

Estimating Volatility of Stock Index Returns by Using Symmetric Garch Models

Mohd. Aminul Islam

¹Department of Computational & Theoretical Sciences,
Faculty of Science, International Islamic University Malaysia,
Bandar Indera Mahkota, Kuantan 25200, Pahang, Malaysia

Abstract: This paper utilizes Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models to estimate volatility of financial asset returns of three Asian markets namely; Kuala Lumpur Composite Index (KLCI) of Malaysia, Jakarta Stock Exchange Composite Index (JKSE) of Indonesia and Straits Times Index (STI) of Singapore. Two symmetric GARCH models with imposing names such as the GARCH (1, 1) and the GARCH-in-Mean or GARCH-M (1, 1) are considered in this study. The study covers the period 02/01/2007 – 31/12/2012 comprising daily observations of 1477 for KLCI, 1461 for JKSE and 1493 for STI excluding the public holidays. We choose to apply GARCH models as they are especially suitable for high frequency financial market data such as stock returns which has a time-varying variance. Unlike the linear structural models, GARCH models are found useful in explaining a number of important features commonly observed in most financial time series such as leptokurtosis, volatility clustering and leverage or asymmetric effects. In this paper, we applied the symmetric GARCH models to examine their capability in explaining the volatility clustering and leptokurtic characteristic of the financial data. In addition, we also empirically tested the positive correlation hypothesis between the expected risk and the expected return usually predicted in financial application. Our results provide strong evidence that daily stock returns can be characterized by these two symmetric GARCH models. From the results of risk-return hypothesis test in GARCH-M model, we found evidence of positive correlation between the risk and return for all markets as expected. However, only for Indonesian market which is found to be more volatile than the other two markets, the estimated coefficient of risk premium appeared to be statistically significant indicating that increased risk leads to a rise in the returns. The risk-premium coefficients for other two markets are positive but statistically insignificant suggesting that increased risk does not necessarily produce higher return.

Key words: Volatility • GARCH models • Financial time series • KLCI

INTRODUCTION

Up and down movement in the daily prices of the securities can be considered as one of the consequences of the stochastic nature of the financial markets. In the face of usual up-down price movements, investors invest their funds in the financial markets particularly in the stocks or stock indices with the expectation of being compensated by risk-premium. The variation in the returns provided by the stocks due to changes in the daily price

is generally termed as volatility which is measured by the standard deviation or the variance. The usual up-down rally of the stock prices may not be bad but it turns out to be bad if the price swings are unusually very sharp or rapid over short time periods as it makes financial planning difficult. Higher fluctuations in the prices obviously increase the uncertainty about the future returns and hence increase the risk. If the market performance is unstable, investors cannot reliably predict the future which may result in further uncertainty about

Corresponding Author: Mohd. Aminul Islam, Department of Computational & Theoretical Sciences, Faculty of Science, International Islamic University Malaysia, Bandar Indera Mahkota, Kuantan 25200, Pahang, Malaysia.

future price movements. Uncertainty in prices in the future may prevent the investors to take risk and fund investment.

In such a volatile market it is difficult for companies to raise funds in the capital markets. Uncertainty causes loss of investor confidence which is important in stock trading particularly in making investment and leverage decision. This uncertainty can aggravate volatility further. Excess volatility may even lead to crashes or crisis in financial markets. Thus more accurate estimation of volatility is pivotal to risk management. Knowledge of volatility is of crucial importance in many areas such as the Value-at-Risk (VaR) models for market risk, valuation of derivatives products such as options [1] and so on. Investors in the stock market are interested in the volatility of stock prices, for high volatility in daily stock price changes could mean to them huge losses or gains and hence greater uncertainty [2]. Similarly, high variation in the volatility in the exchange rates means huge losses or profits for exporters, importers and traders in the foreign exchange markets. Therefore, it is pertinent to select the right volatility model that can estimate and forecast volatility of financial time series more accurately.

Past few years, there has been observed a huge up and down shifts in the stock prices in many markets including developed and emerging markets worldwide. Investors as well as the financial analysts are mostly concerned about the high volatility or sharp up-down movement of asset prices and its resulting effects of uncertainty of returns on their investment assets. The fluctuations in the asset prices are widely believed to be the cause of changes in the economic factors such as interest rates, inflation, variability in speculative market prices, unexpected events (e.g., political unrest, natural calamities) and the instability of market performance. However, the biggest driver of the volatility in the financial market is a drop in the market performance. Volatility typically tends to decline as the stock market rises which in turn reduces the risk. In contrast, volatility tends to increase when the stock market falls and hence increases the risk. The stochastic nature of the financial market thus requires development of quantitative tools to explain and analyze the behavior of stock market returns and hence capable of dealing with such uncertainty in future price movements. In recent, there has been a remarkable progress in developing sophisticated econometrics models which are able to explain and

capture various characteristics of financial time series volatilities and hence to help managing the risks associated with them.

It is found that the financial time series (particularly stock/index prices) often exhibit the phenomenon of volatility clustering [3] that is, the series exhibit sometimes high volatilities and sometimes low volatilities for an extended time periods. However, for a short period of time, there is a strong chance that a day of high volatility will be followed by another day of high volatility. In other words, if a high volatility is observed yesterday, it is more likely that a high volatility will also be observed today. This means that today's volatility is positively correlated with yesterday's volatility and thus we can estimate volatility conditionally on the past volatility.

Volatility can either be historical volatility which is a measure based on past data, or implied volatility which is derived from the market price of a market traded derivative particularly an option. The historical volatility can be calculated in three ways namely; (1) simple volatility, (2) Exponentially Weighted Moving Average (EWMA) and (3) GARCH. In this study, we will apply the most commonly used stochastic volatility model GARCH (1, 1) as it is theoretically superior to and more appealing than the other two approaches. Furthermore, GARCH is also said to be a preferred method for finance professionals as it provides a more real life estimate while forecasting parameters such as volatility, prices and returns.

The GARCH is the extension of the Autoregressive Conditional Heteroscedasticity (ARCH) model¹. These two models are said to be volatility clustering models and are importantly applied to measuring and forecasting the time-varying volatility of high frequency financial data like daily stock or stock index returns [3]. Since the introduction of these two models into the literature, they become very popular and most common predominantly in financial market research as they enable the financial analysts to estimate the variance of a series at a particular point in time [4] more accurately. A large number of empirical studies utilized ARCH and its variations in many markets and their applicability in capturing the dynamic characteristics of stock index returns has been demonstrated successful. Some of the studies who, along with other asymmetric GARCH models, have applied the standard/ basic GARCH models across different countries are Floros [5-8]; Elsheikh and Zakaria [9]; Shamiri and Zaidi [10]; Islam [11] to name a few. A lot of empirical

¹ Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model which is the extension of the Original ARCH model. ARCH model was originally developed by R. Engle (1982), while GARCH was developed by T. Bollerslev (1986).

studies also used the different extensions of the basic GARCH such as the Exponential GARCH (EGARCH) developed by Nelson [12], the Threshold GARCH (TGARCH or ZGARCH) introduced by Zakoian [13], the GJR-GARCH by Glosten, Jaganathan and Runkle [14], the Power GARCH (PGARCH) proposed by Ding, Granger and Engle [15] and so on. These are called asymmetric GARCH as they are capable of modeling asymmetric response and leverage effect.

Floros [5] applied GARCH-type models² to model volatility and to explain financial market risk of two middle-east stock indices: Egypt (CMA General Index) and Israeli (TASE-100 index) over the period 1997 – 2007. Using daily data of these two markets, the study concludes that the GARCH models are capable of characterizing the dynamics of daily stock returns including volatility clustering. By utilizing GARCH-in-Mean model, the study found positive but insignificant relationship between increased expected risk and increased expected return leading to the conclusion that higher expected risk does not necessarily produce higher expected return in these two markets.

In modeling and forecasting of the Malaysian stock market proxied by KLCI over the period 1/1/1998 to 31/12/2008, Shamiri and Zaidi [10] used standard GARCH, EGARCH and non-linear asymmetric GARCH (NAGARCH). The study compared the performance of these three models with six different error distributions³. The study found existence of standard leverage effects in the KLCI index returns. The study concludes that successful volatility model much more depends on the choice of error distribution than the choice of GARCH models. In other words, performance of the GARCH models depends much on the error distribution. Elsheikh and Zakaria [9] used GARCH-type models that include both symmetric and asymmetric models to estimate volatility in the daily returns of the Khartoum (Sudan) Stock Exchange over the period from January 2006 to November 2010. They found evidence that the GARCH models are fit to characterize the daily returns for the case

of Sudan. With respect to risk-return relationship, this study found risk premium coefficient positive and statistically significant implying that increased risk leads to higher return as predicted in financial theory. Islam [11] applied the GARCH-type models including symmetric and asymmetric models to test their applicability in analyzing the stylized facts (e.g., volatility clustering, leptokurtosis and leverage effects) commonly observed in high frequency financial time series such as stock/stock indices for the cases of 4-asian stock indices⁴. The study found strong evidence that the models can characterize the dynamics of daily stock returns in all four markets in the sample. With respect to the risk-return relationship, the study found positive correlation in all cases which is in consistent with the financial theory.

Apart from these, there are many studies also who have investigated the relationship between conditional variance and risk premium using GARCH model. BAC *et al.*, [16] examined the relationship between volatility and risk premium for the case of the New York Stock Exchange (NYSE) index US over the period 1952 – 1999. The study found evidence of positive correlation between volatility and risk premium. Appiah and Menyah [17] investigated the relationship between volatility and risk premium of 12-African⁵ stock markets for the period 1990 – 1994. They found evidence of time-varying risk premium in five most volatile markets⁶. Similarly, Algidede and Panagiotidis [18] from their study on the largest 7-African stock markets⁷ found evidence of positive association between high volatility and high risk premium.

The present paper focuses on 3-Asian markets comprising of Malaysia, Singapore and Indonesia. Out of these three, Malaysia and Indonesia are considered as the emerging markets while Singapore is categorized as the developed market⁸. We aim to examine whether or not the symmetric GARCH models are capable of explaining the dynamics of stock returns behavior in these three countries. In addition, this study also aims to test the positive correlation hypothesis between expected risk and the expected return that is often predicted in the financial

² Includes 7-types of GARCH (Basic GARCH, GARCH-in-Mean, EGARCH, PGARCH, TGARCH, Component- GARCH, and Asymmetric Component GARCH)

³ Includes: Normal distribution, Skew-normal distribution, student-t distribution, generalized error distribution (GED), skewed-t distribution and NIG.

⁴ Includes Kuala Lumpur Composite Index of Malaysia, Straits Times Index of Singapore, Nikkei 225 of Japan and Hang Seng Index of Hong Kong.

⁵ Botswana, Egypt, Ghana, Ivory Coast, Kenya, Mauritius, Morocco, Nigeria, South Africa, Swaziland and Zimbabwe

⁶ Ghana, Ivory Coast, Mauritius, Nigeria and Swaziland

⁷ Egypt, Morocco, Nigeria, Kenya, South Africa, Tunisia and Zimbabwe

⁸ These ranking are based on the FTSE Group's list as of March 2012.

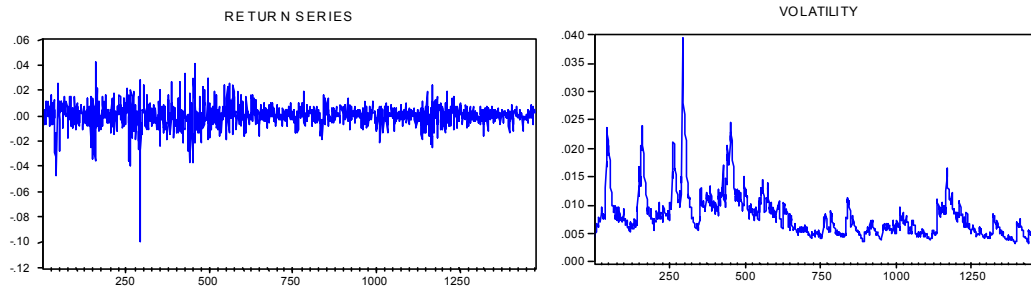


Fig. 1: KLCI: Change in the log of daily index prices and the daily volatility from January 2007 to December 2012

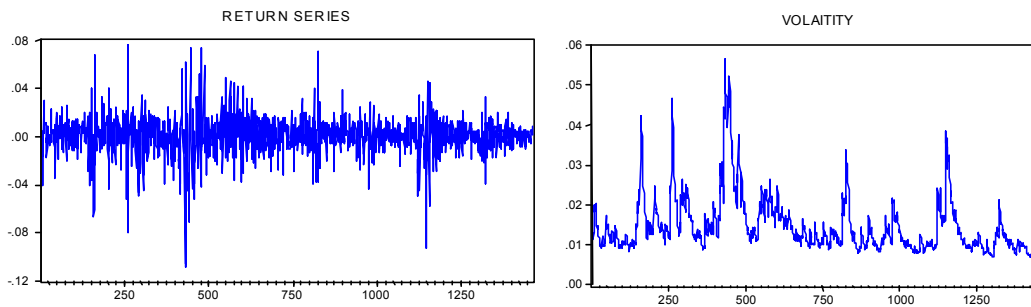


Fig. 2: JKSE: Change in the log of daily index prices and the daily volatility from January 2007 to December 2012

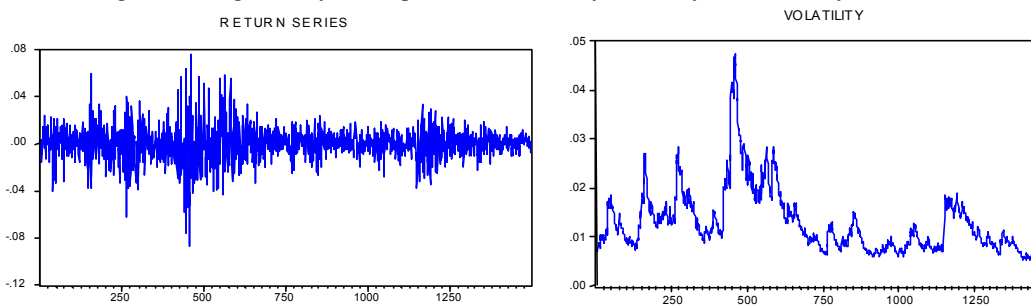


Fig. 3: STI: Change in the log of daily index prices and the daily volatility from January 2007 to December 2012

theory. This may help the investors in making their investment decision. This paper is structured as follows: following the introduction, section 2 briefly discusses the basic statistics about the data. Section 3 outlines methodological framework. Section 4 presents the results. Conclusions are provided in section 5 followed by a list of references used in this study.

Data and Basic Statistics: The daily stock price index data used in this paper is the daily closing prices of stock index of each market collected from online database over the period from January 2007 to December 2012 with daily observations of 1477 for KLCI, 1461 for JKSE and 1493 for STI. The daily index returns are expressed in the continuously compounded returns calculated as $r_t = \log(p_t) - \log(p_{t-1})$ where p_t and p_{t-1} are the index prices on day t and $t-1$ respectively. Before proceeding further for formal statistical tests, we plot the changes in daily index returns

and their volatilities over the study period as exhibited in Figure 1 through Figure 3 in order to get an initial clue about the likely nature of the return series.

It can be seen that the graphs presented above exhibit considerable swings or volatility in the return series over the sample period. The bulges in the return plots are the graphical evidence suggesting that the volatility is time varying. Putting differently, the bulges in the return plots indicate the presence of volatility clustering effect in the series whereby the series exhibit some periods of high volatility and some periods of relatively low volatility. Presence of volatility clustering also implies that there is positive autocorrelation in the squared returns.

The results from descriptive statistics reported in Table 1 below show that during the sample period, Indonesian market observed the highest mean daily return of 0.0585% followed by Malaysian market 0.028% and the

Table 1: Summary statistics

Statistics	Indices		
	KLCI	JKSE	STI
Mean (%)	0.028	0.0585	0.00279
Maximum (%)	4.2587	7.6234	7.5305
Minimum (%)	-9.9785	-10.9539	-8.696
Std. Dev. (%)	0.8861	1.6055	1.416
Skewness	-1.284484	-0.649942	-0.153093
Kurtosis	16.57428	9.627268	7.363519
Jarque-Bera (J-B)	11737.93	2774.63	1189.50
Probability of J-B.	0.00000	0.00000	0.00000
No. of Observations	1476	1460	1492

Table 2: ADF unit root test for the stock return series based on SIC at level.

	Return series at level based on SIC, (max lag=1)		
	KLCI	JKSE	STI
ADF-test statistic	-33.61474 [0]*	-34.54731 [0]*	-37.90829 [0]*
Critical values:			
1% level	-3.434576	-3.434624	-3.434528
5% level	-2.863293	-2.863315	-2.863273
10% level	-2.567752	-2.567763	-2.567741

Singapore market 0.00279%. The corresponding volatilities measured by standard deviation are 1.61% for Indonesian market, 0.8861% for Malaysian market and 1.42% for Singapore market. This implies that Indonesian market is more volatile followed by Singaporean market. The Malaysian market is less volatile. The higher volatility offers the possibility of higher rate of returns, but also poses more risk. Malaysian market (KLCI) seems to have the lowest volatility with lowest rate of return compared to the higher rate of returns and higher volatility for Indonesian and Singapore markets. The return series for all markets show negative skewness suggesting that the distribution have long left tail. The excess values (that is >3) for kurtosis indicate fat tails characteristics of the asset returns distribution. The Jarque-Bera (JB) test of normality clearly rejects the null hypothesis of normality in all cases. The tests suggest that the distributions of the return series are non-normal.

Data Stationarity Test (Unit Root Test): In order to check whether the financial time series (returns) are stationary or not, we have applied the standard Augmented Dickey-Fuller (ADF) test (Dickey and Fuller, 1979)⁹. The ADF test statistic rejected the null hypothesis of the existence of

unit root in the return series at level as the absolute values of ADF statistic exceed the McKinnon¹⁰ critical (absolute) values at 1% significance level for all returns. This ensures that we can use the time series stochastic models to examine the dynamic behavior of volatility of the returns over time. The results are presented in Table 2 as below:

Testing for ARCH-Effect: The linear structural model assumes that the variance of the errors is constant over time. But this assumption is not applicable for many financial data particularly the stock prices or stock indices in which the errors exhibit time-varying heteroskedasticity. Before proceeding to applying GARCH models, it is necessary to ascertain the existence of ARCH effects in the residuals. To test for ARCH effects in the conditional variance of u_t ($\sigma_t^2 = \text{Var}(u_t | \Omega_{t-1})$ ¹¹) we followed two steps: First we consider the AR (1) model for the returns series of each individual index as:

$$r_t = \beta_0 + \beta_1 r_{t-1} + u_t \quad (1)$$

And run the linear regression on it to obtain the residuals u_t . Secondly, we run a regression of squared OLS residuals (u_t^2) obtained from equation (1) on q lags¹² of squared residuals to test for ARCH of order q . The ARCH (q) specification for σ_t^2 is denoted as-

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \dots + \alpha_q u_{t-q}^2 \quad (2)$$

The null hypothesis of 'no ARCH effect'

$$H_0 : \alpha_1 = \alpha_2 = \dots = \alpha_q = 0$$

Is tested against the alternative hypothesis that,

$$H_1 : \alpha_1 \neq 0, \quad \alpha_2 \neq 0, \dots, \alpha_q \neq 0$$

If the value of the LM version of test statistic is greater than the critical value from the $\chi^2_{(q)}$ distribution, or the coefficient of the lagged term is statistically significant, then the null hypothesis is rejected that there is no ARCH effect in equation (1). The same conclusion

⁹ It is an augmented version of the Dickey-Fuller test. The ADF test is a commonly used unit root test. Reader can consult any standard econometrics text for technical details of the method. See for example, *Basic Econometrics*, (Gujarati, 2003; p.817).

¹⁰ MacKinnon (1996)

¹¹ Ω_{t-1} is the publicly available information at time $t-1$.

¹² q Represents the number of autoregressive terms in the model. We use $q=3$ in this model.

Table 3: ARCH-LM test for residuals of returns series

	KLCI	JKSE	STI
ARCH-LM test statistic ($n \cdot R^2$)	68.69987	208.5051	328.7336
Prob. Chi-square {3}	[0.0000]*	[0.0000]*	[0.0000]*
F-statistic	23.95502	80.88619	140.2464
Prob. (F-statistic)	[0.0000]*	[0.0000]*	[0.0000]*

Notes: *Significant at the 1% level. Figures in {.} refer to order of lag and in [] refer to p - values. The LM version of the test statistic is defined as $n \cdot R^2$ (where n is the number of observations and R^2 is the coefficient of correlation).

can be achieved if the F -version of the test is considered. We carried out the test for a lag order of $q=3$. The test results are presented in Table 3.

Methodology

Volatility Modeling Techniques: The GARCH (p, q) Models: The ARCH model introduced by Robert Engle (1982) is one of the particular non-linear models that have proved very useful in the application to many economic time series especially to financial time series analysis. In the ARCH, the conditional variance of the error term u_t is modeled as being normally distributed with mean zero and variance σ_t^2 , where the σ_t^2 is expressed as a function of past squared error values u_t as stated in equation (2). In estimating an ARCH model, it is required that the unknown coefficients ($\alpha_0, \alpha_1, \alpha_2, \dots, \alpha_q$) are non-negative since the variance cannot be negative. If these coefficients are positive and the recent squared residuals are large, the ARCH model predicts that the current squared error will be large in magnitude in the sense that its variance σ_t^2 is large [10]. An extension of ARCH model is the generalized ARCH or GARCH model developed by Bollerslev [5]¹³. In GARCH model, the variance σ_t^2 is allowed to be dependent upon its own past values as well as lags of the squared error terms. The general form of a GARCH (p, q) model is defined as-

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \dots + \alpha_p u_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \dots + \beta_q \sigma_{t-q}^2 \quad (3)$$

where p is the order of the moving average ARCH terms and q is the order of the autoregressive GARCH terms. The simplest and most commonly used GARCH model is the GARCH (1, 1). This basic GARCH model is reasonably a good model for analyzing financial time series as well as to estimating and forecasting the time-varying volatility of returns of financial assets, especially the high frequency financial assets such as daily stock index returns.

The GARCH (1, 1) Model: In financial markets, volatility is known as a measure of uncertainty about the return provided by the stocks or stock indices. The volatility of many economic time series, especially financial time series changes over time. In some periods the daily stock returns exhibit high volatility while in other periods they exhibit low volatility, a commonly observed phenomenon in financial time series which is referred to as volatility clustering. That is volatility comes in cluster. It is assumed that a day of high volatility most likely to be followed by another day of high volatility within each state or over a short period of time. As such, linear models which assume homoscedasticity (constant variance) are inappropriate to explain such unique behavior of financial time series data. It is preferable to use models that examine behavior of financial time series allowing the variance to depend upon its history. GARCH (1, 1) model is capable of capturing the volatility clustering effects in the financial time series data. The GARCH models are especially suitable for financial market data¹⁴ as the GARCH processes are 'fat-tailed' compared to the normal distribution. The GARCH (1, 1) model is defined as:

$$\sigma_t^2 = \gamma V_L + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (4)$$

where, V_L is the long-run average variance rate, γ is the weight assigned to the V_L , α is the weight assigned to u_{t-1}^2 and β is the weight assigned to σ_{t-1}^2 . Weights must be equal to unity as, $\gamma + \alpha + \beta = 1$. Equation (4) can be written by setting $\omega = \gamma V_L$ as,

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2 \quad \text{where } \omega > 0, \alpha, \beta \geq 0, \alpha + \beta < 1 \quad (5)$$

A stable GARCH (1, 1) process requires $\alpha + \beta < 1$. Once the parameters of the GARCH model are estimated, the long-term variance, V_L and γ can be calculated as ω/γ and $1 - \alpha - \beta$ respectively. The GARCH (1, 1) model in equation (5) estimates the current volatility of assets returns based on a linear combination of the last period's squared returns and the last period's volatility. Since the GARCH model is no longer of the usual linear form, the parameters in GARCH (1, 1) model cannot be estimated by the usual OLS method. As such to estimate GARCH parameters, alternative technique is used. The most common method to estimate the GARCH parameter is to take the log likelihood which is the logarithm of the

¹³ GARCH model improves the original specification of ARCH model by adding lagged conditional variance which acts as a *smoothing term*.

¹⁴ It is usually observed that at high sampling frequencies such as daily stock returns data are leptokurtic, whereas Low frequency data are normally distributed.

Maximum Likelihood (ML)¹⁵ method. ML employs trials and errors to determine the optimal values for the parameters that maximize the likelihood of the data occurring.

The GARCH-in-Mean (GARCH-M) Model: The GARCH-in-Mean (GARCH-M) model due to Engle, Lilien and Robins [14] was proposed for modeling risk-return tradeoffs. In financial investment theory, it is predicted that the expected return on an asset is proportional to the expected risk of the asset. In other words, high risk is often expected to lead to high returns as a compensation for taking risk. Engle *et al.* [19] Proposed to extend the basic GARCH model so that the conditional volatility can generate a risk premium which is part of the expected returns. Unlike the basic GARCH (1, 1) model which is subject to the assumption that the conditional mean is time invariant to the risk premium, in GARCH-M model the risk-premium is time-varying. GARCH-M model allows the conditional mean to depend directly on the conditional variance/standard deviation which enters the conditional mean equation as a measure of expected risk. The GARCH-M model extends the conditional mean equation¹⁶ as-

$$r_t = \mu + \lambda \sigma_t + u_t, \text{ [conditional mean equation]} \quad (6)$$

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \beta \sigma_{t-1}^2 \quad (7)$$

The parameter in the conditional mean return λ is the risk premium parameter. The time-varying risk premium is estimated by the significance of the ' λ ' coefficient of ' σ_t ' in the conditional mean equation. If the coefficient of ' λ ' is positive and significant, then the increased expected return is said to be caused by the increased expected risk or conditional variance/standard deviation. GARCH (1, 1) and GARCH-M are considered to be symmetric models which imply that the positive and negative shocks of equal size elicit an equal response from the market.

RESULT AND DISCUSSION

The results of the ML estimates of the GARCH parameters are presented in Table 4. In the GARCH estimation, maximum likelihood method is used assuming student's *t*-distribution for the conditional distribution of

the errors, u_t . The results show that the estimate of the standard GARCH parameters α and β are positive and statistically highly significant for all specifications.

The values of β coefficient are found to be very high ranging between 83% - 90% which implies persistent volatility clustering. The statistical significance of α and β indicates that the news on volatility from the past periods have impact on the current volatility. It can be seen from the results that the sum of the two estimated coefficients (α and β) are above 0.98 but less than one ($\alpha + \beta < 1$) signifying that the GARCH process is mean reverting. This also implies the long periods of volatility clustering as seen in Figure 1 through 3.

The ML estimates of the coefficient (λ) of the conditional standard deviation (σ_t) in the mean equation are found positive for all markets as expected. The positive sign is in consistent with one of the important class of asset pricing models that predicts a positive interaction between conditional expectations of excess returns and their conditional variances¹⁷. The ML estimates of λ for Malaysian and Singaporean markets are appeared with correct sign but are not statistically significant suggesting the lack of evidence of GARCH-in-Mean effect in these two markets. In other words, for these two markets, it seems increased risk does not necessarily lead to a rise in expected returns. In contrast, the ML estimate of λ for Indonesian market is found to be statistically significant in all specifications (variance, log variance and standard deviation) indicating the evidence of a GARCH-in-Mean effect in Indonesian market. In other words, based on the estimate, it seems in Indonesian market, higher variance/volatility produces higher expected return.

Performance Comparison: In order to see the performance of the two GARCH models in removing the autocorrelations, we have performed a Ljung-Box test for the first 15 lags at 99% confidence interval. The results are presented in Table 5. According to this test, both models have removed more than 96% of the autocorrelation in the series. However, in the case of Malaysia, GARCH (1, 1) model did a very slightly better job than the GARCH-in-Mean whereas in the cases of Indonesia and Singapore markets, GARCH-in-Mean seems to show a slightly better performance than GARCH (1, 1) model. Nevertheless, from the test we conclude that both models have done good job in removing the autocorrelation.

¹⁵ For a technical detail of the maximum likelihood method, see Chris Brooks (2008, p. 445-8), Introductory Econometrics for Finance, and also, Eviews 7(econometric software) User's Guide II, Chapter 7, p.198.

¹⁶ The other two variants of the GARCH-M specification in equation (6) are:

$rt = \mu + \lambda \sigma_t^2 + u_t$, and; $r_t = \mu + \log(\sigma_t^2) + u_t$

¹⁸ See for example, Merton, (1980) and also Glosten, L. *et al.* (1993)

Table 4: Estimation results of GARCH (1, 1) and GARCH-M (1, 1) Models

	GARCH (1, 1)		
	KLCI	JKSE	STI
ω	0.00000120*	0.00000586*	0.00000112**
α	0.137538*	0.141572*	0.094392*
β	0.855505*	0.841951*	0.902370*
$\alpha + \beta$	0.993043	0.983523	0.996762
ARCH-LM test statistic ($n \cdot R^2$)	0.060367	0.418093	2.183543
Prob. CHSQ[2]	0.9703	0.8114	0.3356
Long-term variance rate, V_L	0.000172488	0.000355647	0.000345892
Long-term volatility:			
Per-day	1.313%	1.886%	1.860%
Per year	20.85%	29.94%	29.53%
GARCH-M (1, 1)			
ω	0.00000118*	0.00000648*	0.00000113**
α	0.137211*	0.152528*	0.094705*
β	0.856012*	0.829707*	0.901977*
$\alpha + \beta$	0.993223	0.982235	0.996682
σ	0.051169[0.3852]	0.16327[0.029]**	0.04519[0.5026]
σ^2	2.624892[0.4992]	3.7413[0.068]***	1.24133[0.6043]
$\text{Log}(\sigma^2)$	0.000252[0.3082]	0.00121[0.021]**	0.00025[0.4914]
ARCH-LM test statistic ($n \cdot R^2$)	0.068921	0.347181	2.073109
Prob. CHSQ[2]	0.9661	0.8406	0.3547
Long-term variance rate, V_L	0.000174118	0.000364762	0.000340566
Long-term volatility:			
Per-day	1.320%	1.910%	1.845%
Per year	20.95%	30.32%	29.30%

Notes: *, ** and *** refer to significance levels at 1%, 5% and 10% respectively. Figures in [.] refer to p -values. The LM version of the test statistic is defined as $n \cdot R^2$ (where n is the number of observations and R^2 is the coefficient of correlation)

Table 5: Autocorrelations before and after the implementation of GARCH (1, 1) and GARCH-M (1, 1) models: Ljung-Box (LB) test¹⁸ (at a lag of 15)

	KLCI	% ↓	JKSE	% ↓	STI	% ↓
LB statistic _{critical value}	30.58					
LB statistic _(autocorrelation)	133.59		677.40		1616.70	
LB statistic _{GARCH(1, 1)}	4.7633	96.43	5.5795	99.18	21.8770	98.647
LB statistic _{GARCH-M(1, 1)}	4.8395	96.38	5.5008	99.19	21.7630	98.654

CONCLUSION

In this paper, we have utilized two of the GARCH family models with imposing names GARCH (1, 1) and GARCH-in-Mean or GARCH-M (1, 1). The main objectives of this paper are to estimate volatility of financial time series and to empirically test the existence of risk-return tradeoff in the financial application for the cases of Malaysia, Indonesia and Singapore Markets. The key results are as follows: both models are found sufficiently capable of capturing the dynamics of the financial time series particularly with respect to volatility clustering and the leptokurtic characteristic of the distribution of the daily return series. Secondly, we found the evidence of the existence of positive interactions between the expected risk and

the expected return as predicted in the investment theory that increased risk is to be compensated by increased return. In this regard, we found Indonesian market as more volatile or riskier for investors compared to Malaysian and Singaporean markets but also promising of being compensated by higher return as evident from the empirical test. The two models that we have applied in this study are called symmetric models and they are not capable of capturing many other characteristics such as leverage and asymmetry of the financial time series. Some of the important variations of GARCH models that have been developed since Engle [20] and Bollerslev [5] such as TGARCH, PGARCH and EGARCH can be considered well worth in capturing the extent of the effect of positive and negative shocks on conditional future volatility.

REFERENCES

1. Hull, J.C., 2012. Options, Futures and Other Derivatives. 8th Ed., Pearson Education Limited, pp: 498-500.
2. Gujarati, D.N., 2003. Basic Econometrics, 4th Ed. McGraw-Hill Publishing, pp: 813.
3. Stock, J.H. and M.M. Watson, 2012. Introduction to Econometrics. 3rd Ed., Pearson Publishing: UK.
4. Enders, W., 2004. Applied Econometric Time Series. 2nd Ed., Wiley Series in Probability and Statistics.
5. Floros, C., 2008. Modeling Volatility using GARCH Models. Evidence from Egypt and Israel. Middle Eastern Finance and Economics, 2: 30-41.
6. Abou-Deif, M.H., M.A. Rashed, M.A.A. Sallam, E.A.H. Mostafa and W.A. Ramadan, 2013. Characterization of Twenty Wheat Varieties by ISSR Markers, Middle-East Journal of Scientific Research, 15(2): 168-175.
7. Kabiru Jinjiri Ringim, 2013. Understanding of Account Holder in Conventional Bank toward Islamic Banking Products, Middle-East Journal of Scientific Research, 15(2): 176-183.
8. Muhammad Azam, Sallahuddin Hassan and Khairuzzaman, 2013. Corruption, Workers Remittances, Fdi and Economic Growth in Five South and South East Asian Countries. A Panel Data Approach Middle-East Journal of Scientific Research, 15(2): 184-190.
9. Elsheikh, A. and S. Zakaria, 2011. Modeling Stock Market Volatility Using GARCH Models Evidence from Sudan. International Journal of Business and Social Science, 23(2): 114-128.
10. Shamiri, A. and Z. Isa, 2009. Modeling and Forecasting Volatility of the Malaysian Stock Markets. Journal of Mathematics and Statistics, 5(3): 234-240.
11. Islam, M.A., 2013. Modeling Univariate Volatility of Stock Returns using Stochastic GARCH Models. Evidence from 4-Asian Markets. International Research Journal of Finance and Economics (Forthcoming).
12. Nelson, D.B., 1991. Conditional Heteroscedasticity in asset returns, a new approach. Econometrica, 59: 347-370.
13. Zakoian, J.M., 1994. Threshold Heteroskedastic Models. Journal of Economic Dynamics and Control, 18: 931-944.
14. Glosten, L., R. Jaganathan and D. Runkle, 1993. On the relationship between the expected value and the volatility of the normal excess returns on stocks. Journal of Finance, 48: 1779-1801.
15. Ding, X., C.W.J. Granger and R.F. Engle, 1993. A Long Memory Property of Stock Market Returns and a New Model. Journal of Empirical Finance, 1: 83-106.
16. Bae, J., C.J. Kim and C.R. Nelson, 2007. Why are stock returns and volatility negatively correlated? Journal of Applied Finance, 14: 41-58.
17. Appiah-Kusi, J. and K. Menyah, 2003. Return predictability in African stock markets. Review of Financial Economics, 12: 247-270.
18. Alagidede, P. and T. Panagiotidis, 2009. Modelling stock returns in Africa's emerging equity markets. International Review of Financial Analysis, 18: 1-11.
19. Engle, R.F., D.M. Lilien and R.P. Robins, 1987. Estimating Time Varying Risk Premia in the Term Structure. The ARCH-M Model. Econometrica, 55: 391-407.
20. Engle, R.F., 1982. Autoregressive Conditional Heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica, 50: 987-1007.