

Methods of Processing Video Polarimetry Information Based on Least-Squares and Fourier Analysis

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Abstract: The polarization state of light is a very significant source of information in biomedical imaging and diagnosis. Processing video polarimetry information can be quite difficult to measure. Effective calibration minimizes systematic errors in measuring intensity that appear due to movement of the optical components. In this article, we presented a comparison of two methods, namely, least square methods and Fourier analysis for the detection of Stokes parameters of a reflected by a biological tissue beam.

Key words: Tissue imaging • Video polarimetry • Stokes vector • Least square method

INTRODUCTION

The detection and treatment of cancer remains one of the biggest problems in modern medicine. The probability of survival significantly increases with early detection. Until now, biopsy has been the standard for cancer diagnosis. Skin cancer is suspected if an area of the skin changes color, shape, or size; or it does not heal after injury. However, this procedure can miss tumors if they are still in an early stage due to overlapping of tissue layers.

Polarization methods (Figure 1) can improve the accuracy of the diagnosis, on one hand and simplify hardware expenses on the other [1, 2]. Analysis of video polarimeters can be found in literature [3], from which we can conclude that they are potentially powerful tools, but it is quite difficult to measure the polarization of the light reflected by the sample. Effective calibration minimizes systematic errors that appear when one moves the optical elements.

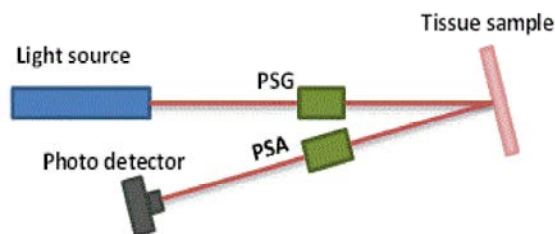


Fig. 1: Video polarimeter scheme (PSG Polarization state generator, PSA Polarization state analyzer)

MATERIALS AND METHODS

Modulation schemes allow determination of the state of polarization of the beam reflected by tissues. In these schemes, the optical elements rotate with a given frequency and the state of polarization is obtained by multiple measurements [4]. One of the most common ways of modulation uses a rotating quarter-wave plate and two fixed linear polarizers. This approach has the advantage that different polarization states can be defined on the same pixel.

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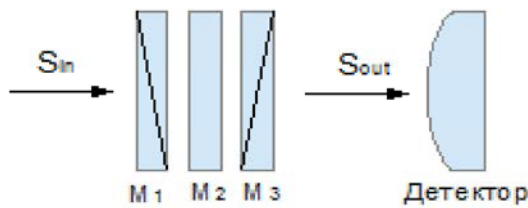


Fig. 2: Analyzer: M_1, M_3 Mueller matrices of linear polarizer, M_2 Mueller matrix of quarter wave plate.

The state of polarization of light is described by the Stokes parameters $S = [S_0, S_1, S_2, S_3]^T$; $S_0 = I_0 + I_{90}$; $S_1 = I_0 - I_{90}$; $S_2 = I_{45} - I_{135}$; $S_3 = I_R - I_L$; where $I_0, I_{90}, I_{45}, I_{135}$ correspond to the intensity of linear polarization states and I_R, I_L to the intensity of circular polarization states. Partially polarized light is characterized by the degree of polarization DOP, equation 1. The degree of polarization can be determined as the ratio of the intensity of polarized light to the total beam intensity. For ideal polarized light, DOP is equal to 1, whereas totally unpolarized light has a DOP equal to 0.

$$DOP = \frac{\sqrt{Q^2 + U^2 + V^2}}{I} \quad \text{Eq. 1}$$

The Stokes vector may be defined by four independent measurements due to the relationship $I = I_0 + I_{90} = I_{45} + I_{135} = I_R + I_L$, where I is the light intensity measured without analyzer.

The Mueller matrix MM allows the description of the effect of the optical elements on the polarization of the light beam. This is a square matrix, which has a 4x4 dimension and all of its elements are real numbers equation 2.

$$MM = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \quad \text{Eq. 2}$$

The result of the interaction of the incident optical radiation described by Stokes vector S_{in} with the optical system described by Mueller matrix MM can be obtained by the product $S_{out} = MM * S_{in}$. The effect of a set of optical elements on the polarization state of the incident beam is described by the product $S_{out} = M_3 * M_2 * M_1 * S_{in}$, wherein the matrixes of elements, successively traversed by the light beam, are arranged in reverse order, Figure 2.

A rotation matrix $R(\theta)$ allows calculating Mueller matrix at any azimuth by the use of Mueller matrix of the simple optical elements at an azimuth of zero, equation 3.

$$MM(\theta) = R(\theta) * MM(0) * R(-\theta) \quad \text{Eq. 3}$$

$$R(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\theta) & -\sin(2\theta) & 0 \\ 0 & \sin(2\theta) & \cos(2\theta) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Not all matrices are physically realizable Mueller matrices. A Mueller matrix must satisfy two criteria: 1) the matrix cannot give outgoing Stokes vectors with the degree of polarization greater than one and 2) The matrix cannot give outgoing Stokes vectors with intensity greater than the intensity of the input Stokes vector [5].

We modeled the Stokes vector and polarization state analyzer for a modulation scheme of the phase plate taking into account the above-mentioned intensity relationships. The Mueller matrix of the analyzer MA was found using equations 3 and the corresponding Mueller matrices of the linear polarizer PL and phase plate QWP .

$$MA(\theta) = \begin{bmatrix} 0.5 & 0.5 * \cos(2\theta)^2 & 0.5 * \cos(2\theta) * \sin(2\theta) & -0.5 * \sin(2\theta) \\ 0.5 & 0.5 * \cos(2\theta)^2 & 0.5 * \cos(2\theta) * \sin(2\theta) & -0.5 * \sin(2\theta) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Eq. 4}$$

The intensity recorded by the detector is determined by the first parameter of the Stokes vector and is calculated by $I(\theta) = S_{1out}(\theta) = MA_1(\theta) * S_{in}$, where MA_1 is the first row of the analyzer Mueller matrix. The modulation scheme may be expressed with the following linear system of equations, equations 5, or in matrix form $I(\theta) = A * S_{in}$, where each row of the matrix A is the first row of analyzer Mueller matrix for each measurement.

$$\begin{aligned}
 I_1 &= ma_{11}(1) * S_{in0} + ma_{12}(1) * S_{in1} + ma_{13}(1) * S_{in2} + ma_{14}(1) * S_{in3} \\
 I_2 &= ma_{21}(2) * S_{in0} + ma_{22}(1) * S_{in1} + ma_{23}(1) * S_{in2} + ma_{24}(1) * S_{in3} \\
 &\vdots \\
 I_N &= ma_{N1}(1) * S_{in0} + ma_{N2}(1) * S_{in1} + ma_{N3}(1) * S_{in2} + ma_{N4}(1) * S_{in3}
 \end{aligned}
 \tag{Eq. 5}$$

As result, matrix A has dimensions N×4. Input Stokes vector may be obtained by solving the previous linear system of equations. Since A is not square and the number of equations is larger than the number of unknowns, the system is over determined. In the presence of errors, this system does not have a solution. In this case, instead of the exact solution we should look for a solution that best satisfies all equations. The method of least squares is commonly used to find the solution.

Intensity becomes a continuous periodic function, which can be expanded with a Fourier series, due to the modulation scheme. In this case, the parameters of the Stokes vector may be determined from the coefficients of the series, equation 6.

$$I(\theta) = \left(\frac{1}{2}S_0 + \frac{1}{4}S_1 \right) - \frac{1}{2}\sin(2\theta)S_3 + \frac{1}{4}\cos(4\theta)S_1 + \frac{1}{4}\sin(4\theta)S_2
 \tag{Eq. 6}$$

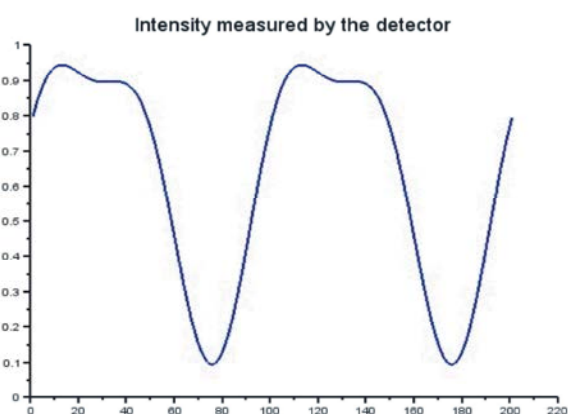


Fig. 3: Modeling of the reflected light: Intensity vs. angle

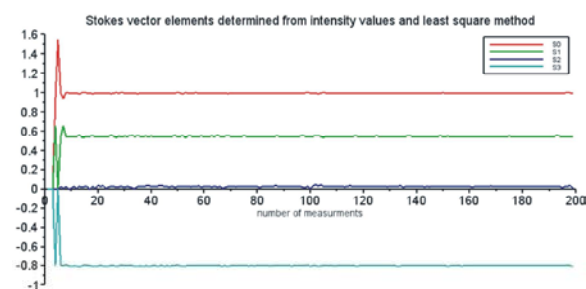


Fig. 4: Stokes parameters values by least squares methods

RESULTS

We modeled the detected intensity, Figure 3, considering the standard accuracy for stepper motors and we performed calculations of the Stokes parameters using two methods and then we compared the results.

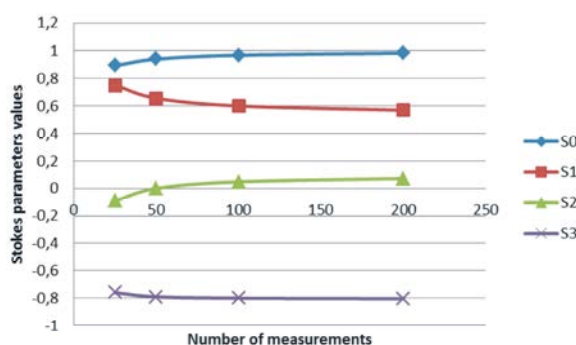


Fig. 5: Stokes parameters values by Fourier analysis

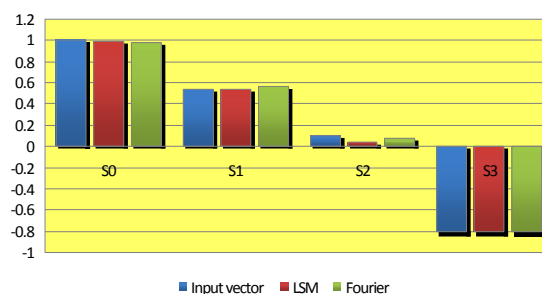


Fig. 6: Comparison of results of the two methods for 200 measurements

Figures 4 and 5 show the dependence of the values of the Stokes parameters of the number of measurements.

The results in Figures 3, 4, 5 y 6 showed that for a large number of measurements both methods provide accurate values of the parameters of the Stokes vector. However, for a small number of measurements the results obtained by Fourier analysis strongly diverges from expected values.

Summary and Future Research: We have simulated a beam of light reflected by biological tissue, described the mathematical model for a state of polarization analyzer and determined the Stokes vector parameters using the least square methods and the Fourier analysis. We found that the least squares method allows the obtainment of accurate results for a small number of measurements. This research allows setting calibration schemes to be set in video polarimetry information processing. In this research, we considered standard accuracy for stepper motors. In a future research, it would be interesting to include strong misalignments of optical elements.

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