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Integration of Shewhart and EWMA Control Charts through P-value Approach for Better Detection

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Abstract: P-value is the probability of obtaining a test statistic at least as extreme as the one that was actually observed, assuming that the null hypothesis is true. Considering Statistical Process Control (SPC) as a series of hypothesis tests, the P-value can be used to monitor the states of the process under study through P-value charts. This paper describes P-value approach towards SPC and shows that P-value control charts present better graphical diagrams which are interpreted more easily by users and improve their ability to realize out-of-control processes for sensitive values. Here, it is shown that P-value control charts can be replaced or used as complement of the presented control charts, including Shewart's or non-Shewart's charts. It is also shown that the integration of P-value chart with EWMA can provide better graphic chart and as a result of this user can detect small and large changes simultaneously in a unique chart.

Key words: P-value control charts, SPC, quality control, EWMA, Xbar, Shewart's charts

INTRODUCTION

Statistical Process Control (SPC) was born in the field of engineering and, later, statisticians turned their attention towards it. So, some of its statistical aspects such as the multiplicity are not yet appropriately known. One of the reasons of the poor and non-predicting performance of the conventional statistical process control methods is the neglecting of the multiplicity aspect. When going beyond one step in the number of sampling times, the resulting control charts with multiple sampling are more sensitive for detecting the changeability. Hence, using a suitable chart in such problems is very important [1].

P-value is an important statistical concept which is useful in multiple-control processes. To achieve the concept of P-value in the frame of SPC, the hypothesis test should be considered. In other words, the SPC control charts can be thought of as repeated hypothesis tests of the null hypothesis that the process is under control [1].

Meanwhile, to the great amazement of many statisticians, it is still a matter of debate whether SPC can be seen as a series of statistical tests. Woodall's review [2] on this nonconformity has renewed a discussion of the matter which was intensified soon after his paper. He believes that this confusion is a result of the "inability to distinguish between the uses of phases I and II" [3, 4] for more information about the phases). Woodall believes phase I as a descriptive structure in which nothing can be tested; because there is no information on the underlying distribution. Hoerl [5] looks at this problem as a hypothesis-generation phase after which the hypothesis tests of phase II occur. In many cases, these hypotheses are true. Thus, instead of looking for a convenient method, the researchers have mostly preferred to claim that there is no relevant to the hypothesis test. Since Shewhart's charts are known as suitable ones, it is common belief that "practically, the relation between control chart and hypothesis test has a negligible influence" [2, 6, 7].

When a new point is added to an SPC chart, two different cases are possible. Either there is not enough evidence that the process parameters are outside the defined "acceptable" zone, or, there is enough evidence that a modification is essential. Knowing this, Benjamini and Kling [1] focused on decision-making points. The main challenge is to specify the hypothesis for these points and to appropriately test it. Woodall and Montgomery [8] mentioned the relation between hypothesis test and control charts through testing the main debates on the SPC research. Even when not explicitly specified, the corresponding errors of type I and type II are usually neglected during the report of the above-mentioned hypothesis [9, 10]. Assuming that the process is under control and that the tests are statistically independent, the type I error which is the probability of false alarm in a particular time, is the inverse of ARL0 [10].

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It should not be unexpected that the concept of P-value makes the relationship of the results and the statistical analysis simpler for statisticians. In many research fields, the results of the statistical test are arbitrarily reported according to the P-value. This is in contrary to the conventional "accepted" and "not accepted" reports which have a constant significance level. The P-value can be defined as the smallest significance level in which the related hypotheses can be rejected through having the particular detected value [11]. Hence, it is seen that the observations over the null hypothesis are powerful. Almost all statistical software report the results according to the P-value and avoid to define the significance level, α. Of course there is a huge debate on the advantages, limits and false interpretations of the P-value [12-15]. Therefore, P-value has become the most common method of using and reporting the results of statistical tests.

To use the P-values for decision-making in SPC, we do such that the observations that have P-values less than the significant level, α, be detected as out of control. The arbitrary evaluation of α has made the statisticians able to verify SPC at arbitrarily significant levels. Clearly P-value control chart has some properties that help the user to determine whether the point in an SPC is out of control or not. For example, let the chart in Fig. 1 be the Xbar control chart of an imaginary process (The data are shown in Table 1).

The horizontal line in this chart shows the run number and the vertical line shows the Xbar values. The upper and lower control limits are different because of the variety of sample sizes. The disorder of control limits attracts us the most, so at the first sight we cannot determine whether the points are between control limits or not. This problem has a high importance when there are many points in the process. Moreover, the points that almost have the same value in the chart may contain different results as it is shown in Fig. 1. In this chart, points 7 and 9 have the same level, but point 7 is under control, while point 9 is out of control. The P-value charts can overcome most of these problems and present more useful demonstrations of information from the data. The cost of all of this usefulness may possibly be the complexity of drawing them. But this problem is not a big matter of concern when using computer systems.

In the process of drawing the P-value control chart, some interesting results about the points that are near the control limits are gained. It will be discussed completely in section 2. Multiple testing and combinations of P-value charts with other control charts such as EWMA will be discussed in section 3.

In this connection, it is worth mentioning that all of the calculations are done and the graphical charts are drawn by the statistical software R (www.r-project.org).

Table 1: Data of an imaginary process

Subgroup	Observations					
1	18.39	23.52	21.18	22.41	20.36		
2	22.42	18.17	22.35	14.02	22.41		
3	23.53	19.83	18.81	15.06	24.4	18.41	
4	22.71	23.28	22.03	20.5	21.75	21.4	
5	17.39	19.21	24.45	21.74	20.52	18.43	20.14
6	18.66	20.78	20.67				
7	21.67	26.335	25.965				
8	20.53	19	21.96	20.94			
9	26.51	21.64	24.9	25.98	24.26		
10	18.55	21.89	18.98	21.31	20.76	20.77	
11	17.34	18.95	18.48	19.94			
12	24.85	23.4	21.64	22.09	26.89	23.26	
13	18.94	16.41	19.22	18.33			
14	20.78	19.26	22.08	23.57	18.95		
15	23.75	21.03	22.66	24.7			

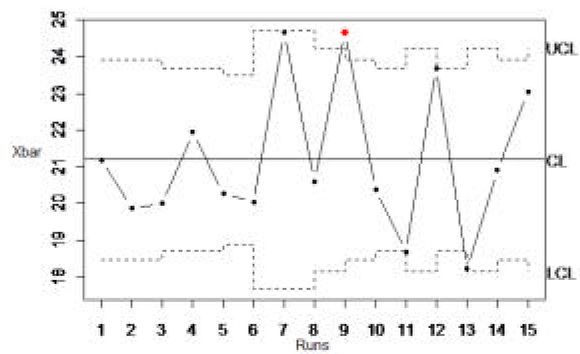

Fig. 1: Xbar control chart of data in table

Using P-values in SPC charts: The P-value chart is explained with the conditions in which the sample size can change from one sample to another, so the SPC charts can have variable action lines. The chart in Fig. 1 shows an Xbar chart that demonstrates this property, the disorder in the control limits confuse the user of the chart. Moreover, there are points that have the same value but different results.

Note that in Fig. 1, run number 9 is out of control, while run number 7 which has the same value is under control. The only reason of this is due to the less observation of run number 7 in comparison to that of run number 9, so the user must find the reason why run number 9 is out of control, but run number 7 is not. This means that the user must verify the sample sizes in subgroups 7 and 9 again to find out whether the reason of difference between them is because of the sample sizes or not.

Now to solve this problem we construct a P-value control chart. With the value of the center line of the Xbar chart, which is the average of all process points and is equal to 21.19726 and with the following formula, we can calculate the value of statistics Z_t (t being the run number) for all of the points. The total standard deviation, S, of the process is 2.137171.

$$Z_t = \frac{\bar{X}_t - \bar{\bar{X}}}{\frac{S}{\sqrt{n_t}}} = \frac{\bar{X}_t - 21.19726}{\frac{2.137171}{\sqrt{n_t}}} \quad (1)$$

In this formula $\bar{X}_t$ and n_t are the value and sample size of the run number t, respectively. As we know Z_t is normal distributed with mean 0 and standard deviation 1 We can calculate P-value for every run using:

$$\text{P-value}_t = 2(1 - \Phi(|Z_t|)) \quad (2)$$

After calculating the p-values for all of the runs of the Xbar chart, we compare them with the predefined significance level, α and then we can decide whether they are under control or not.

The P-value chart for data in Table 1 is shown in Fig. 2. The circles are replaced by the points in the original Xbar chart, so the circles show the value of Xbar for that point as before. The black color in the circles in the chart is related to the P-value for that run. The circumference of all circles is equal to the predefined significance level. In this example let $\alpha = 0.0027$. Hereafter we consider this type of charts as P-value charts.

The black color in the circle shows the out of control signal. For a chart with predefined significant level, α, filling a point with the black color is done when the radius of the inner circle is equal or more than that of α. Filling the circle has indirect relationship to the P-value, which means, the darker the circle is the less the p-value in terms of the significance level α. When the P-value is equal or less than the significance level, α, the circle is totally filled which means that the point is out of control. For better detection of the points that are out of control, they marked with red color at the center of the circle.

Circles used because they naturally draw the attention to their centers [1]. The comparison of circle surfaces is easier than that of bars in rectangular systems. The P-value calculation considers sample sizes and by this, it simplifies decision-making at the level of being out of control or not. Again see the different behavior that appears in run numbers 7 and 9 in Fig. 2.

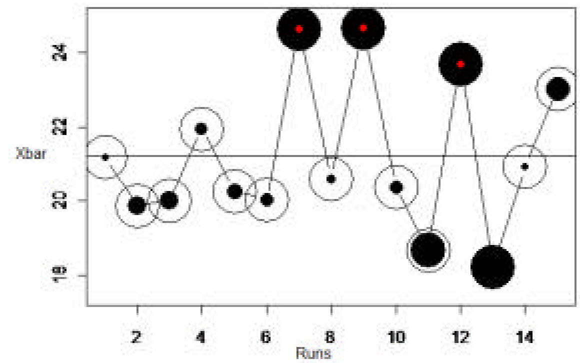

Fig. 2: The P-value chart for data of Table 1

Their derivation is much more obvious than before. We found that run numbers 7 and 12 that were under control in Xbar chart are detected out of control in the P-value chart.

This issue is remarkable; for the verification and comparison of the points that are out of control and the points that are near the control limits, we chose run numbers 7, 9, 11, 12 and 13.

As shown in Table 2, being out of control, run number 9 is detected by both charts. But the two runs of 7 and 12 which are detected as out of control by the P-value chart are under control by the Xbar chart. After verification of the values of the Xbar, sample size and limit controls, it is found out that both of these points are near the control limits, but they are both between the control limits and because of this, they are detected as under control by the Xbar chart. Our curiosity becomes more as the run numbers 11 and 13 have the same property and their Xbar values were near the control limits, but they were not detected out of control by the control charts. To find out the reason why run numbers 7 and 12 are out of control, we investigate the P-value of these two runs. After investigation we found out that these points are detected out of control by the P-value chart. This sample showed that since P-value is a very good and proper criterion for decision-making, drawing the P-value control chart is better alternative to conventional control charts since they are very useful in the detection of being under control or not.

[16] have three important principles for SPC charts that the P-value control charts also have them in common:

- The original measurement scale should be maintained,
- Ink should be proportional to the size of the warning signal,
- Their interpretation should be relatively intuitive.

Table 2: Comparison of the Xbar chart and the P-value chart for some runs

Run no.	Xbar	Subgroup sample size	Limits: Lower	Limits: Upper	P-value	Out of control detection: Xbar	Out of control detection: P-value
7	24.65667	3	17.49560	24.89892	0.0025264340		*
9	24.65800	5	18.32997	24.06455	0.0001467992	*	*
11	18.67750	4	17.99153	24.40299	0.0091860325		
12	23.68833	6	18.57979	23.81473	0.0021511015		*
13	18.22500	4	17.99153	24.40299	0.0027055016		

Table 3: Milk bag data

Subgroup	Sample	Obs.	Subgroup	Sample	Obs.	Subgroup	Sample	Obs.	Subgroup	Sample	Obs.
1	1	1.042	9	1	1.032	17	1	1.022	25	1	1.028
	2	1.041		2	1.036		2	1.020		2	1.022
	3	1.040		3	1.020		3	1.019		3	1.033
2	1	1.027	10	1	1.046	18	1	1.042	26	1	1.022
	2	1.032		2	1.047		2	1.051		2	1.022
	3	1.032		3	1.054		3	1.045		3	1.028
3	1	1.030	11	1	1.033	19	1	1.041	27	1	1.029
	2	1.031		2	1.035		2	1.044		2	1.030
	3	1.031		3	1.039		3	1.025		3	1.030
4	1	1.031	12	1	1.045	20	1	1.030	28	1	1.030
	2	1.021		2	1.053		2	1.028		2	1.034
	3	1.042		3	1.049		3	1.029		3	1.023
5	1	1.031	13	1	1.042	21	1	1.042	29	1	1.025
	2	1.030		2	1.034		2	1.023		2	1.019
	3	1.032		3	1.037		3	1.023		3	1.030
6	1	1.036	14	1	1.046	22	1	1.022	30	1	1.035
	2	1.039		2	1.031		2	1.044		2	1.028
	3	1.035		3	1.044		3	1.041		3	1.042
7	1	1.036	15	1	1.037	23	1	1.031			
	2	1.031		2	1.018		2	1.030			
	3	1.030		3	1.039		3	1.024			
8	1	1.034	16	1	1.037	24	1	1.023			
	2	1.033		2	1.041		2	1.043			
	3	1.035		3	1.037		3	1.045			

When the sample size varies, most of the SPC charts have variable control limits. This makes the user confused, especially when there are many runs. Moreover, the user must verify the sample sizes to find out whether the runs are out of control or not. Figure 2 shows how drawing the P-value chart simplifies the interpretation of the Xbar chart in Fig. 1. This simply shows the out of control signal. With the use of the p-value in the above method, we can design simpler charts visually that do not have control limits. Moreover, since the P-value is considered as a proper sensitive and powerful criterion for decision-making, it is very useful to find out whether the points near the control limits are under control or not.

Integration of EWMA chart and Xbar chart by using P-values: The Exponentially-Weighted Moving Average (EWMA) chart is a control chart useful for detecting small process shifts or for working with non-normal data [17]. The EWMA chart is sensitive to small shifts in the process mean, but does not match the ability of Shewhart-style charts (namely Xbar-R and Xbar-S charts) to detect larger shifts. A good way to further improve the sensitivity of the procedure to large shifts without sacrificing the ability to detect small shifts is to combine a Shewhart chart with the EWMA. These combined Shewart-EWMA control procedures are effective against both large and small shifts [18].

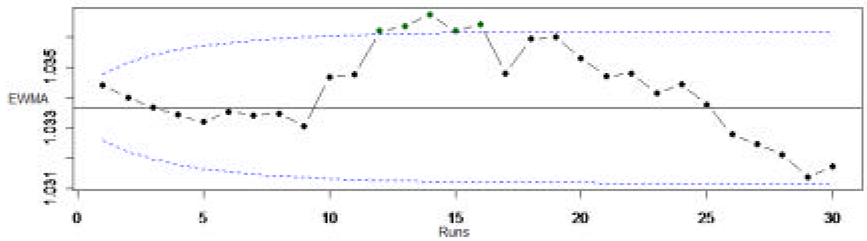

Fig. 3: The EWMA control chart for the data of the mean weight of Milk Bag subgroups

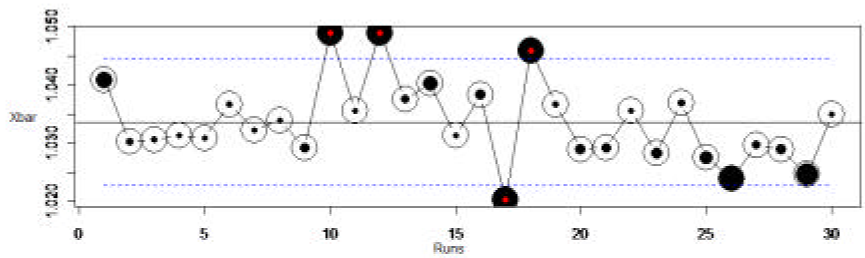

Fig. 4: The Xbar p-value chart for the weight of Milk bags

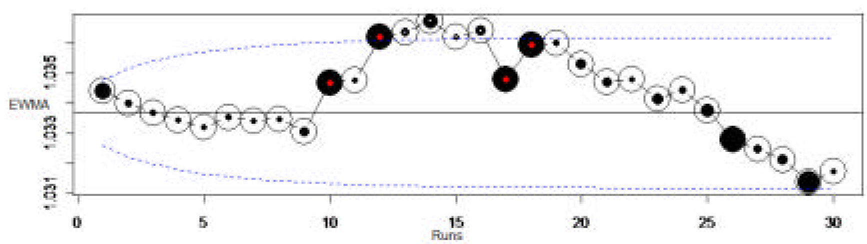

Fig. 5: The EWMA_P-value chart for the weight of Milk bags

In the previous section, after drawing the P-value control chart and the Xbar control chart simultaneously, it is found out that the P-value control chart is a proper chart to be combined with other control charts. One of our suggestions for combination of this chart with other control charts is its combination with the EWMA control chart. The combination of the control chart EWMA with the control chart Xbar is a common method for the combination of Shewart-EWMA charts. But the existence of control limits on these charts makes the user confused. P-value charts introduce a very good Shewart-EWMA combined chart because of the deletion of the control limits of the Xbar chart and they also work more precisely in addition to better graphic view. This property can be further displayed in a case study.

In the product line of Milk Bag in the Iran Dairy Industrial Co. Pegah, the weights of 3 Milk Bags are measured every 30 minutes (Table 3). The manager of the Company believes that detecting both the small and the large changes are important because of the importance of the weight critical to quality for the factory. So, the type of control charts that demonstrate small and large varieties simultaneously are desired.

To detect small changes in this process, we first draw EWMA charts for the average of subgroups with size 3.

In Fig. 3, the run numbers 12, 13, 14, 15 and 16 are above the upper limit of the chart which indicate shift in the mean of the Milk Bags. But since the EWMA chart is constructed for the determination of small changes, we draw a simultaneous Xbar and P-value charts for large changes (Fig. 4). The run numbers 10, 12, 17 and 18 are detected as out of control which means the existence of large changes in these points.

For the points that are detected out of control with EWMA chart and Xbar_P-value chart, just the point 12 is detected by both charts. Because of this, we have to draw both charts to show small and large changes. But this is hard for the user, so we are going to present a better graphic chart by the integration of EWMA and P-value charts and as a result of this, we are able to detect small and large changes simultaneously in a unique chart (Fig. 5). As it is shown in Fig. 5, the points 10, 17 and 18 have large shifts which the EWMA chart could not detect them alone. But with this new chart, we can show these large varieties in addition to the small ones.

CONCLUSION

This study was an attempt to investigate the P-value control charts. For this investigation, P-value control charts, which present better graphical diagrams and are interpreted more easily by user, were introduced. Moreover they improve the user's ability to realize out-of-control processes for sensitive values and finally they provide suitable charts for combining control charts in the field of multiple testing. In section 2, it was shown that when the sample size varies, variable control limits make user confused specially when there are many runs and proposing a P-value chart simplifies the interpretation of Xbar chart because we can design simpler charts visually that don't need conventional control limits. Moreover since the P-value is considered as a proper sensitive and powerful criterion for decision-making, it is very useful to find out whether the points near the control limits are under control or not.

Combination of P-value chart with other control charts such as EWMA to present a better graphic chart were introduced in section 3. As a result of this we are able to detect small and large changes simultaneously in a unique chart. This better alternative of graphical presentation not only can be used in various control charts but also integration of P-value control chart with other control charts of different properties can make powerful results in decision-making of SPC.

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