Middle-East Journal of Scientific Research 14 (5): 712-719, 2013
ISSN 1990-9233

DOI: 10.5829/idosi.mejsr.2013.14.5.335

A New Technique for Multi Criteria Decision Making Based on Modified Similarity Method

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Abstract: Multiple attribute decision making (MADM) is the most well-known branch of decision making. It is a branch of a general class of operation research models that deal with decision problems under the presence of a number of decision criteria. This paper proposes a New MADM method. This similarity-based method effectively makes use of the ideal solution concept in such a way that the most preferred alternative should have the highest degree of similarity to the positive ideal solution and the lowest degree of similarity to the negative-ideal solution. The overall performance index of each alternative within all criteria is determined based on the concept of the degree of similarity between each alternative and the ideal solution using alternative gradient and magnitude. In this paper Deng's similarity-based method is modified. His method also turns out to be subject to significant drawbacks. Finally, a numerical example is given, to verify the feasibility and effectiveness of the method proposed.

Key words: Similarity-Based method · Multi-Criteria Decision Making · Multi-Attribute Decision Making

INTRODUCTION

In real world, many decision problems are required simultaneous attention to several aspects of one certain criterion [1]. Decision making that deals with several aspects of a finite set of available alternatives in a given situation is often referred to as multi criteria decision making (MCDM). In the literature, there are two basic methods to multiple criteria decision making (MCDM) problems: multiple attribute decision making (MADM) and multiple objective decision making (MODM). MADM problems are distinguished from MODM problems, which involve the design of a "best" alternative by considering the tradeoffs within a set of interacting design constraints. MADM refers to making selections among some courses of action in the presence of multiple, usually conflicting, attributes. In MODM problems, the number of alternatives is effectively infinite and the tradeoffs among design criteria are typically described by continuous functions.

MADM is the most well-known branch of decision making. It is a branch of a general class of operation research models that deal with decision problems under the presence of a number of decision criteria. Many efforts has been made and several methods have been effectively developed for (MADM) problems, which in literature, have been resulted in very successful application of these methods [2-5] One of the most commonly used methods in this regard is the technique for order preference by similarity to ideal solution (TOPSIS) [6-8]. The TOPSIS method is developed based on the perception that a preferred alternative be close to

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the positive ideal solution and far from the negative ideal solution as much as possible which is simple and understandable [7]. As a result, numerous applications of such an method have been reported in the literature for addressing various practical multicriteria analysis problems in the real world setting. Besides, according to the simulation comparison from [9], TOPSIS has the fewest rank reversals among the eight methods in the category. Thus, TOPSIS is chosen as the main body of development. Under some circumstances counter-intuition outcomes may occur while comparing two alternatives just simply based on the distance between them and the ideal solution. Mathematically, the relative similarity between each alternative and the ideal solution is better represented by the magnitude of the alternatives and the degree of conflict between them [3]. To avoid the existing concern of TOPSIS method [10] presented a similarity based method for solving the general multicriteria analysis problem. This method effectively uses the concept of ideal solution and in a way in which strongly preferred variable must have highest similarity degree to the positive ideal solution and the lowest similarity to the negative solution. The overall performance index of any variables for all criteria is based on the concept of similarity degree between each variable and ideal solution using gradient and magnitude. Unfortunately, such a method also turns out to be subject to some significant drawbacks.

To provide a valid yet practical method for MADM problems, this paper proposes a new similarity-based method. Therefore the problem of Deng's method will be identified and solved. The paper is organized as follow. Section 2 reviews the Concept of Similarity method and Deng's similarity- based method. An example is made at the end to indicate the incorrect result. Section 3 proposes the new method and shows how to solve Deng's similarity- based method's problem and the proposed method is illustrated with a numerical example. In section 4, the result of solving numerical example is presented and also the accuracy of obtained result is investigated. Finally, the conclusions are given in Section 5.

MATERIALS AND METHODS

Similarity-based approach for rating multi criteria variables is for solving interrupted multi criteria problems and effectively uses the concept of ideal solution and in a way in which strongly preferred variable must have highest similarity degree in positive ideal solution and the lowest similarity in negative similarity solution.

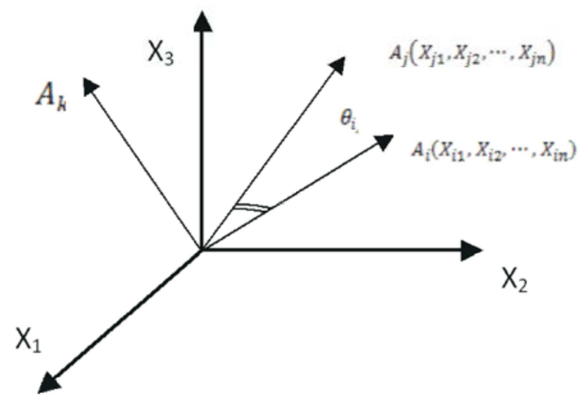

Fig. 1: Degree of conflict between alternatives by gradients

Concept of Similarity Method: There are several methods for expressing conflict among two variables in multi criteria analysis problems [11-13]. Among them, the notion of variable's gradient explains conflict between decision criteria in multi criteria analysis problems, which is very common [14]. Using this method, a conflict index is calculated between two alternatives to show the degree of conflict between the alternatives. Assuming that A_i and A_j are the two alternatives concerned in a given multicriteria analysis problem, these two alternatives can be considered as two vectors in the m-dimensional real space. The angle between Ai and Aj in the m-dimensional real space is a good measure of conflict between them. As shown in Figure 1, A_i and A_j are in no conflict if $\theta_{ij} = 0$, the conflict is possible if $\theta_{ij} \neq 0$, i.e. $\theta_{ij} \in (0, \pi/2)$. This is so because when $\theta_{ij} = 0$ the gradients of both the alternatives A_i and A_j are simultaneously in the same increasing direction and there is no conflict between them. The situation of conflict occurs when $\theta_{ij} \neq 0$, i.e. when the gradients of A_i and A_j are not coincident. The degree of conflict between alternatives A_i and A_j is determined by

$$\cos\theta_{ij} = \frac{\sum_{k=1}^{m} x_{ik} k_{jk}}{\left(\sum_{k=1}^{m} x_{ik}^2\right)^{0.5} \left(\sum_{k=1}^{m} x_{jk}^2\right)^{0.5}} \quad (1)$$

where θ_{ij} is the angle between the gradients of the two alternatives and $(X_{j1}, X_{j2}, ..., X_{jn})$ and $(X_{j1}, X_{j2}, ..., X_{jn})$ are the gradients of two alternatives A_i and A_j respectively.

The conflict index equals to one characterized by $\theta_{ij} = 0$ as the corresponding gradient vectors lie in the same direction of improvement. Similarly, the conflict index is zero characterized by $\theta_{ij} = \pi/2$ which indicates that their gradient vectors have the perpendicular relationship between each other.

Deng's Similarity-Based Method: In this paper it has used Deng's similarity-based method [10]. to rank processes. Deng described this method as follow.

$$X = \begin{bmatrix} x'_{11} & x'_{12} & \cdots & x'_{1m} \\ x'_{21} & x'_{22} & \cdots & x'_{2m} \\ \vdots & \vdots & & \vdots \\ x'_{n1} & x'_{n1} & \cdots & x'_{mn} \end{bmatrix} \quad (2)$$

$$W = (w_1, w_2, \ldots, w_m) \quad (3)$$

The concept of the ideal solution is used in such a way that the most preferred alternative should have the highest degree of similarity to the positive ideal solution and the lowest degree of similarity to the negative-ideal solution. The ranking method starts by normalizing the decision matrix to ensure all the criteria involved are advantageous ones based on Eq.(4), described as:

$$x'_{ij} = \frac{x_{ij}}{(\sum_{k=1}^{n} x_{ik}^{1})^{1/2}} \quad (4)$$

As a result, a normalized decision matrix can be determined as

$$X' = \begin{bmatrix} x'_{11} & x'_{12} & \cdots & x'_{1m} \\ x'_{21} & x'_{22} & \cdots & x'_{2m} \\ \vdots & \vdots & & \vdots \\ x'_{n1} & x'_{n1} & \cdots & x'_{nm} \end{bmatrix} \quad (5)$$

The weighted performance matrix which reflects the performance of each alternative with respect to each criterion is determined by multiplying the normalized decision matrix in Eq. (5) by the weight vector, given as Eq. (3)

$$X' = \begin{bmatrix} w_1 x'_{11} & w_2 x'_{12} & \cdots & w_m x'_{1m} \\ w_1 x'_{21} & w_2 x'_{22} & \cdots & w_m x'_{2m} \\ \vdots & \vdots & & \vdots \\ w_1 x'_{n1} & w_2 x'_{n1} & \cdots & w_m x'_{nm} \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1m} \\ y_{21} & y_{22} & \cdots & y_{2m} \\ \vdots & \vdots & & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{nm} \end{bmatrix} \quad (6)$$

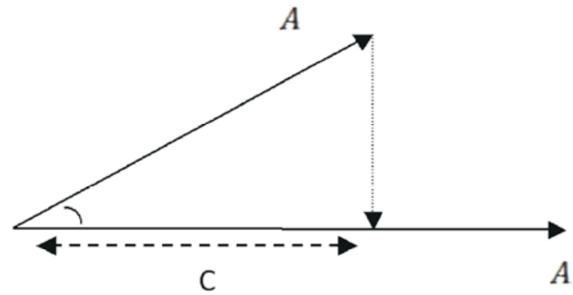

Fig. 2: Degree of conflict between A_i and A^+

The positive (or negative) ideal solution consists of the best (or worst) criteria values attainable from all the alternatives if each criterion takes monotonically increasing or decreasing values [7]. This concept has been widely used in various multicriteria analysis models for solving practical decision problems [6]. This is due to (a) its simplicity and comprehensibility in concept, (b) its computational efficiency and (c) its ability to measure the relative performance of the decision alternatives in a simple mathematical form. Based on this concept, the positive ideal solution and the negative ideal solution can be determined from the performance matrix in Eq. (6), given as

$$\begin{cases} A^{+} = (y_1^{+}, y_2^{+}, \ldots, y_m^{+}) \\ A^{-} = (y_1^{-}, y_2^{-}, \ldots, y_m^{-}) \end{cases} \quad (7)$$

where

$$\begin{cases} y_j^{+} = \max_{i=1,2,\ldots,n} y'_m \\ y_j^{-} = \min_{i=1,2,\ldots,n} y'_j \end{cases} \quad (8)$$

And

$$A_i = (y'_1, y'_2, \ldots, y'_m)$$

The degree of conflict between each alternative A_i and the positive ideal solution (the negative ideal solution) can be determined based on Eq. (1), given as

$$A_i, A^{\mp} = |A_i||A^{\mp}| \cos\theta_i^{\mp}$$

$$A_i, A^{\mp} = \sum_{j=1}^{m} y'_{ij} y_j^{-+}$$

$$|A_i| = \left(\sum_{j=1}^{m} {y'_{ij}}^{2}\right)^{0.5}$$

$$A^{\mp}| = \left(\sum_{j=1}^{m} y_{ij}^{\mp 2}\right)^{0.5}$$

$$\begin{cases} \cos\theta_i^{+} = \dfrac{\sum_{j=1}^{m} y'_{ij} y_j^{+}}{\left(\sum_{j=1}^{m} y_{ij}^{'2}\right)^{0.5} \left(\sum_{j=1}^{m} y_{ij}^{+2}\right)^{0.5}} \\ \cos\theta_i^{-} = \dfrac{\sum_{j=1}^{m} y'_{ij} y_j^{-}}{\left(\sum_{j=1}^{m} y_{ij}^{'2}\right)^{0.5} \left(\sum_{j=1}^{m} y_{ij}^{-2}\right)^{0.5}} \end{cases} \tag{9}$$

As a consequence, the degree of similarity between each alternative A_i and the positive and the negative ideal solution can be determined by Eq. (10)

$$|C_i| = \cos\theta_i^{-+} * |A_i|$$

$$|C_i| = \frac{\sum_{j=1}^{m} y'_{ij} y_j^{-+}}{\left(\sum_{j=1}^{m} y_{ij}^{'2}\right)^{0.5} \left(\sum_{j=1}^{m} y_{ij}^{-+2}\right)^{0.5}} * \left(\sum_{j=1}^{m} y_{ij}^{'2}\right)^{0.5}$$

$$S_i^{-+} = \frac{|C_i|}{|A^{-+}|} = \frac{\cos\theta_i^{-+} * |A1|}{} = \frac{\cos\theta_i^{-+} * \left(\sum_{j=1}^{m} y_{ij}^{'2}\right)^{0.5}}{\left(\sum_{j=1}^{m} y_{ij}^{-+2}\right)^{0.5}} \tag{10}$$

The larger the S_i is, the higher the degree of similarity between alternative A_i and A_j. An overall performance index can then be calculated for each alternative across all criteria based on the degree of similarity of alternative Ai relative to the ideal solution as

$$P_i = \frac{S_i^{+}}{S_i^{+} + S_i^{-}}, \quad i = 1, 2, ..., n \tag{11}$$

The larger the index value, the more preferred the alternative.

An example has been presented to show details of this method as following.

Example 1: For better and more accurate analysis, first alternative completely equals to positive ideal and then the results will be examined. Consider the following decision matrix:

It can be noticed with a little consideration that first alternative has all the quantity from the point of view of all criteria and it is completely equivalent and similar to the positive ideal solution but it has been selected as the second option with the help of the Similarity method. It seems that there is a problem. Why the first Alternative which had the best quantity hold the most similarity with the negative ideal solution?

Table 1: decision matrix

X	C1	C2	C3	C4	C5
A1	3.00	3500.00	25000.00	8.00	9.00
A2	2.00	2000.00	17000.00	6.00	4.00
A3	1.00	1500.00	5000.00	3.00	2.00
A4	0.50	750.00	3000.00	2.00	5.00
A5	1.25	2200.00	6500.00	4.00	1.00
W_i	0.22	0.20504	0.19596	0.23	0.15

Table 2: normalized decision matrix

X'	C1	C2	C3	C4	C5
A1	0.754	0.716	0.794	0.704	0.799
A2	0.503	0.409	0.540	0.528	0.355
A3	0.251	0.307	0.159	0.264	0.177
A4	0.126	0.153	0.095	0.176	0.444
A5	0.314	0.450	0.207	0.352	0.089
wj	0.220	0.205	0.196	0.229	0.150

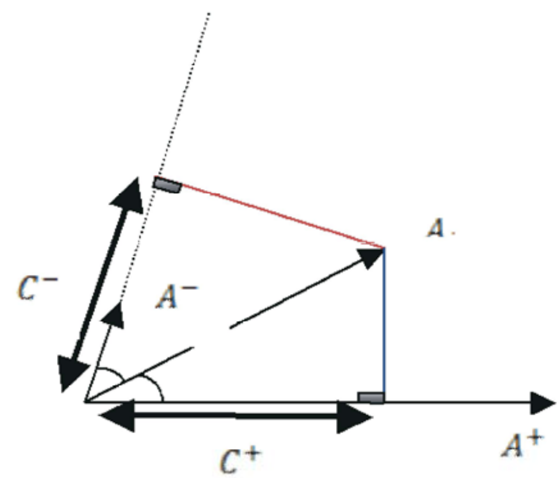

Fig. 3: Degree of conflict between A_i and $A^{\mp}$

In the next section, an in-depth critique of the problems, a way to fix them and finally the complete and modified method will be presented for solving MADM Problems.

Theory/Calculation: In this section, the main problem of Deng's method is identified, removed and the new method will be offered.

Proposed Method: In the Deng's Similarity based method, the formula (10) is used to obtain S_i^{-} and S_i^{+}. According to this method, if an alternative has the most similarity to positive ideal solution and least similarity to the negative ideal solution, then it will be the best. However, regarding obtained result of Example1 can be seen that the best real alternative has the most similarity with negative ideal alternative. It is concluded that Deng has made a mistake in the formula negative similarity (S_i^-). Whereby the Deng's Similarity method hasn't been indicated in this point will be performed as below.

$$A_i = (y_1', y_2', ..., y_m')$$

$$A^- = (y_1^-, y_2^-, ..., y_m^-)$$

$$A^+ = (y_1^+, y_2^+, ..., y_m^+)$$

$$\cos\ \theta_i^{\pm} = \frac{\sum_{j=1}^{m} y_{ij}' y_j^{\pm}}{\left(\sum_{j=1}^{m} y_{ij}'^{2}\right)^{0.5} \left(\sum_{j=1}^{m} y_{ij}^{\pm 2}\right)^{0.5}}$$

$$| C_i^{\pm} | = \cos\theta_i^{\pm} * | A_i |$$

$$| C_i^{\pm} | = \frac{\sum_{j=1}^{m} y_j' y_j^{-+}}{\left(\sum_{j=1}^{m} y_j'^{2}\right)^{0.5} \left(\sum_{j=1}^{m} y_j^{-+2}\right)^{0.5}} \left(\sum_{j=1}^{m} y_j'^{2}\right)^{0.5}$$

$$S_i^+ = \frac{| C_i^+ |}{| A^+ |} = \frac{\cos\theta_i^+ * | A1 |}{| A+ |} = \frac{\cos\theta^+ * \left(\sum_{j=1}^{m} y_{ij}'^{2}\right)^{0.5}}{\left(\sum_{j=1}^{m} y_{ij}^{+2}\right)^{0.5}} \quad (12)$$

$$S_i^+ = \frac{| A^- |}{| C_i^- |} = \frac{| A^- |}{\cos\theta_i^- * | A1 |} = \frac{\left(\sum_{j=1}^{m} y_{ij}^{-2}\right)^{0.5}}{\cos\theta * \left(\sum_{j=1}^{m} y_{ij}'^{2}\right)^{0.5}} \quad (13)$$

Another point that should be considered is a noticeable difference between this proposed method and TOPSIS. Overall performance of TOPSIS method is designed based on a logic which comes to one (when $A_i = A^+$) in the best situation and comes to zeros (when $A_i = A^-$) in the worst situation [15]. In proposed method due to the reason that the alternative i_{th} has an unclear angle with negative(positive) ideal solution when it is equal to the positive(negative) ideal solution, therefore Overall performance doesn't equal one (zero).

The Step by Step Explanation of New Method for Solving MADM Problems (Modified Similarity) is as follows:

Step 1: Determine the decision matrix as in Eq. (2).

Step 2: Determine the weighting vector as in Eq. (3).

Step 3: Normalize the decision matrix as in Eq. (5) which has been obtained by Eq. (2) and Eq. (4)

Table 3: The values of ±, $S^{\pm}$ and P for all alternatives

Y	C1	C2	C3	C4	C5	cos θ+	cos θ−	S+	S−	P	Ranking
A1	0.166	0.147	0.156	0.161	0.120	1.000	0.959	1.000	5.175	0.161	2
A2	0.110	0.084	0.106	0.121	0.053	0.989	0.969	0.642	3.395	0.158	3
A3	0.055	0.063	0.031	0.060	0.027	0.966	0.991	0.318	1.762	0.153	4
A4	0.028	0.031	0.019	0.040	0.067	0.873	0.817	0.234	1.182	0.165	1
A5	0.069	0.092	0.040	0.081	0.013	0.926	0.979	0.404	2.303	0.148	5
A(+)	0.166	0.147	0.156	0.161	0.120						
A(-)	0.028	0.031	0.019	0.040	0.013						

Table 4: Comparing TOPSIS and Proposed method

TOPSIS [14]	Proposed method
$C_i = \frac{D_i^-}{D_i^- + D_i^+}$	$P_i = \frac{S_i^+}{S_i^+ + S_i^-}$
$A_i = A^+, D^+ = 0, C_i = 1$	$A_i = A^+, S^+ = 1, P_i = \frac{1}{1 + S_i^-}$
$A_i = A^+, D^+ = 0, C_i = 1$	$A_i = A^-, S^- = 1, P_i = \frac{S_i^+}{1 + S_i^+}$
$0 \leq C_i \leq 1$	$0 < P_i < 1$
D_i^- = Euclidean distance between A_i and A^-	$0 \leq \theta < 90°$
D_i^+ = Euclidean distance between A_i and A^+	

Table 5: Comparison of the proposed new method results with Deng's Similarity-Based Method

Proposed new method		Deng's Similarity-Based Method		TOPSIS	
Pi	Rank	Pi	Rank	Pi	Rank
0.84	1	0.161	2	1.000	1
0.69	2	0.158	3	0.608	2
0.36	4	0.153	4	0.041	4
0.22	5	0.165	1	0.023	5
0.48	3	0.148	5	0.150	3

Table 6: Decision matrix

Bank	x_1	x_2	x_3	x_4
A	2500(million)	160,000	6	12
B	2300(million)	120,000	8	17
C	1900(million)	150,000	5	18
D	3100(million)	100,000	7	14
E	2800(million)	130,000	7	10
Weights	0.3	0.2	0.25	0.25

Table 7: Normalized decision matrix

Bank	x_1	x_2	x_3	x_4
A	0.806	1	0.75	0.667
B	0.742	0.75	1	0.944
C	0.613	0.938	0.625	1
D	1.000	0.625	0.875	0.778
E	0.903	0.813	0.875	0.556
Weights	0.3	0.2	0.25	0.25

Step 4: Calculate the performance matrix as expressed in Eq. (6)

Step 5: Determine the positive ideal solution and the negative ideal solution by Eq. (7) and Eq. (8).

Step 6: Calculate the degree of conflict between each alternative and positive ideal solution and negative ideal solution by Eq. (9).

Step 7: Calculate the degree of similarity between alternatives and the positive ideal solution and the negative-ideal solution by Eq. (12) and Eq. (13).

Step 8: Calculate the overall performance index for each alternative across all criteria by Eq. (11).

Step 9: Rank the alternatives in the descending order of the index value.

Now resolve the example1 in section 2.2 with the new method and compare with Deng's similarity based method result.

A Numerical Example: In this section we are going to propose a numerical example to illustrate an application of the proposed method in the previous section.

Assume the bank evaluation problem can be described as follows. Suppose the criteria of evaluating banks can be represented by investment income (x_1), number of customers (x_2), brand image (x_3) and branch numbers (x_4). Let the five banks and the corresponding evaluation ratings [14] be described as shown in Table 6:

First, the normalized preferred ratings should be calculated, as shown in Table 7, to transform the scale into [0, 1].

RESULTS AND DISCUSSION

Final results of the Modified Similarity Method are shown in Table 9. The proposed method scores the Bank D as best ranked among the 5 alternatives.

To better examine new method, results are compared with TOPSIS method in Table 9. It is observed that both rankings are the same and Bank D has been selected as the best bank in both methods.

Table 8: The values of cos θ±, S± and P for all alternatives

Bank	x_1	x_2	x_3	x_4	θ^+	θ^-	cos θ^+	cos θ^-	S^+	S^-	P
A	0.242	0.200	0.188	0.167	7.611	6.138	0.991	0.99	0.789	0.764	0.508
B	0.223	0.150	0.250	0.236	7.682	8.865	0.991	0.99	0.856	0.708	0.547
C	0.184	0.188	0.156	0.250	13.05	14.99	0.974	0.97	0.762	0.800	0.488
D	0.300	0.125	0.219	0.195	8.726	8.606	0.988	0.99	0.856	0.706	0.548
E	0.271	0.163	0.219	0.139	9.996	7.61	0.985	0.99	0.797	0.754	0.514

Table 9: Comparison of the Modified Similarity Method results with TOPSIS method

	Rank by Modified Similarity Method		TOPSIS		Deng's Similarity-Based Method	
Bank	Pi	Rank	Pi	Rank	Pi	Rank
A	0.508	4	0.412	4	0.37602	4
B	0.547	2	0.644	2	0.37745	2
C	0.488	5	0.326	5	0.37874	1
D	0.548	1	0.652	1	0.37667	3
E	0.514	3	0.468	3	0.37524	5

With respect to table 8 can be seen that the vector relating bank D has angular size 8.7 degrees with the positive ideal solution and 8.6 degrees with the negative ideal solution. This alternative has the highest Similarity with the positive ideal and the lowest Similarity with the negative ideal compared with other options and it is fully in accordance with the main concept of method. It can be seen that this method can provide reliable results as one of the MADM techniques.

CONCLUSIONS

This paper presents a new method using the concept of alternative gradient and magnitude for solving the general multicriteria analysis problem effectively. The proposed method is capable of addressing the concern of the TOPSIS method that the comparison of the alternatives cannot be determined solely by the distance between the alternatives. This method could be replaced with TOPSIS method. The concept of the degree of similarity between the alternatives and the ideal solution is combined to derive an overall performance index of each alternative for the general multicriteria analysis problem which has shown some potential. The advantages of this method are named as: (i) a sound logic that represents the rationale of human choice; (ii) a scalar value that accounts for both the best and the worst alternatives simultaneously; (iii) a simple computation process that can be easily programmed into a spreadsheet; and (iv) the performance measures of all alternatives on attributes can be visualized on a polyhedron, at least for any two dimensions. As a consequence, the proposed multicriteria analysis method is of practical use in solving real multicriteria analysis decision problems.

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