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# Mathematical Modeling, Moisture Diffusion and Energy Consumption in Thin Layer Drying of Alfalfa

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**Abstract:** The present study investigated the influence of microwave power on the drying kinetics of alfalfa during microwave dehydration at 180, 360, 540, 720 and 900 W. The Midilli model gave better prediction than the Page's model and satisfactorily described drying characteristics of alfalfa. Results indicated that drying took place in the falling rate period. It was found that the overall effective diffusion coefficient and drying constant varied from $3.10\times10^{-9}$ to $8.21\times10^{-9}$ $m^2/s$ and 0.0142 to 0.210/min, respectively. An Arrhenius relation with an activation energy value of 6.595 W/g expressed effect of microwave power on the diffusivity. The least specific energy consumption (3.87 MJ/kg water) was at microwave power of 180 W and the highest (9.98 MJ/kg water) was at 900 W.

**Key words:** Drying kinetics · Modeling · Effective diffusion · Alfalfa · Energy consumption

## INTRODUCTION

Alfalfa is one of the most important forage that is often called "Queen of forages" because it provides high levels of energy, protein and nutrients for livestock. Fresh alfalfa must be dried to moisture contents of 8~10% for safely stored and baled [1, 2].

Most of the alfalfa is dehydrated by traditional technique, open sunlight in the field. This process is very long, making many active components lost. Furthermore, the rehydration time for the traditional solar-dried alfalfa is form two to six days. For thesis reasons, several researchers have studied the drying to better control the final moisture and quality of alfalfa [1-6]. Hot air method has a number of disadvantages such as very long-lasting drying period and high energy consumption. In addition, improving drying processes by reducing energy consumption and providing high quality with minimal increase in economic input has become the goal of modern drying.

Most of the above studies examined on hot air, sun and solar drying kinetics of alfalfa. But, limited study concerning microwave drying kinetics and consumption energy of alfalfa has been performed up to now. Therefore, the objectives of this study was to study microwave dehydration characteristic of alfalfa and discuss the influence of drying power on drying time, moisture diffusion and energy consumption and to estimate the activation energy for microwave drying of alfalfa.

## MATERIALS AND METHODS

**Sample Preparation:** Alfalfa samples were harvested from an alfalfa farm in the Ilam, Iran and were stored in the refrigerator at temperature of 4±1°C until the experiments were carried out. The initial moisture content of the alfalfa samples was determined by using standard oven method at 103±1°C. These experiments were replicated thrice to obtain a reasonable average. Average moisture content was found to be 81±2 % w.b.

**Experimental Equipment and Procedure:** Drying studies were carried out with a domestic digital microwave oven (M945, Samsung Electronics Ins) with the technical feature of 230V, 50 Hz and 1000 W. The oven had the dimensions of 51×44×31 cm with a rotating glass plate having 300 mm in diameter. The oven has a fan for air flow in drying chamber and cooling of magnetron. The moisture from drying chamber was removed with this fan by passing it through the openings on the right side of the oven wall to the outer atmosphere. Furthermore, it was able to work at various microwave outputs and had a digital control facility into adjust the processing time.

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In order to carry out the drying experiments, 30 g of alfalfa samples were weighed using a digital balance (Sartorius GP3202, Germany) with a precision of 0.01 g. Alfalfa samples were removed from the oven periodically (every 30 s) during the drying period and the moisture loss was determined by weighing the plate using digital balance. Five different microwave power outputs, 180, 360, 540, 720 and 900 W, were used to dry the samples and three replications were performed at each of these power outputs. Drying was carried out until the final moisture content reaches to a level less than 8% (w.b.).

**Modeling of Drying Process:** The moisture ratio of alfalfa samples during the thin layer drying experiments was calculated using the following equation:

$$MR = \frac{X_t - X_0}{X_0 - X_e} \quad (1)$$

where MR is the moisture ratio (dimensionless), $X_t$ is the moisture content at drying time t (% d.b.) and $X_0$ is the initial moisture content (% d.b.). The values of $X_e$, are relatively small compared to $X_t$ or $X_0$. Thus, Eq. (1) can be reduced to $MR=X_t/X_0$ [7].

The drying rate of samples was calculated by using Eq. (2):

$$DR = \frac{X_{t+\Delta t} - X_t}{\Delta t} \quad (2)$$

where $X_{t+\Delta t}$ is moisture content at time t+Δt (% d.b.), t is the time (min) and DR is the drying rate (% d.b./min).

The experimental sets of (MR, t) were fitted to two empirical model from the literature, using IBM SPSS Statistics 19. (SPSS, Inc.).

Page's model [8]:

$$MR = \exp\left(-kt^n\right) \quad (3)$$

Midilli model [9]:

$$MR = a\exp\left(-kt^n\right) + bt \quad (4)$$

where k is the drying rate constant (1/min) and a, b and n are equation constants model.

There are several criteria such as coefficient of determination ($R^2$) and chi-square ($\chi$) are used to determine the quality of the fit. The model is said to be good if $R^2$ value is high and $\chi^2$ value is low. These parameters are defined as follows [19]:

$$R^2 = 1 - \left( \frac{\sum_{i=1}^{N}\left(MR_{\exp,i} - MR_{pre,i}\right)^2}{\sum_{i=1}^{N}\left(\overline{MR}_{\exp} - MR_{pre,i}\right)^2} \right) \quad (5)$$

$$\chi^2 = \frac{\sum_{i=1}^{N}\left(MR_{\exp,i} - MR_{pre,i}\right)^2}{N - z} \quad (6)$$

where $MR_{pre,i}$ is the ith predicted moisture ratio, $MR_{exp,i}$ is the ith experimental moisture ratio, N is the number of observations and z is the number of constants in drying model.

**Moisture Diffusivity:** The moisture and/or vapour migration during drying period is controlled by diffusion. The rate of moisture movement is described by an effective diffusivity a lumped value. Fick's second law of diffusion is used to describe a moisture diffusion process. The solution of Fick's second law in slab geometry, with the assumptions of moisture migration being by diffusion, negligible shrinkage and constant diffusion coefficients [20] was as follows:

$$MR = \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\exp\left(-(2n+1)\frac{D_{eff}\pi^2 t}{4L^2}\right) \quad (7)$$

where $D_{eff}$ is the effective diffusivity ($m^2/s$) and L is the L is the half-thickness of samples (m), n is a positive integer. For long drying times, a limiting of Eq. 7 is obtained and expressed in a logarithmic form [19]:

$$\ln(MR) = \ln\left(\frac{8}{\pi^2}\right) - \left(\frac{\pi^2}{4L^2}D_{eff}t\right) \quad (8)$$

From Eq. 8, a plot of ln MR versus drying time gave a straight line with a slope (S) of:

$$S = \frac{\pi^2}{4L^2}D_{eff} \quad (9)$$

**Activation Energy:** Inasmuch as temperature is not precisely measurable inside the microwave drier, the activation energy is found as modified from the revised Arehnious equation. In a first method it is assumed as related to drying kinetic constant rate (k) and the ratio of sample weight to microwave output power (m/p) instead of to air temperature. Then Equation (10) can be effectively used as follows [10]:

$$k = k_0 \exp\left(-\frac{E_a m}{P}\right) \quad (10)$$

In the second method, the correlation between the effective diffusion coefficient and (m/p) is used for calculation of the activation energy [11].

$$D_{eff} = D_0 \exp\left(-\frac{E_a m}{P}\right) \quad (11)$$

where k is the drying rate constant obtained by using best model (1/min), $k_0$ is the pre-exponential constant (1/min), $E_a$ is the activation energy (W/g), m is the mass of raw sample (g) and $D_0$ is the pre-exponential factor ($m^2$/s).

**Energy Consumption:** The energy consumption of microwave could be calculated as follows:

$$Ec = P \times t \quad (12)$$

The specific energy consumption is calculated using Eq. (13) [12].

$$E_s = \frac{P \times t}{m_w} \quad (13)$$

where $E_s$ is the specific energy consumption (J/ kg water); P is the microwave power (W); and $m_w$ is the total mass of evaporated water (kg).

**Statistically Analyze:** The values of drying parameters for three replications were taken for statistical analysis. Analysis of variance (ANOVA) was carried on to study the effect of microwave power on moisture diffusivity, drying time and energy consumption.

## RESULTS AND DISCUSSION

**Experimental Drying Curves:** The moisture content versus time curves for microwave drying of alfalfa samples as influenced by microwave powers are shown in Fig. 1. As the microwave power decreased, the time required to achieve a certain moisture content increased and water loss slowly. On the other hand, mass transfer within the sample was more rapid during higher microwave power heating because more heat was generated within the sample creating a large vapor pressure difference between the centre and the surface of the product due to characteristic microwave volumetric heating. The drying time required to reach the final moisture content of samples were 9, 7.5, 6, 5.5 and 4 min at the drying powers of 180, 360, 540, 720 and 900 W, respectively.

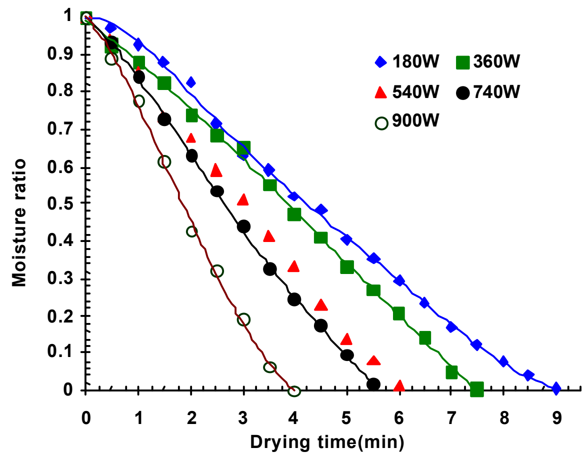


Fig. 1: Influence of microwave power level on the drying curves of alfalfa

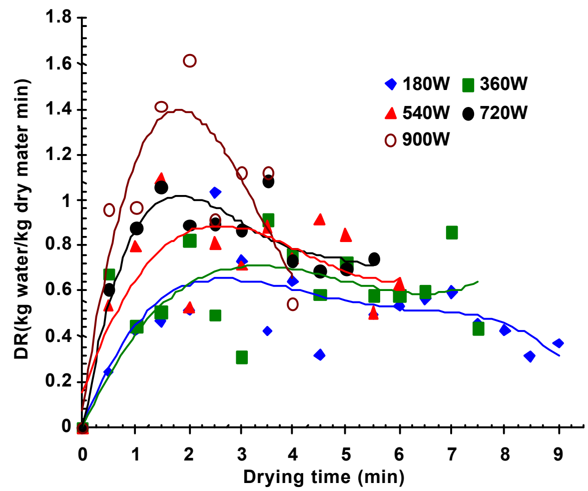


Fig. 2: Drying rates for alfalfa at different microwave powers

Fig. 2 shows the changes in drying rate as a function of drying time at the different powers. It is clear that the moisture content and drying rate decrease continuously with drying time. The drying rate was rapid during the initial period but it became very slow at the last stages during the drying process. The moisture content of the material was very high during the initial phase of the drying which resulted in a higher absorption of microwave power and higher drying rates due to the higher moisture diffusion. As the drying progressed, the loss of moisture in the product caused a decrease in the absorption of microwave power and resulted in a fall in the drying rate. Similar results were reported by Soysal *et al.* [13], Al-Harahsheh *et al.* [14] and Evin [7]. Constant drying rate period was not observed during the drying of alfalfa. This shows that diffusion in dominant physical mechanism governing moisture movement in the samples.

Table 1: Statistical results obtained from Page's and Midilli thin-layer drying models

| P(w) | Model constants | $R^2$ | $\chi^2$ |
|---|---|---|---|
| | Page's | | |
| 900 | k=0.239, n=1.811 | 0.993 | 0.00780 |
| 720 | k=0.143, n=1.675 | 0.993 | 0.00085 |
| 540 | k=0.117, n=1.702 | 0.987 | 0.00159 |
| 360 | k=0.072, n=1.734 | 0.983 | 0.00132 |
| 180 | k=0.058, n=1.746 | 0.992 | 0.00093 |
| | Midilli | | |
| 900 | k=0.210, n=1.564, a=0.994, b=-0.041 | 0.998 | 0.00022 |
| 720 | k=0.127, n=1.366, a=1.002, b=-0.045 | 0.999 | 0.00017 |
| 540 | k=0.084, n=1.279, a=1.003, b= -0.072 | 0.999 | 0.00020 |
| 360 | k=0.040, n=1.389, a=0.990, b=-0.068 | 0.996 | 0.00006 |
| 180 | k=0.0142, n=1.200, a=1.011, b=-0.097 | 0.998 | 0.00033 |

**Evaluation of the Models:** The statistical results from models are summarized in Table 1. The $R^2$ values of two models were all above 0.98. The statistical parameter estimations showed that $R^2$ and $\chi^2$ values were ranged from 0.983 to 0.999 and 0.00006 to 0.00780, respectively. The Midilli model showed the best fitting due to higher value of $R^2$ and lower values of $\chi^2$ and RMSE. To account the effect of the microwave power on the Midilli model, the constants k, a, b and n were regressed against those of drying microwave powers using regression analysis. Based on the regression analysis, the accepted model and their constants are as follows:

$$MR(t, P) = a \exp(-kt^n) + bt \quad k = 3 \times 10^{-6} P^{1.6545} \tag{14}$$

$$n = 6 \times 10^{-9} P^3 - 6 \times 10^{-8} P^2 + 0.0044 P + 0.6816 \tag{15}$$

$$b = 0.03298 \ln(P) - 0.2643 \tag{16}$$

$$a = -6 \times 10^{-10} P^3 + 1 \times 10^{-6} P^2 - 0.0005 P + 1.07 \tag{17}$$

Fig. 3 shows the comparison between experimental moisture ratio at different drying powers and that predicted by the Midilli model. As can be seen, the dots in Fig. 3 are closely banding around at a 45° straight line - a very good agreement between calculated and experimental data, which indicates that the Midilli model could adequately describe the drying behavior of alfalfa samples.

**Moisture Diffusivity:** The variation in ln (MR) and drying time (t) for different powers have been plotted in Fig. 4 to obtain the slope S (Eq. 9) which can give the effective moisture diffusivity ($D_{eff}$). The values of effective moisture diffusivity were calculated using Eq. 9 and are shown in Fig. 5. It was noted that $D_{eff}$ values increased greatly with increasing drying microwave power.

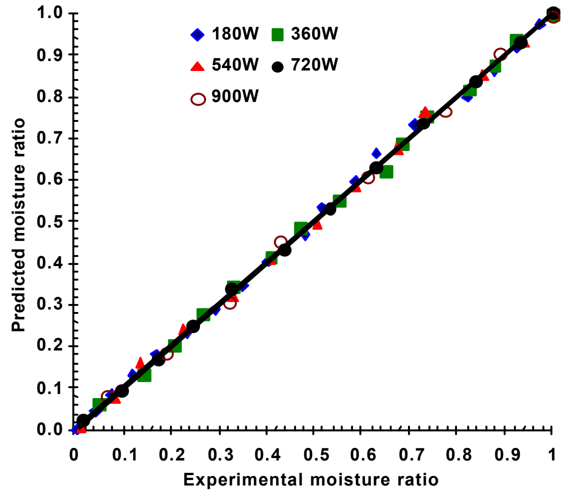


Fig. 3: Experimental and predicted moisture ratio values at different microwave powers for the Midilli model

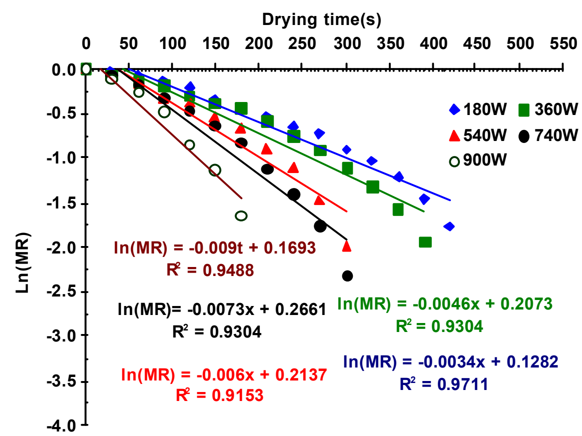


Fig. 4: Variation in ln (MR) and drying time (in seconds) for alfalfa dried at different microwave powers

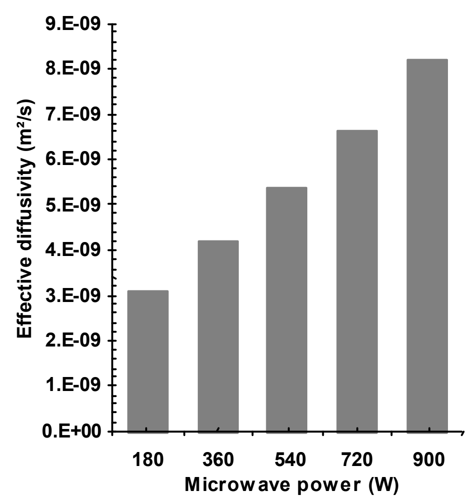


Fig. 5: Variation of effective diffusivity with drying microwave power

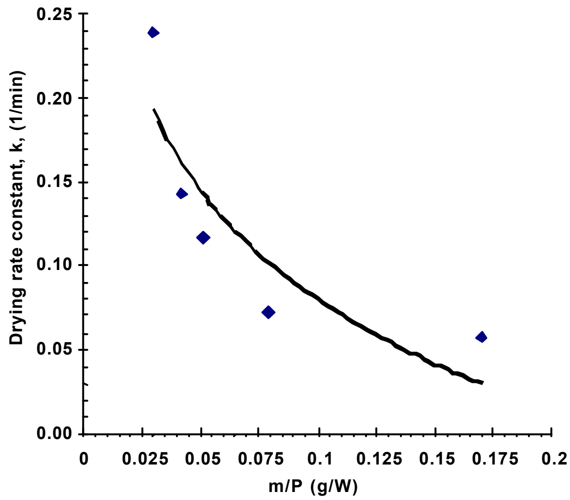


Fig. 6: Variation of drying rate constant with sample weight/microwave operating power for alfalfa

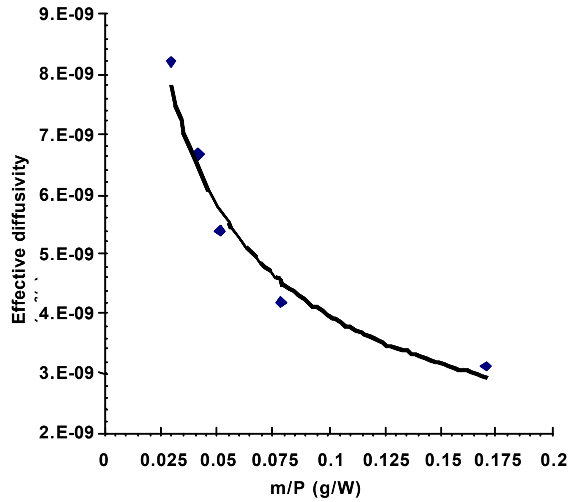


Fig. 7: Relationship between $D_{eff}$ and sample weight/microwave operating power for alfalfa

When samples were dried at higher microwave power, increased heating energy would increase the activity of water molecules leading to higher moisture diffusivity. The effective of diffusivity during drying of alfalfa varied from $3.10\times10^{-9}$ to $8.21\times10^{-9}$ $m^2/s$ in the microwave power range from 180 to 900 W. The values of $D_{eff}$ obtained from this study lie within in general range $10^{-12}$ to $10^{-8}$ $m^2/s$ for drying of food materials. Equation (18) shows the effect of microwave power on $D_{eff}$ of samples with the following coefficients:

$$R^2 = 0.995 \quad D_{eff} = 3\times10^{-12} P + 8\times10^{-10} \qquad (18)$$

The values of $D_{eff}$ are comparable with the reported values of $6.3\times10^{-11}$ to $2.19\times10^{-10}$ $m^2/s$ mentioned for coriander leaves microwave drying in the range of 180–900 W [15]; $3.43\times10^{-11}$ to $1.714\times10^{-10}$ $m^2/s$ for celery leaves microwave drying at 180–900 W [16], $5.5\times10^{-8}$ to $3.5\times10^{-7}$ $m^2/s$ for Gundelia tournefortii microwave drying at 90-800W [7].

**Activation Energy:** Activation energy can be calculated from the (K-m/P) curve (Fig. 6) and Eq. (10). Based on statistical analysis and Page's model coefficients, $k_0$ and $E_a$ values were estimated as 0.208 (1/min) and 8.46 (W/g). The activation energy can be interpreted as the minimum energy required to break solid-water or water-water interactions and to move water molecules from inside to the surface of a solid. A smaller Ea value of a sample indicates that water molecules can move more readily in the solid. Another method for calculation of activation energy is the calculation of the coefficients

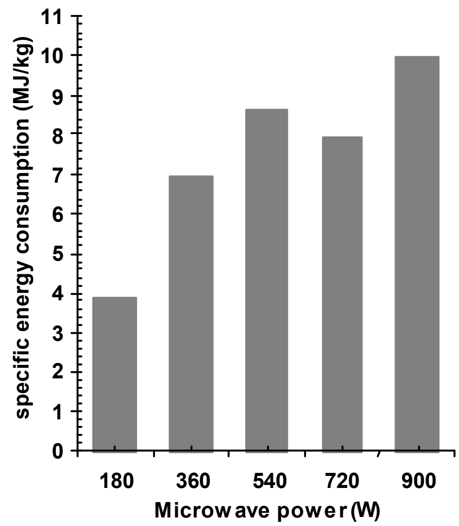


Fig. 8: Specific energy consumption at different microwave power for alfalfa

for Equation (11) from ($D_{eff}$) versus (m/P) curve (Fig. 7), which would yield activation energy value of 6.595 W/g. The values of activation energy are comparable with the reported values of 5.54 W/g mentioned for okra [17], 13.6 W/g for pandanus leaves [18], 12.284 for mint leaves [10], 16.675 W/g and 24.222 W/g for sweet and sour pomegranate, respectively, [11].

**Specific Energy Consumption:** The specific energy consumption ($E_s$) for alfalfa ate selected powers is shown in Fig.8. The $E_s$ to reach the final moisture content were 3.8, 6.96, 8.62, 7.90 and 9.98 MJ/kg at 180, 360, 540, 720 and 900 W, respectively. The best result with regard to energy consumption was obtained for 180 W with 3.8 MJ/kg water. The $E_s$ values increased with the increase in microwave power, because the total drying times required reaching the final moisture content 9, 7.5, 6, 5.5 and 4 min at microwave power range, respectively. Similar trends were also observed by Evin [7] for microwave drying of Gundelia tournefortii L. and Wang *et al*. [19] for microwave drying of apple pomace.

## CONCLUSIONS

In this study, microwave drying of alfalfa was investigated. Drying of alfalfa occurred in the falling rate period; no constant rate period of drying was observed. To explain the drying behaviour of alfalfa two thin-layer drying models (Page's and Midilli models) were applied. The Midilli model showed better fit with high $R^2$ and low $\chi^2$ values. The values of calculated effective diffusivity varied from $3.10\times10^{-9}$ to $8.21\times10^{-9}$ $m^2/s$ over microwave power range. The effective diffusivity increases as microwave power increases. Minimum specific energy consumption (3.8MJ/kg water) was obtained at 180W microwave levels.

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