Temperature Distribution in Long Porous Fins in Natural Convection Condition

Seyfolah Saedodin and Siamak Sadeghi

Department of Mechanical Engineering, Science and Research Branch, Islamic Azad University, Semnan, Iran

Abstract: This work introduces a simple method of analysis to study the performance of porous and solid fins in a natural convection environment. The method is based on using energy balance and Darcy’s model to formulate the heat transfer equation. This study is based on long fin tip. The porous fin allows the flow infiltrate through it. The theory section addressed the derived governing equation. The effects of the porosity parameter \( S_p \), different convection parameter \( m \) on the dimensionless temperature distribution, compare the solid fin with porous fin and heat transfer rate are discussed. It is found that the heat transfer rate from porous fin could exceed that of a solid fin. Kind of fins to used in research are Titanium, Ni and Steel and heat transfer convectivity coefficients are \( h_{wa} \) and \( h_{water} \).

Keywords: Porous fin · Solid fin · Darcy’s model · Temperature distribution · Heat transfer · Natural convection

INTRODUCTION

High rate of heat transfer with reduced size and cost is in demand for a number of engineering applications such as heat exchangers, economizers, superheaters, conventional furnaces, gas turbines etc. Some engineering applications also require lighter fin with higher rate of heat transfer where they use high thermal conductivity metals in applications such as airplane and motorcycle applications. However, cost of high thermal conductivity metals is also high. Thus, the enhancement of heat transfer can be achieved by increasing the heat transfer rate and decreasing the size and cost of fin. The major heat transfer from surface to surrounding fluid takes place by convection process. Therefore, the rate of heat transfer depends mainly on the following three parameters:

- Heat transfer coefficient (\( h \))
- Surface area available
- Temperature difference between surface and surrounding fluid

The value of ‘\( h \)’ depends mainly upon the properties of surrounding fluid and average velocity of fluid over the surface. Thus, it can be assumed as a constant in certain cases. Most of the times, the temperature difference is prescribed in a given application. Fins are frequently used in many heat transfer applications. Meyer [1] in his famous book with a simple manner describes the temperature distribution in conventional fins. But in the recent years, porous fins consider as a potential field for increasing heat transfer. The basic philosophy behind using porous fins is to increase the effective area through which heat converted to ambient fluid. Extensive research has been done in this area and many references are available especially for heat transfer in porous fins.

Review of the Literature: Described below are a few papers relevant to the study described herein.

Kiwan and Al-Nmr [2] conducted thermal analysis of natural convection porous fins. They grouped all the geometric and flow parameters that influence the temperature distribution in to one parameter called \( S_p \). Three cases of fin types were considered: the infinite fin, finite fin with insulated tip and finite fin with...

**Governing Equations:** As shown in Fig. 1 a cylindrical fin profile is considered. The dimensional of the fin are length L and the radius R. the cross-sectional area of the fin is constant.

For the porous fin, Due to this fact that the fin being porous, It allow for the flow to infiltrate through it. In order to simplify the solution, the following assumptions are considered: (1) the porous medium is homogeneous, isotropic and saturated with a single-phase fluid, (2) both the fluid and the solid matrix have constant physical properties, (3) the surface radiant exchanges are neglected, (4) the solid matrix and the fluid are assumed to be at local thermal equilibrium with each other, (5) the temperature inside the fin is only function of $x$, (6) no temperature variation across the fin thickness and (7) the interactions between the porous medium and the clear fluid can be simulated by the Darcy formulation.

Apply an energy balance to the slice segment of the fin of thickness $\Delta x$, shown in Figure 1, requires that

$$q(x) - q(x + \Delta x) = mc_p(T(x) - T_\alpha) + h(T(x) - T_\alpha)$$

(1)

In the right hand of equation (1), the first term represents the heat transfer lost to the fluid passing through the porous media and the second terms represent the natural convection around the fin. This fluid is induced by the buoyancy force created due to the temperature difference between the fin and the surroundings. It should be noted that Equation (1) assumes that the fluid enters the fin at $T_i$ and leaves at $T(x)$.

The mass flow rate of the fluid passing through the porous material can be written as,

$$m = 2\pi R \Delta x \rho v_w$$

(2)

The flow in the porous medium shall be considered next to account for the value of $v_w$. Referring to assumption (7) above, Darcy’s model gives,

$$v_w = \frac{\alpha k \beta}{\nu}(T(x) - T_\alpha)$$

(3)

By substitution of equations (2) and (3) into equation (1), the result is:

$$\frac{q(x) - q(x + \Delta x)}{\Delta x} = \frac{2\pi \rho c_p g k \beta}{\nu}(T(x) - T_\alpha)^2 + h(T(x) - T_\alpha)$$

(4)

If $\Delta x \to 0$, equation (4) transfer to
According to Fourier’s law of conduction,

$$q = -k_{eff} \frac{dT}{dx}$$  \hspace{1cm} (6)

In this equation, $A$ is the cross-sectional area of the fin ($A=\pi R^2$) and $k_{eff}$ is the effective thermal conductivity of the porous fin. Substitution Equation (6) in to Equation (5) yields,

$$\frac{d^2T}{dx^2} - \frac{2\rho c_p gk_B}{R_k A_{eff}} (T(x) - T_{inf})^2 - \frac{h_p}{k_{eff} A} (T(x) - T_{inf}) = 0$$  \hspace{1cm} (7)

By introducing the non-dimensional temperature function, $\theta = \frac{T(x) - T_{inf}}{T_b - T_{inf}}$ into equation (7) becomes

$$\frac{d^2\theta}{dx^2} - S_h \theta^2 - m^2 \theta = 0$$  \hspace{1cm} (8)

The constants, $S_h = \frac{2\rho c_p gk_B}{R_k A_{eff}}$ and $m^2 = \frac{h_p}{k_{eff} A}$ input all the geometric and flow parameters that influence the solution of the problem into definite parameters. Here $S_h$ is a porous parameter that indicates the effect of the porous medium as well as buoyancy effect so higher value of $S_h$ indicates higher permeability of the porous medium or higher buoyancy forces and $m$ is a convection parameter that indicate the effect of surface convecting of the fin.

For solid fin, by applying energy balance equation at steady state condition we have:

$$\frac{d^2\theta}{dx^2} - m^2 \theta = 0$$  \hspace{1cm} (9)

It is clear that in solid fins we would not have $S_h$.

Equation (8) represents an ordinary nonlinear second-order differential equation. To solve this equation we need two boundary condition. One boundary condition here is that the temperature at the base of the fin is $T_b$. Then:

$$\theta(0) = 1$$  \hspace{1cm} (10)

The second boundary condition depends on the condition of the fin at the tip. The case that considered in this study is long fin tip. In this case, the fin length is considered to be very long such that its tip temperature is essentially equal to the temperature of the surrounding fluids. The second boundary condition can therefore be written as

$$\theta(x \rightarrow \infty) = 0$$  \hspace{1cm} (11)

The governing equation is solved numerically using the Runge–Kutta fourth-order method. Depending on the tip condition of the fin, we have three different types of cases, that in this research we only study long fin.

Values of $h$, $k$ and constants to used in research are:

$$h_{air} = 25 \frac{w}{m^2k}, h_{water} = 100 \frac{w}{m^2k}, k_{tt} = 7.44 \frac{w}{mk},$$

$$k_{st} = 16.27 \frac{w}{mk}, k_{Ni} = 91.74 \frac{w}{mk},$$

$L = 100m, R = 0.5m, k_s = 204 \frac{w}{mk}, h_s = 15 \frac{w}{m^2k}$

Equation (8) along with the boundary conditions given by Equations (10) and (11) are solved for several values of $S_h$ and $m$. Fig. 2 shows the variation of dimensionless temperature distribution with the axial distance along the fin when the value of $S_h$ is varying and of $m$ was kept constant. From fig. 2 we can see that the value of dimensionless temperature decreases along the fin length and it is clear that the fin tip reaches the surrounding temperature faster as the value of $S_h$ increases.

Figs. 3 and 4 show the results for the effect of variation of $m$ by varying the values of $k$ on dimensionless temperature distribution. The values of $m$ were varied from 0.26, 0.62 and 0.92 with $h_s$ and 1.44, 2.48 and 3.67 with $h_{water}$ while keeping $S_h$ constant. From Figs. 3 and 4 it is observed that by increase $k$, the convection parameter $m$ will increase. Hence, by increase of $m$ and $S_h=1$, the temperature distribution rapidly decreasing process to go and figure quickly reaches the surrounding temperature.

From Fig. 5, by comparison Figs. 3 and 4, we can see that by increase $h$, the convection parameter $m$ will increase. Hence, by increase of $m$ and $S_h=1$, the
Fig. 2: The distribution of the axial non-dimensional temperature along the infinite fin for different values of $S_w$.

Fig. 3: The distribution of the axial non-dimensional temperature along the infinite fin for different values of $m$ with $h_w=25$.

Fig. 4: The distribution of the axial non-dimensional temperature along the infinite fin for different values of $m$ with $h_w=100$.

Fig. 5: The distribution of the axial non-dimensional temperature along the infinite fin for different values of $m$.

Fig. 6: The distribution of the axial non-dimensional temperature along the infinite fin for different values of $m$ with $h_w=25$.

And now for solid fins, equation (9) was solved using this two boundary conditions given by equations (10) and (11) for different values of $m$. Fig. 6 shows the results for the effect of variation of $m$ by varying the values of $k$ on dimensionless temperature distribution. The values of $m$ were varied from 0.26, 0.62 and 0.92 with $h_w$ for solid fin. From Fig. 6 it is observed that by increase $k$, the convection parameter will increase and the fin cool down rapidly.

Figs. 7, 8 and 9 comparing temperature distribution in porous and solid fins. From this figures observed that by $h_w=25\text{w/m}^2\text{K}$ and different $k$ in according to porosity parameter, velocity of temperature decrease in porous fin was more than solid fin and fin quickly reaches the surrounding temperature and the fin cool down rapidly.

In order to make a comparison between the heat transfer from a porous fin with that from a solid fin, the ratio of heat transfer rate between the two fins are given by
Fig. 7: The distribution of the axial non-dimensional temperature along the infinite fin for different values of m.

Fig. 8: The distribution of the axial non-dimensional temperature along the infinite fin for different values of m.

\[ \frac{q_p}{q_s} = -k_{eff} \frac{A_p}{A_s} \frac{dT}{dx} \bigg|_{x=0} \]  \hspace{1cm} (12)

where, \( q_p \) is the maximum possible heat transfer rate obtained using porous fin and \( q_s \) is the maximum possible heat transfer rate obtained using solid fin. Writing the above equation in terms of the dimensionless temperature and axial distance, yields.

\[ \frac{q_p}{q_s} = -\frac{A_p}{A_s} \frac{k_{eff}}{hL} \frac{d\Theta}{dx} \bigg|_{X=0} \]  \hspace{1cm} (13)

Fig. 10 shows porous fin to solid fin heat transfer rate with convection and without convection. The figure also shows the variation ratio of porous fin to solid fin heat transfer rate with Kr. It is clear that variation of ratio of porous to solid fin heat transfer rate increases as Kr increases for both the cases. However comparing both the cases of with convection and without convection, heat transfer rate is more with convection than without convection as shown in the figure.

Fig. 9: The distribution of the axial non-dimensional temperature along the infinite fin for different values of m.

Fig. 10: The variation of the ratio of porous fin to solid fin heat transfer rate with Kr.

CONCLUSION

This work introduces a simple method to analyze the performance of a porous fin and solid fin. It is found that the problem of heat transfer through the porous fin is governed by a second order nonlinear-ordinary-differential equation. It is also found that all the geometric and flow parameters that influence the temperature distribution are grouped into two parameters called \( S_h \) and \( m \). This thermal analysis was performed on one type of fin case: the infinite fin. The effect of these two parameters were investigated. Is found that increasing \( S_h \) by increasing either Da or Ra increases the heat transfer from the fin. Also increasing \( m \), increases the heat transfer from the fin. Finally, The ratio of heat transfer rate for porous fin to solid fin is compared for both the cases of with convection and without convection. It is found from this analysis that with convection transfers more heat than that dissipates heat without convection.

Nomenclature:

- \( c_p \): Specific heat
- \( Da \): Darcy number, \( k/\mu \)
Gravity constant

Grashoff number, \( Gr = \frac{g\beta(l_h - T_\infty)}{v^2} \)

Thermal conductivity

Thermal conductivity ratio, \( K_r = \frac{k_{eff}}{k_f} \)

Permeability of the porous fin

Nusselt number, \( Nu = \frac{hL}{k_f} \)

Convection parameter

Prandtl number, \( Pr = \frac{\nu}{\alpha} \)

Heat transfer rate

Rayleigh number, \( Ra = Gr.Pr \)

Porous parameter

Temperature at any point

Temperature at the fin base

Velocity of the fluid passing through the fin at any point

Radius of the fin

Axial coordinate

Dimensionless axial coordinate, \( \gamma_L \)

Perimeter of the fin

Greek Symbols:

Thermal diffusivity

Coefficient of volumetric thermal expansion

Temperature difference

Porosity or void ratio

Dimensionless temperature, \( \theta = \frac{T(x) - T_{\infty}}{T_h - T_{\infty}} \)

Base temperature difference, \( (T_h - T_{\infty}) \)

Dynamic viscosity

Kinematics viscosity

Density

Subscripts:

Solid properties

Fluid properties

Porous properties

REFERENCES


