et al*. [9] estimator.

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