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Model of Quantum Measurement, which Leads to Reduction of Wave Function

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Abstract: In this paper, a mathematical model of reduction of the wave function is proposed. The model is based on ideas developed by Klimontovich and apply the method of stochastic quantization in the formulation of Haken. The equation takes into account the stochastic nature of the interaction of a quantum system and the measuring device during the measurement. From this equation, an equation of the Fokker-Planck was received. The solution of which show a reduction of wave function.

Key words: Quantum information science . Schrödinger equation . wave function reduction

INTRODUCTION

In connection with the development of quantum information [1-7], the problem of the quantum theory of measurement, set as one of the three great problems of theoretical physics of the 21st century still Ginzburg, became relevant again. In the field of applied research-quantum information technology, working principles underlying the theory of quantum measurements. Applied nature of quantum information and the promising prospects of building devices based on it have led to the fact that in recent times the quantum theory of measurement and interpretation of quantum mechanics again actively developing.

Since the days of the famous paper by Einstein, Podolsky and Rosen is becoming increasingly clear that to give an interpretation of quantum mechanics-the means to explain how to understand the reality of quantum mechanics. Explain quantum reality in some way means using a particular interpretation. This can be done in different ways. Simple ways (which include the Copenhagen interpretation) is pretty easy to understand and convenient for practical use, but they do not accurately convey the meaning of quantum reality. More sophisticated techniques [8] pass this sense precisely, are difficult to understand, but in practical work (for common quantum-mechanical problems) hinder rather than help. This explains why Everett's interpretation has not been used. In recent decades, it has become popular, particularly with the advent of quantum computing.

One of the first to criticize the Copenhagen interpretation of quantum mechanics based on the analysis of the process of quantum measurement, made H. Everett [8]. He noted that the time variation of the state of a quantum system, consisting of a system of elementary particles and the measuring device, is described by the Schrödinger equation. Since the system consists only of a large number of elementary particles. The solution of this equation is a single-valued function of time and, therefore, the measurement must also be unique. This is contrary to the Copenhagen interpretation of quantum mechanics [9]. According to which the wave function reduction in the measurement process is random. Over time were developed, the extended Everett concept [10].

Experimental data support the stochastic nature of reduction of quantum state in the measurement process. In this connection there is the problem of constructing a mathematical model of quantum measurement in which this contradiction would be eliminated. Everett proposed a mathematical model based on the search for such averaging algorithm solutions of the Schrödinger equation, which would lead to a reduction of the state vector. He acted in the same way as did the Poincare, trying average trajectory of classical dynamical systems in order to obtain an increase in entropy. Since the process of increasing entropy is an objective reality, it can not depend on the measure in the phase space of a dynamical system with the help of which is the average. Therefore, attempts to enter into the mechanics of the Poincare entropy failed. There is much more successful

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approach, based on the stochastic quantization of dynamical systems, proposed by Langevin, Einstein, Planck and Fokker [11, 12]. Note also the approach developed by Prigozhin and Klimontovich [13, 14].

BASIC EQUATIONS

During the measurement between the measuring device and quantum system an interaction is exist. Which in our model is reduced to correlation between fluctuations of ket vector of the quantum state $\delta|t, a>$ and δH (a). Here $\delta H(a)$ describes influence of the random force from measuring device on the quantum system in the measurement process. As a result of this interaction, the Schrödinger equation

$$i\hbar\frac{d|t>}{dt} = H|t> \qquad (1)$$

takes the form:

$$i\hbar\frac{d(|t>+\delta|t,a>)}{dt} = (H+\delta H(a))(|t>+\delta|t,a>) \qquad (2)$$

This equation is the original equation of our work. It describes how we see this, the process of quantum measurement. The fluctuations $\delta H(a)$ and $\delta|t, a>$, which are included in the expression (2) impose conditions

$$<\delta H(a)> = <\delta|t,a>> = 0 \qquad (3)$$

Here, averaging vector or operator of the random function f (a) is understood as

$$<f(a)> = \int_{\Omega} P(da) f(a) \qquad (4)$$

where $\Omega = \{a\}$ is a set of random events which are observing the dynamics of (2) by the same measuring device for equally prepared (as accurately as possible) initial states. P (...)-the probability measure on the set. After spending an average of (2) in the sense of (4), we obtain

$$i\hbar\frac{d|t>}{dt} = H|t> + <\delta H(a)\delta|t,a>> \qquad (5)$$

To use equation (5) for the description of quantum measurement, we expand vectors $|t>$ and $<\delta H(a)\delta|t,a>$ in the sum of the eigenvectors vectors observed, which measures the instrument

$$|t> = \sum_i c_i(t)|i>, <\delta H(a)\delta|t,a>> = \sum_i \Phi_i(t)|i> \qquad (6)$$

In fact, the last formula is the definition of the measurement process.

Substituting the results of the expansion in equation (5), we get

$$\sum_j i\hbar\dot{c}_j|j> = \sum_{k,j}|j><j|H|k>c_k + \sum_j \Phi_j(t)|j> = \sum_j i\hbar f_j(c)|j>$$

that is

$$\dot{c}_k = f_k(c) \qquad (7)$$

In the derivation of (7) has been suggested that $\Phi_j(t)$ depend on t only through c_n. Let the equation (7) has Hamiltonian form, that is $\{c_k\} = \{x^k\}\cup\{p_k\}$-canonical coordinates and momentums and (7) can be written as the Cartan equation [15]:

$$0 = \frac{\partial\Omega}{\partial dx^k} = \frac{\partial\Omega}{\partial dp_k}$$

where

$$\Omega = dp_i \wedge dx^i - dH \wedge dt$$

Turning to the variable action angle J_i, α^i:

$$\Omega = dJ_i \wedge d\alpha^i - d(H_0(J_s) + H_1(J_s,\alpha^s)) \wedge dt$$

and Cartan equations take the form:

$$0 = \frac{\partial\Omega}{\partial dJ_i} = \frac{\partial\Omega}{\partial d\alpha^i} = d\alpha^i - \left(\frac{\partial H_0}{\partial J_i} + \frac{\partial H_1}{\partial J_i}\right)dt = dJ_i + \frac{\partial H_1}{\partial\alpha^i}dt$$

or:

$$\dot{\alpha}^i = \frac{\partial H_0(J_s)}{\partial J_i} + \frac{\partial H_1(J_s,\alpha^s)}{\partial J_i}, \dot{J}_i = -\frac{\partial H_1(J_s,\alpha^s)}{\partial\alpha^i} \qquad (8)$$

Solving the system of equations (8) by perturbation theory [13]:

$$J_i = J_i^0 + J_i^1 + ..., \ \alpha^i = \alpha_0^i + \alpha_1^i + ...$$

In the first order we get:

$$\dot{J}_i^1 = -\frac{\partial H_1(J_s^0, \alpha_0^s + \omega^s t)}{\partial\alpha^i} = \sum_n c_{n_k}\exp(i(n_k\omega^k)t) \qquad (9)$$

Here

$$\omega^k \equiv \frac{\partial H_0}{\partial J_k}$$

and we expand the right side of (9) in a Fourier series. Thus, in the first order perturbation theory, the variable "action" would be:

$$J_i^1 = \sum_n \frac{c_{n_k} \exp(i(n_k\omega^k)}{i(n_k\omega^k)} \quad (10)$$

We assume that the series (10) for which the resonance condition: $n_i\omega^i = 0$ is not equal to 0, then, as shown by numerical experiments, the motion becomes random [13].

This allows equation (7) is replaced by the equivalent Langevin equation:

$$dc_k = -\frac{\partial U(c_s)}{\partial c_k}dt + dW_k \quad (11)$$

where dW_n-«random force" satisfying the conditions [12]:

$$< dW_n >= 0, < dW_n dW_m >= Q\delta_{mn}dt \quad (12)$$

Equation (11), in turn, is equivalent to the Fokker-Planck equation for density of probability $\rho(c_m,t)$ [12]:

$$\dot{\rho}(c_k) = \sum_j \left\{ \frac{\partial}{\partial c_j}(\frac{\partial U(c_k)}{\partial c_j}\rho(c_k)) + \frac{1}{2}Q\frac{\partial^2}{\partial c_j^{\,2}}\rho(c_k) \right\} \quad (13)$$

Indeed: for the time derivative of the probability density $\rho(c_m,t)$ of finding the system at c_m obtain

$$\frac{d}{dt}\int dc\rho(c_m,t)f(c_m) = \int dc\frac{\partial\rho(c_m,t)}{\partial t}f(c_m)$$
$$= \frac{1}{dt}\int dc\rho(c_m,t)df(c_m)$$
$$= \frac{1}{dt}\int dc\rho(c_m,t)(\frac{\partial f}{\partial c_p}dc_p + \frac{1}{2}\frac{\partial^2 f}{\partial c_p\partial c_s}dc_p dc_s + ...)$$
$$= \frac{1}{dt}\int dc\rho(c_m,t)\left\{ \begin{array}{l} \sum_p(\frac{\partial f}{\partial c_p}(-\frac{\partial U}{\partial c_p}dt + dW_p)) + \\ \sum_{p,s}(\frac{1}{2}\frac{\partial^2 f}{\partial c_p\partial c_s}(-\frac{\partial U}{\partial c_p}dt + dW_p) \\ \qquad (-\frac{\partial U}{\partial c_s}dt + dW_s)) + ... \end{array} \right\}$$
$$= \int dc\frac{\partial}{\partial c_p}(\rho\frac{\partial U}{\partial c_p})f + \frac{1}{2dt}\int dc\frac{\partial^2 f}{\partial c_p\partial c_s}\rho dW_p dW_s + ...$$

Here we used the integration of the initial value of the trajectory of the system and takes into account the fact that the system is Hamiltonian type. Now we average this expression over all values of the random force dW_n, all random trajectories of the system. Then we obtain the following expression

$$\int dc\frac{\partial\rho}{\partial t}f(c_m) =$$
$$\int dc\left\{ \sum_{p,s}(\frac{1}{2}\frac{\partial^2\rho}{\partial c_p\partial c_s}\frac{< dW_p dW_s >}{dt}) + \sum_p(\frac{\partial}{\partial c_p}(\frac{\partial U}{\partial c_p}\rho)) \right\} f$$

which by the arbitrariness of the function f and the condition (12) is the Fokker-Planck equation (13). Its stationary solution has the form:

$$\rho(c_n) = C\exp(-\frac{2U(c_n)}{Q}) \quad (14)$$

If we choose $U(c_n)$ such that with

$$\tilde{c}_n \;\; (U'(\tilde{c}_n) = 0, \text{and}, U(c_n) > U(\tilde{c}_n)$$

for $\forall c_n$) is equal to (1, 0,...), (0,1,...),..., it had a min and

$$\rho(\tilde{c}_n) = C\exp(-\frac{2U(\tilde{c}_n)}{Q}) = |c_n(0)|^2 \quad (15)$$

Thus, we obtain a simple mathematical model of reduction ket quantum state during the of measurment,since the equation | t›,will give | k› with probability (15). If the depth of the "potential wa ll" >>Q, the process of reduction leads to a unique result.

CONCLUSION

The study showed that for the reduction we had to modify the original equation (1) and submit it in the form (2). The mathematical model also shows that the reduction of the wave function is specified stochastic nature of the interaction between the quantum system and the measuring device. This interaction leads to a response δ|t,a>. Account of the correlations between, this process leads to the equation (7), which leads to a reduction.

REFERENCES

1. Shumacher, B.W., 1995. Quantum coding. Phys. Rev., pp: 2738-2747.
2. Feinman, R.P., 1986. Quantum Mechanical Computer. Foundations of Physics, 16 (6): 507-531.
3. Shor, P.W., 1994. Algorithms for quantum computation: Discrete logarithms and factoring. Proc. 35th Annual Symposium on Foundations of Computer Science (IEEE Press), pp: 124-134.
4. Zalka, C., 1998. Simulating quantum systems on a quantum computer. Proc. Roy. Soc. London, pp: 313-322.

5. Ozhigov, Y., 1997. Quantum Computer Can Not Speed Up Iterated Applications of a Black Box. quant/ph/9712051.
6. Cory, D.G., A.F. Fahmy and T.F. Havel, 1996. Nuclear Magnetic Resonance spectroscopy: An experimentally accessible paradigm for quantum computing. Proc. of the 4th Workshop on Physics and Computation (Complex Systems Institute, Boston, New England).
7. Chivilihin, S.A., 2009.Quantum informatcs. Information Technologies, St. Petersburg, pp: 80.
8. Everett, H., 1957. "Relative State" Formulation of Quantum Mechanics. Revieus of Modern Physics, 29 (3): 454-462.
9. Landau, L.D. and E.M. Lifshitz, 1990. Quantum mechanics. M.: Science, pp: 752.
10. Mensky, M.B., 2007. Quantum measurements, the phenomenon of life and time arrow: The three "great problems" (Ginzburg's terminology). UFN, 177 (4): 415-425.
11. Haken, H., 1980. Synergetics. Academic Press, pp: 193-197.
12. Haken, H., 1985. Synergetics (hierarchy of instabilities in self-organizing systems and devices). Academic Press, pp: 177-188.
13. Prigozhin, I., 1984. From Being to Becoming (time and complexity in the physical sciences.). M.: Science, pp: 287.
14. Klimontovich, Yu.L., 1982. Statistical Physics. M.: Science, pp: 608.
15. Kartan, E., 1940. Integral invariants. M., L.: GITL, pp: 215.