

Application of Genetic Programming to Modeling of Uniaxial Compressive Strength

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Abstract: Uniaxial Compressive Strength (UCS) is the most important rock parameter required and determined for rock mechanical studies in most civil and mining projects. In this study, two soft computing approaches, which are known as neuro-fuzzy inference system (ANFIS) and Genetic Programming (GP), are used in strength prediction of uniaxial compressive strength (UCS). Block Punch Index (BPI), porosity (n), P-wave velocity (V_p), Density (ρ) were used as inputs for both methods and were analyzed to obtain training and testing data. Of all 130 data sets, the training and testing sets consisted of randomly selected 110 and 20 sets, respectively. Results showed that the ANFIS and GP models are capable of accurately predicting the uniaxial compressive strength (UCS) used in the training and testing phase of the study. The GP model results better prediction compared to ANFIS model.

Key words: Neuro-Fuzzy • Genetic Programming • Uniaxial Compressive Strength

INTRODUCTION

Measures and estimates of Uniaxial Compressive Strength (UCS) of rock materials are widely used in rock engineering, they are important for intact rock classification and rock failure criteria. In addition, analytical and numerical solutions require UCS. The procedure for measuring this parameter has been standardized by both the American Society for Testing and Materials (ASTM) and the International Society for Rock Mechanics (ISRM). High-quality core samples are needed for the application of UCS test in laboratories; a careful execution of this test is very difficult, time consuming, expensive and involves destructive tests. In order to overcome these difficulties, encountered during core sample preparation and execution of these tests, some predictive models considering simple index parameters such as Schmidt hammer, point load index, P-wave velocity and physical properties were developed by many investigators [1-11], because these indexes test require a relatively small number of samples, are quick and easy to execute, with portability and low costs, compared

with uniaxial compressive strength tests. Despite some deficiencies, index tests, when coupled with experienced judgment, can provide initial estimates of rock properties, required at the feasibility and design stage [12, 13]. Traditionally, statistical methods used in rock engineering, such as simple and multiple regression techniques are employed to establish predictive models [14]. In recent years, new techniques such as genetic programming and fuzzy inference systems have been employed for developing predictive models to estimate the required parameters [15-22]. (Baykasog and etal) used genetic programming to predict uniaxial compressive strength (UCS) for limestone and It is figured out that genetic programming techniques are able to provide good prediction equations for strength prediction.

The objective of this study is to investigate the usability of neuro-fuzzy inference system (ANFIS) and genetic programming (GP) in predicting the uniaxial compressive strength (UCS) by use five rock types and make comparison of prediction levels between developed models by using the related prediction values and results. The ANFIS and GP approaches were used to predict

Table1: Basic descriptive statistics of the established dataset according to the rock type.

Rock Type	Case #	$\rho(\text{gr/cm}^3)$			n(%)			$V_s(\text{km/s})$			BPI			UCS(MPa)		
		Max	$\bar{x} \pm$	Min	Max	$\bar{x} \pm$	Min	Max	$\bar{x} \pm$	Min	Max	$\bar{x} \pm$	Min	Max	$\bar{x} \pm$	Min
limestone	26	2.72	2.610.08	2.45	4.91	2.141.5	0.29	49.83	38.616.13	25.87	33.01	2.266.48	12.16	173.76	96.2532.31	34.78
Andesite	26	2.72	2.600.09	2.35	16.87	7.054.83	0.29	49.83	31.687.89	16.69	38.1	2.0511.1	3.01	173.76	53.0140.32	9.5
Hornfels	26	2.81	2.750.02	2.68	1.2	0.430.25	0.09	57.76	43.212.51	37.93	58.36	31.4313.6	11.85	335.82	264.755.96	133.58
Sandstone	26	2.82	2.230.25	1.7	37.69	17.918.8	6.51	52.02	32.956.57	21.56	10.99	3.192.7	0.27	99.72	39.329.12	5.33
Travertine	26	2.53	2.040.06	2.27	14.21	9.033.61	3.12	53.15	49.762.14	44.29	20.82	11.444.04	4.96	99.72	52.8319.00	23.26

$\bar{x} \pm$ refers to average values with standard deviation

the uniaxial compressive strength (UCS). Complex relationship between the parameters affecting the UCS can be easily modeled by use ANFIS and GP approach unlike statistical models. Experimental UCS data were collected from various samples to be included in training and testing phase of ANFIS and GP approaches.

Experimental Study and Test Results: In this study, a dataset generated by Saedi (2006) were used for constructing the neuro network for prediction of uniaxial compressive strength. Various types of rock cores including Limestone, Hornfels, Travertine andesite and Sandstone were gathered from different mine sites in Iran. A reliable predictive model requires a sufficiently large number of high-quality data. For this purpose 10 block samples were collected from the mine sites and 130 sample sets were obtained for rock mechanical tests. Followings the core retrieving, rock samples were prepared and some related laboratory rock tests such as Block Punch Index(BPI), porosity (n), P-wave velocity (Vp), Density (ρ), uniaxial compressive strength (UCS) were carried out in accordance with ISRM. The basic descriptive statistics of the dataset according to the rock type and data are summarized in Table 1.

Genetic Programming: Koza [12] proposed genetic programming (GP) technique which is an extension to Genetic algorithms. In genetic programming, populations of hundreds or thousands of computer programs are genetically bred. This breeding is done using the Darwinian principle of survival and reproduction of the fittest along with a genetic recombination (crossover) operation appropriate for mating computer programs [23]. GP breeds computer programs to solve problems by executing the following three steps: (1) Generate an initial population of random computer programs composed of the primitive functions and terminals of the problem. (2) Iteratively perform the following sub-steps until the termination criterion is satisfied: (a) Execute each problem in the population so that a fitness measure indicating how well the program solves the problem can be computed for the program. (b) Create a new population of programs by selecting programs in the population with a probability

based on fitness and then applying the following primary operations:

- Reproduction: Copy an existing program to the new population.
- Crossover: Create new computer programs by crossover.
- Mutation: Create new computer programs by mutation.
- Choose an architecture-altering operation to one selected program.
- The single best computer program in the population produced during the run (best solution so far) is designated as the result of genetic programming [23-25].

GP Model Development: An aim of this study is to obtain an explicit formulation for Uniaxial Compressive Strength (UCS) using genetic programming based on experimental results. Details of the experimental procedure have been explained in Section 2. The details of the experimental database including the parameters and their range are presented in Table 2. To achieve generalization capacity for the formulations, the experimental database is divided into two sets as training and test sets. The formulations are based on training sets and are further tested by test set values to measure their generalization capability. In the literature, this type of studies includes test sets as 20–30% of the training set. The patterns used in testing and training sets are selected randomly. Among the experimental data, 110 sets were used for GP training and 20 sets for GP testing. Parameters of the GP models are presented in Table 3. The purpose of this section is to obtain the explicit formulation of Uniaxial Compressive Strength (UCS) as a function of Block Punch Index(BPI), porosity (n), P-wave velocity (Vp), Density (ρ). Explicit formulations based on GP for UCS was obtained as a function of experimental parameters as

$$UCS = f(BPI, n, v_p, \rho)$$

(Fig. 1) shows the expression tree of GP models, whose explicit formulation is:

$$UCS = e^{\cos\left(\cos\left(\cos\left(\log\left(\left(\left(\rho^{\sin s-BPI}\right)\right)\times\left(\log 10^{vp}-vp\times BPI^5\right)\right)\right)-\rho\right)\right)\times 4e^{\cos\left(\log\left(\left(\left(\rho^{\sin s(d)}\right)\times 3\right)\right)\right)}+BPI^d} \quad (1)$$

Table 2: Parameters of the GP model

Population size	50
Maximum number of evaluated individuals	1000
Maximum depth	14
Reproduction	0.1
Initial prob type	fixed
Num back gen	3
Probability of crossover	0.02
Probability of mutation	0.97
Percent change	0.25
Function set	+, -, *, /, power, exp, ln(x), log, p, X ² , X ³ , (1/X).

Table 3: Variables used in model construction.

Variables	Code	Range
Density	X1	1.72-2.82
Porosity	X2	0.09-37.69
Wave Length	X3	1669.84-5776.21
Box-Punch Index	X4	0.27-58.36
UCS	-	5.33-335.82

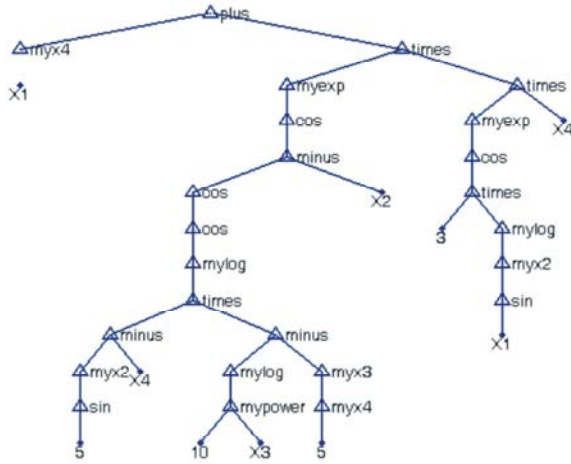


Fig. 1: Expression Tree

It should be noted that proposed GP formulations in Eq. (1) is valid for the ranges of training set given in Table 3.

Neuro-Fuzzy Inference System (ANFIS): In classical set theory, there is a crisp definition as to whether a variable belongs to a set or not. However, the fuzzy theory introduced by Zadeh [26] does not give a sharp answer to questions. In this approach, the belongings of a variable to different sets are defined partially by continuous membership functions that vary between 0 and 1 [27, 28]. Mamdani and Tagagi–Sugeno (TS) models are two types of fuzzy approach commonly-used [29]. The main

difference between these approaches is that Mamdani model uses the human expertise and linguistic knowledge to design the membership functions and if–then rules whereas TS model uses optimization and adaptive techniques to establish the system modeling and also uses less number of rules. TS model preferred mostly for mathematical analysis and its computational efficiency seems to be more advantageous than Mamdani model [30]. Also, the output membership function in TS model is simply designed as either linear or constant [31]. Jang [32] proposed a new fuzzy logic model called ANFIS which uses learning and parallelism properties of artificial neural network (ANN). Fuzzy rules and membership functions are also generated adaptively by neural training process using given data set. So, ANFIS employs method of grid partitioning and subtractive clustering [33-35]. First-order Sugeno type fuzzy inference system is used for linear function and zero-order Sugeno type fuzzy inference system is used for constant function. A typical two if then rules used in first-order Sugeno type is given in the following form:

$$\text{If } x = A_1 \text{ and } y = B_1 \text{ then } f_{1(x,y)} = p_1 x + q_1 y + k_1 \quad (2)$$

$$\text{If } x = A_2 \text{ and } y = B_2 \text{ then } f_{2(x,y)} = p_2 x + q_2 y + k_2 \quad (2)$$

where x (or y) is i th input node, p, q and k are training parameters, A and B are fuzzy membership function labels.

The membership function is updated by backpropagation learning algorithm [36, 37]. The basic structure of an ANFIS model is shown in (Fig. 2). As can be seen there are five layers in which the mathematical computations are performed. The mathematical computations in fuzzy approach are performed in five stages. The value of the i th node of the first stage is computed as below;

$$U_{1,i} = \eta A_i(x) \text{ for } i = 1, 2 \text{ or} \quad (4)$$

$$U_{1,i} = \eta B_{i-2}(x) \text{ for } i = 3, 4 \quad (5)$$

where η is the membership function.

In second stage, the nodes are represented as the fire strength of the rule and the output $U_{2,i}$ which is the product of the incoming signals is computed as follow;

$$U_{2,i} = w_i = \eta A_i(x) \eta B_i(y), \quad i = 1, 2 \quad (6)$$

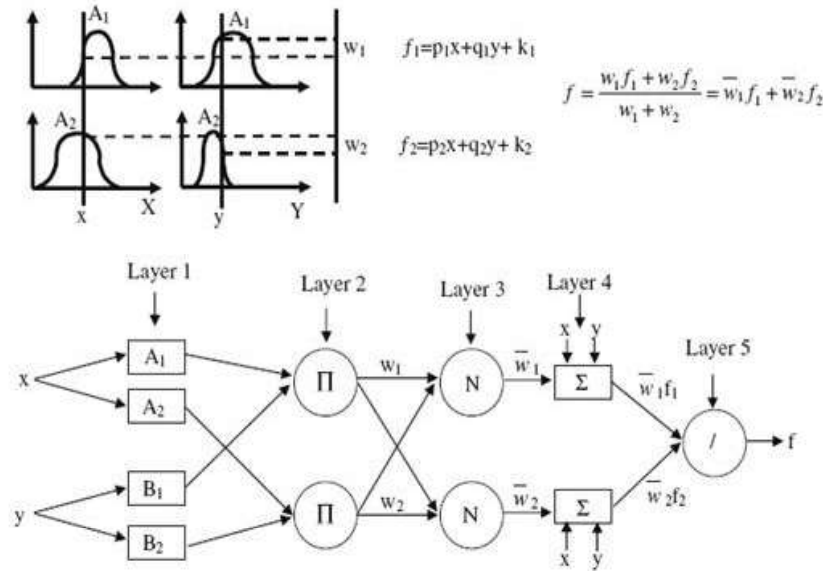


Fig. 2: First order TS model reasoning and basic ANFIS architecture [36].

In third stage, the normalized firing strengths which shows the ratio of the i th rule's firing strength versus all rule's firing strength are computed by following equation;

$$U_{3,i} = U_{3,i} = \bar{w}_i = \frac{w_i}{w_i + w_z} \quad (7)$$

The subsequent stage performs a calculation for determination of the contribution of the i th rule to output;

$$U_{4,i} = \bar{w}_i f_i = \bar{w}_i (p_i x + q_i y + k_i) \quad (8)$$

\bar{w} indicates the normalized firing strength found from layer 3, p_i, q_i and k_i are the consequent parameters. In last stage, the final output of the ANFIS is computed by following the equation;

$$U_{s,i} = \sum_i \bar{w}_i f_i = \frac{\sum_i \bar{w}_i f_i}{\sum_i \bar{w}_i} \quad (9)$$

Development of ANFIS Model: ANFIS model was developed using identical inputs for as in GP, for generation of the membership functions associated with each input variable, the grid partition method was employed for ANFIS model. In the model, the Gaussian membership function was assigned. The hybrid learning algorithm was used for optimizing the parameters allows a fast identification of parameters and substantially reduces the time needed to reach convergence [38]. The minimum validation error is

used as the stopping criterion to avoid over fitting. The ANFIS model has 80 linear parameters, 24 nonlinear parameters, 55 nodes and 16 fuzzy rules. The MATLAB Software was used for the models development.

RESULTS AND DISCUSSION

In this study, it was basically aimed to explore the applicability of the GP and ANFIS for prediction of the UCS value of some rocks that have great significance for rock mechanics and foundation engineering. This section comparatively presents the analyses results obtained from these approaches and quantitative assessments of the model's predictive abilities. Of the 130 data sets, 110 were used for training the models and 20 which are not used in training stage were presented for testing of the models. In order to find out how accurate the results of the developed models are, a statistical verification criteria was utilized as coefficient of correlation (R). As can be seen in Table 4 the comparisons between GP and ANFIS indicate that the best results in terms of the R value generated from the GP analyses that are shown in (Fig. 3, 4) This implies that GP models produce good performance. In statistics, the overall error performances of the relationship between two groups can be interpreted from the R values. According to Smith (1986), if a proposed model gives $R > 0.8$, there is a strong correlation between measured and predicted values overall the data available in the database.

Table 4: Coefficients of correlation obtained for the predictions made by ANFIS and GP

UCS	R
GP	0.96
ANFIS	0.87

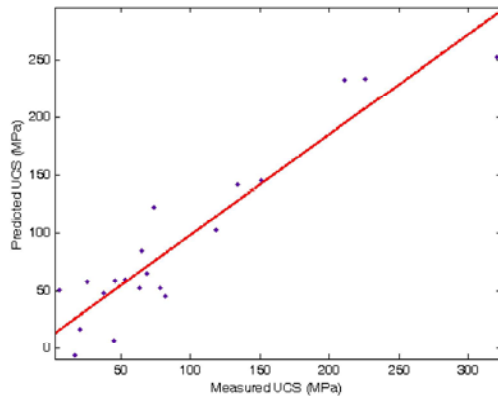


Fig. 3: Predicted UCS by ANFIS model vs. measured UCS for testing set.

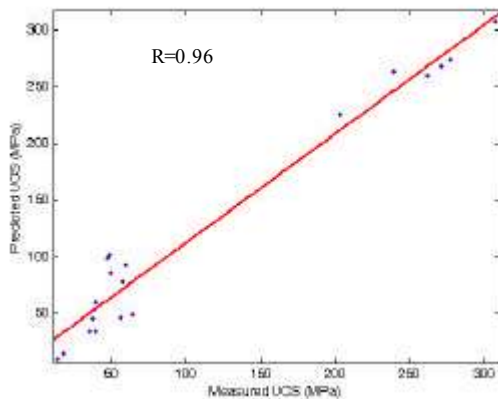


Fig. 4: Predicted UCS by GP model vs. measured UCS for testing set.

CONCLUSIONS

This study demonstrates the efficiency of GP and ANFIS models to predict UCS. The developed models were able to predict the UCS for Block Punch Index(BPI), porosity (n), P-wave velocity (V_p), Density (ρ) used in training and testing processes. Predicting of UCS as a function of parameters is a difficult task to achieve. However, a successfully trained GP and ANFIS models can predict the UCS easily and accurately. So, the GP and ANFIS models can be a powerful alternative approach to traditional statistical methods used in developing the relationship between the UCS and the parameters affecting it. Although the performance of the developed GP and ANFIS models is limited to the range of input data

used in training process, the model can easily be retrained to expand the range of input variables by providing additional new set of data. GP and ANFIS models also have the minimum degree of scatter and maximum ability of trend capture compared to other equations. But as mentioned in section five, the GP model in the paper results better prediction compared to ANFIS model. We believe that genetic programming based techniques will gain much more popularity for strength prediction applications in the literature and applications in the future.

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