

Prediction of Bias-Ply Tire Contact Area Based on Overall Unloaded Diameter, Inflation Pressure and Vertical Load Using Linear Regression Model

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Abstract: This study was conducted to predict contact area (A) of bias-ply tire based on overall unloaded diameter (d), inflation pressure (P) and vertical load (W) using linear regression model. For this purpose, contact area of four bias-ply tires with different overall unloaded diameters was measured at three levels of inflation pressure and four levels of vertical load. Results of contact area measurement for bias-ply tires No. 1, 2 and 3 were utilized to determine regression model and three-variable linear regression model $A_p = 155.24 - 1.2587 d - 2108.6 P + 33.429 W$ with $R^2 = 0.907$ was obtained. Also, results of contact area measurement for bias-ply tire No. 4 were used to verify model. The paired samples t-test results indicated that the contact area values predicted by model were more/less than the contact area values measured by test apparatus. To check the discrepancies between the contact area values predicted by model with the contact area values measured by test apparatus, RMSE and MRPD were calculated. The amounts of RMSE and MRPD were 11.7 cm² and 11.7%, respectively. Corrigible amounts of RMSE and MRPD confirmed that the three-variable linear regression model may be used to predict contact area of bias-ply tire based on overall unloaded diameter, inflation pressure and vertical load. On the other hand, to calculate actual contact area values or contact area values measured by test apparatus (A_M) based on contact area values predicted by model (A_p) the linear regression model $A_M = 0.920 A_p - 2.194$ with $R^2 = 0.916$ can be strongly recommended.

Key words: Bias-ply tire • Contact area • Overall unloaded diameter • Inflation pressure • Vertical load
• Modeling • Prediction

INTRODUCTION

A rule of thumb which can be used for estimation of tire contact area is shown by equation 1 [1]:

$$A = bL \quad (1)$$

where:

A = Contact area (m²)
b = Section width (m)
L = Contact length (m)

Wong [2] and Bekker [3] gave an approximate method for calculating contact length as equation 2:

$$L = 2(d\delta - \delta^2)^{0.5} \quad (2)$$

where:

d = Overall unloaded diameter (m)
 δ = Deflection (m)

Contact area is a key parameter and many equations have been developed based on it to evaluate the tractive performance of bias-ply and radial-ply tires operating in

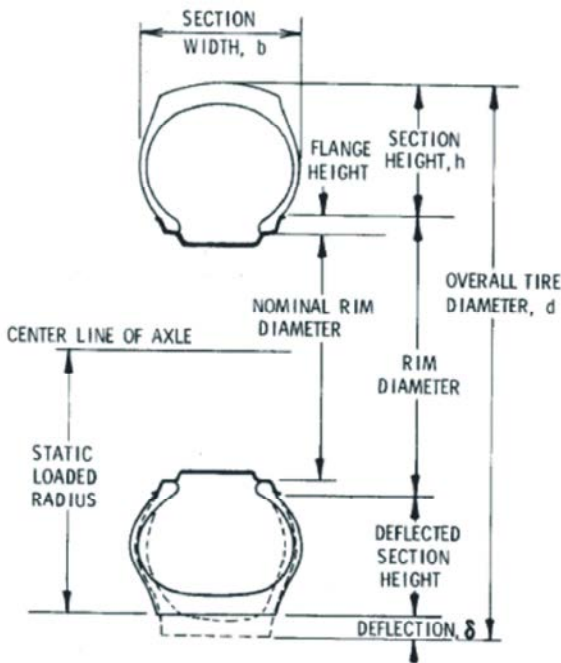


Fig. 1: Tire dimensions, adapted from Brixius [4]

cohesive-frictional soils. Gross traction, motion resistance, net traction and tractive efficiency are predicted as a function of soil strength, tire load, tire slip, tire size, tire deflection and tire contact area [4].

Fig. 1 shows the tire dimensions (b , d and δ) used. The tire dimensions can be obtained from tire data book or by measuring the tire [4]. The section width (b) is the first number in a tire size designation (i.e., nominally 18.4 inches for an 18.4-38 tire). The overall unloaded diameter (d) can be obtained from the tire data handbooks available from off-road tire manufacturers. The tire deflection (δ) on a hard surface is equal to $d/2$ minus the measured static loaded radius. The static loaded radius for the tire's rated load and inflation pressure is also standard tire data from the tire data handbooks. It can also be obtained by measuring the tire.

As contact area for a given tire size, inflation pressure and vertical load are significantly different between bias-ply and radial-ply tires, this study was conducted to predict contact area (A) of bias-ply tire based on overall unloaded diameter (d), inflation pressure (P) and vertical load (W) using linear regression model.

MATERIALS AND METHODS

Tire Contact Area Measurement Apparatus: A tire contact area measurement apparatus (Fig. 2) was designed and constructed to measure contact area of tires with



Fig. 2: Tire contact area measurement apparatus

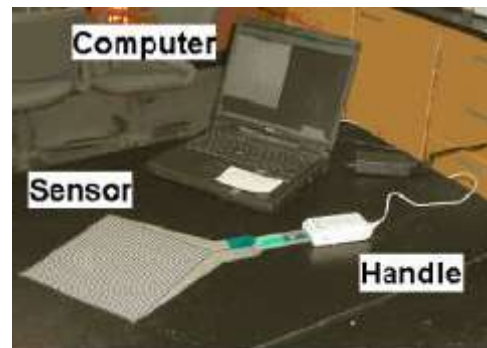


Fig. 3: Contact area measurement system, i.e. tekscan sensor, tekscan USB handle and computer equipped with I-Scan software, adapted from Anderson [5]

different sizes at diverse levels of inflation pressure and vertical load. The contact area measurement system (Fig. 3) consisted of tekscan sensor (Fig. 4), tekscan USB handle and computer equipped with I-Scan software (Fig. 5).

Experimental Procedure: Contact area of four bias-ply tires with different dimensions was measured at three levels of inflation pressure and four levels of vertical load. The dimensions of four bias-ply tires are given in Table 1. Results of contact area measurement for bias-ply tires No. 1, 2 and 3 (Tables 2, 3 and 4) were utilized to determine three-variable linear regression models and results of contact area measurement for bias-ply tire No. 4 (Table 5) were used to verify selected model.

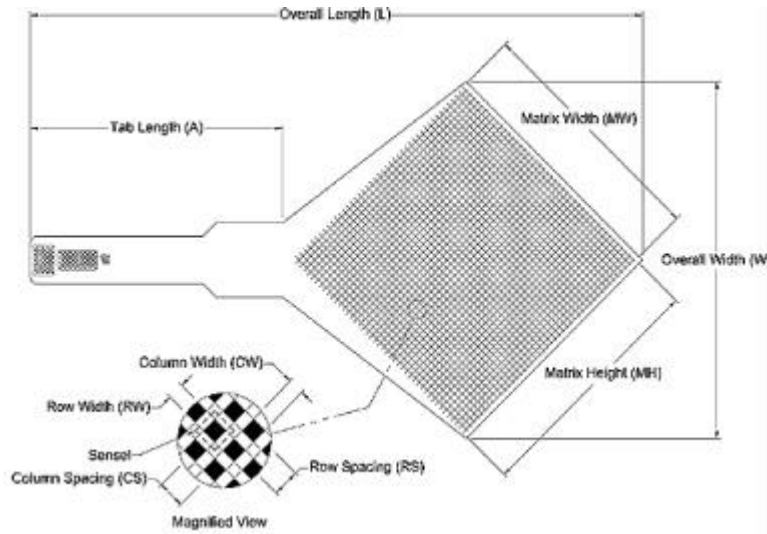


Fig. 4: Tekscan sensor, adapted from Tekscan [6]

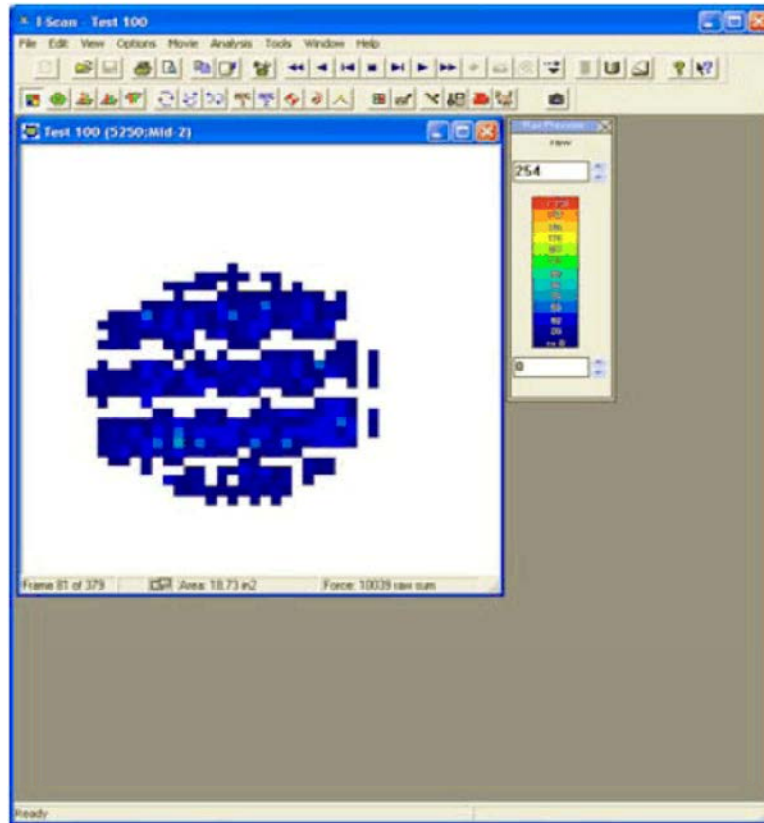


Fig. 5: I-Scan software screenshot for tire contact area measurement

Table 1: Dimensions of the four bias-ply tires used in this study

Tire No.	Section width b (cm)	Overall unloaded diameter d (cm)
1	5.00	33.00
2	6.00	35.56
3	16.5	33.00
4	15.0	50.00

Table 2: Overall unloaded diameter, inflation pressure, vertical load and contact area (three replications) for bias-ply tire No. 1

Tire No.	Overall unloaded diameter d (cm)	Inflation pressure P (MPa)	Vertical load W (kN)	Contact area A (cm ²)		
				R ₁	R ₂	R ₃
1	33.00	0.025	1.67	120.1	118.9	119.6
			2.02	129.8	129.6	129.9
			2.42	139.7	139.6	140.1
			2.92	158.0	159.3	160.1
		0.030	1.67	109.9	108.6	109.9
			2.02	119.9	119.5	120.5
			2.42	132.7	133.0	132.4
			2.92	150.6	150.4	150.0
		0.035	1.67	100.8	100.9	100.8
			2.02	108.8	106.9	107.1
			2.42	125.6	125.0	125.6
			2.92	133.0	134.8	134.9

Table 3: Overall unloaded diameter, inflation pressure, vertical load and contact area (three replications) for bias-ply tire No. 2

Tire No.	Overall unloaded diameter d (cm)	Inflation pressure P (MPa)	Vertical load W (kN)	Contact area A (cm ²)		
				R ₁	R ₂	R ₃
2	35.56	0.025	1.67	118.4	118.0	119.0
			2.02	126.0	126.7	126.0
			2.42	133.0	133.8	134.0
			2.92	153.9	153.3	153.7
		0.030	1.67	105.7	105.7	106.0
			2.02	112.8	113.5	113.0
			2.42	121.7	123.0	126.0
			2.92	137.0	136.1	136.7
		0.035	1.67	104.1	105.0	105.0
			2.02	111.0	110.9	111.0
			2.42	118.0	117.0	117.5
			2.92	127.0	128.2	129.0

Table 4: Overall unloaded diameter, inflation pressure, vertical load and contact area (three replications) for bias-ply tire No. 3

Tire No.	Overall unloaded diameter d (cm)	Inflation pressure P (MPa)	Vertical load W (kN)	Contact area A (cm ²)		
				R ₁	R ₂	R ₃
3	33.00	0.025	1.67	101.5	102.2	101.0
			2.02	128.0	127.8	126.0
			2.42	152.5	154.0	154.5
			2.92	165.8	166.0	165.9
		0.030	1.67	94.60	94.00	94.80
			2.02	114.9	115.5	115.6
			2.42	135.4	136.0	135.4
			2.92	151.5	151.7	152.0
		0.035	1.67	91.00	89.90	90.90
			2.02	100.0	101.1	100.5
			2.42	117.9	118.0	117.5
			2.92	139.9	140.2	140.0

Table 5: Overall unloaded diameter, inflation pressure, vertical load and contact area (three replications) for bias-ply tire No. 4

Tire No.	Overall unloaded diameter d (cm)	Inflation pressure P (MPa)	Vertical load W (kN)	Contact area A (cm ²)		
				R ₁	R ₂	R ₃
4	50.00	0.025	1.67	77.80	78.00	77.40
			2.02	94.00	93.00	94.00
			2.42	103.7	103.6	103.7
			2.92	123.0	123.7	124.0
	0.030	1.67	70.10	69.00	69.90	
		2.02	86.80	86.60	85.00	
		2.42	102.0	101.5	101.7	
		2.92	113.3	112.7	113.5	
	0.035	1.67	66.00	66.00	65.90	
		2.02	80.00	80.20	80.10	
		2.42	100.1	99.90	99.80	
		2.92	109.9	109.8	110.0	

Regression Model: A typical three-variable linear regression model is shown in equation 3:

$$Y = C_0 + C_1X_1 + C_2X_2 + C_3X_3 \tag{3}$$

where:

Y = Dependent variable, for example contact area of bias-ply tire

X₁, X₂, X₃ = Independent variables, for example overall unloaded diameter, inflation pressure and vertical load, respectively

C₀, C₁, C₂, C₃ = Regression coefficients

In order to predict contact area of bias-ply tire based on overall unloaded diameter, inflation pressure and vertical load, a three-variable linear regression model was suggested and all the data were subjected to regression analysis using the Microsoft Excel 2007.

Statistical Analysis: A paired samples t-test was used to compare the contact area values predicted by model with the contact area values measured by test apparatus. Also, to check the discrepancies between the contact area values predicted by model with the contact area values measured by test apparatus, root mean squared error (RSME) and mean relative percentage deviation (MRPD) were calculated using the equations 4 and 5, respectively [7-14]:

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (A_{Mi} - A_{Pi})^2}{n}} \tag{4}$$

where:

RMSE = Root mean squared error (cm²)

A_{mi} = Contact area measured by tire contact area measurement apparatus (cm²)

A_{pi} = Contact area predicted by three-variable linear regression model (cm²)

$$MRPD = \frac{100 \times \sum_{i=1}^n \frac{|A_{Mi} - A_{Pi}|}{A_{Mi}}}{n} \tag{5}$$

where:

MRPD = Mean relative percentage deviation, %

RESULTS AND DISCUSSION

Three-variable linear regression model, p-value of independent variables and coefficient of determination (R²) of the model are shown in Table 6. In this model contact area of bias-ply tire can be predicted as a function of overall unloaded diameter (d), inflation pressure (P) and vertical load (W). The p-value of independent variables (d, P and W) and R² of the model were 0.008025, 2.03E-28, 6.56E-50 and 0.907, respectively. Based on the statistical results, the three-variable linear regression model was initially accepted, which is given by equation 6:

$$A_p = 155.24 - 1.2587 d - 2108.6 P + 33.429 W \tag{6}$$

Contact area of bias-ply tire No. 4 was then predicted at three levels of inflation pressure and four levels of vertical load using the three-variable linear regression model. The contact area values predicted by model were compared with the contact area values measured by test

Table 6: Three-variable linear regression model, p-value of independent variables and coefficient of determination (R²)

Model	p-value			R ²
	d	P	W	
A = 155.24 - 1.2587 d - 2108.6 P + 33.429 W	0.008025	2.03E-28	6.56E-50	0.907

Table 7: Overall unloaded diameter, inflation pressure, vertical load and contact area for bias-ply tire No. 4 used in evaluating three-variable linear regression model

Overall unloaded diameter d (cm)	Inflation pressure P (MPa)	Vertical load W (kN)	Contact area A (cm ²)	
			Measured by test apparatus	Predicted by model
50	0.025	1.67	77.73	95.22
		2.02	93.66	107.1
		2.42	103.7	120.4
		2.92	123.6	137.1
	0.030	1.67	69.66	84.68
		2.02	86.66	96.51
		2.42	102.0	109.9
		2.92	113.2	126.6
	0.035	1.67	66.96	74.13
		2.02	80.10	85.97
		2.42	99.33	99.34
		2.92	109.9	116.1

Table 8: Paired samples t-test analyses on comparing contact area determination methods

Determination methods	Average difference (cm ²)	Standard deviation of difference (cm ²)	p-value	95% confidence intervals for the difference in means (cm ²)
Test apparatus vs. model	10.5	5.37	1.0000	7.09, 13.9

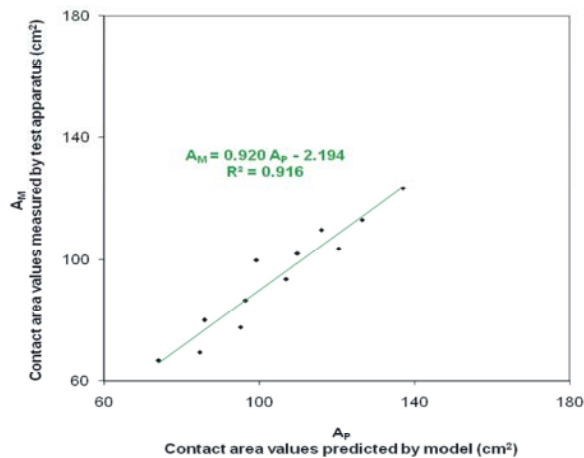


Fig. 6: Curve of contact area values measured by test apparatus (A_M) based on contact area values predicted by three-variable linear regression model (A_p) for bias-ply tire No. 4

cm² (95% confidence interval for difference in means: 7.09 cm² and 13.9 cm²; p-value = 1.0000). The standard deviation of the contact area difference was 5.37 cm² (Table 8). To check the discrepancies between the contact area values predicted by model with the contact area values measured by test apparatus, RMSE and MRPD were calculated. The amounts of RMSE and MRPD were only 11.7 cm² and 11.7%, respectively. Corrigible amounts of RMSE and MRPD confirmed that the three-variable linear regression model $A_p = 155.24 - 1.2587 d - 2108.6 P + 33.429 W$ with $R^2 = 0.907$ may be used to predict contact area of bias-ply tire based on overall unloaded diameter, inflation pressure and vertical load. On the other hand, as it is indicated in Fig. 6, our attempts to relate contact area values predicted by three-variable linear regression model (A_M) to contact area values measured by test apparatus (A_p) using a linear equation resulted in very good agreements ($R^2 = 0.916$) as equation 7:

$$A_M = 0.920 A_p - 2.194 \tag{7}$$

apparatus and are shown in Table 7. The paired samples t-test results indicated that the contact area values predicted by model were more/less than the contact area values measured by test apparatus. The average contact area difference between two methods was 10.5

It means that actual or measured contact area (A_M) can be computed in two steps. At first step predicted contact area (A_p) can be calculated based on overall

unloaded diameter (d), inflation pressure (P) and vertical load (W) using the three-variable linear regression model, i.e. equation 6. Second step is calculating actual or measured contact area (A_M) based on predicted contact area (A_p) using the linear model, i.e. equation 7.

CONCLUSION

It can be concluded that actual or measured contact area (A_M) of bias-ply tire can be computed in two easy steps. At first step, predicted contact area (A_p) can be calculated based on overall unloaded diameter (d), inflation pressure (P) and vertical load (W) using the three-variable linear regression model $A_p = 155.24 - 1.2587 d - 2108.6 P + 33.429 W$ with $R^2 = 0.907$. Second step is calculating actual or measured contact area (A_M) based on predicted contact area (A_p) using the linear equation $A_M = 0.920 A_p - 2.194$ with $R^2 = 0.916$.

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