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Dynamics of a Particle, Constraint Surface and Generalized Uncertainty Principle

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Abstract: Gauge invariant action play an important role in the dynamics of particles. In this road, the equation of motion of a particle on a constraint surface obeys from a gauge invariant action. In this article, using a gauge invariant action, a solution of Jacobi identity is presented on the surface of a manifold. In continue, in the 8-dimensional manifold with a non-trivial topology, a generalized uncertainty relationship a generalized version of space-time uncertainty principle is obtained.

Key words: Gauge invariant action · Generalized space-time uncertainty principle · Jacobi identity

INTRODUCTION

Recently there has been a great deal of interest to study the microscopic origin of space-time [1-15]. It was shown that at the Planck scale regime, the classical perspective of space-time receives a modification and at a high-energy probes, the usual Heisenberg uncertainty receives an unusual correction by adding a new term $\Delta x \geq \frac{\hbar}{\Delta p} + \alpha' \frac{\Delta p}{\hbar}$. Where $\sqrt{\alpha'}$ is Planck distance. This relation is invariant under,

$$\frac{\Delta p\sqrt{\alpha'}}{\hbar} \leftrightarrow \frac{\hbar}{\Delta p\sqrt{\alpha'}} \tag{1}$$

that has a kind of inversion symmetries [15]. However the generalized space-time uncertainty is studied in the string theory [1-3] black hole physics [4] quantum mechanic [5] (anti) de Sitter space time [6, 7] quantum cosmology [8] *etc*… and is applied to probing the physical phenomena [9, 12] but study of this unusual behavior micro space time in the gauge fields perspective may be an alternative. In this letter, we have obtained a generalized space-time uncertainty using a gauge invariant action on a constraint surface.

Manifolds and Constraint Surface: Consider a symplectic super manifold which has coordinates $x^1, x^2, \ldots, x^{2N}$ with $\varepsilon^i = \varepsilon(x^i)$, the non-degenerate simplistic two form $\omega_{ij}(x)$ is $d\omega = 0$. Using the Jacobi identity, we can write [16],

$$\partial_i \omega_{jk}(x)(-1)^{(\varepsilon^i+1)\varepsilon^k} + Cycle(i,j,k) = 0 \tag{2}$$

Where $\omega_{ij}(x) = \omega_{ji}(x)(-1)^{(\varepsilon^i+1)(\varepsilon^j+1)}$ and $\varepsilon(\omega_{ij}) = \varepsilon^i + \varepsilon^j$. In this frame Poisson bracket is,

$$\{A(x), B(x)\} = A(x)\bar{\partial}_i \omega^{ij}(x)\vec{\partial}_j B(x) \tag{3}$$

Where $\varepsilon(\omega^{ij}) = \varepsilon(\omega_{ij})$ and $\omega^{ij}(x) = -\omega^{ij}(x)(-1)^{\varepsilon^i \varepsilon^j}$. Eq. (3) satisfies the Jacobi identity, since. Eq. (2) implies,

$$\omega^{ij}\bar{\partial}_i \omega^{ik}(x)(-1)^{(\varepsilon^i)\varepsilon^k} + Cycle(i,j,k) = 0 \tag{4}$$

Where x^i is the canonical coordinate and ω^{ij} is a constant. Consider a Hamiltonian *H(x)* with *2M < 2N*, irreducible the second class constraint $\theta^\alpha(x)$ which satisfy regularity condition as [16],

$$Rank\theta^\alpha(x)\frac{\bar{\partial}}{\partial x^i}|_{\theta=0} = 2M \tag{5}$$

and

$$Rank\{\theta^\alpha(x), \theta^\beta(x)\}\frac{\bar{\partial}}{\partial x^i}|_{\theta=0} = 2M \tag{6}$$

Consider a generic constraint surface Γ as a sub-manifold of M (manifold) if a continuous function $\bar{x}^i(x): \mathrm{M} \to \Gamma$ exist, then $\bar{x}$ is set to be a retraction and Γ a retract of M. Furthermore, if there exist a continuos map *H* : M × *I* → M, with the interval *[0,1],* we can write,

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H(x,0)=x *H(x,1)*Γ *for any x*εM (7a)
H(x,s)=x *for any* xεF *for any S*ε*I* (7b)

Eq. (7) implies that the identity function on M is hemotopic to the function $\bar{x}$.

Therefore, M and Γ have the same homotopy type and our sub-manifold *M* must have the same fundamental group as,

$$\pi_1(M) = \pi_1(\Gamma) \tag{8}$$

Therefore, a generic constraint surface Γ as the sub-manifold has the same homotopy type with the manifold *M* and we can consider the constraint surface as a manifold. Batalin and Marnelius [16] advance the quantization of Hamiltonian systems with second-class constraints. In this scenario, the equation of motion of a particle obeys from a gauge invariant action.

A Gauge Invariant Action: In the paper by Lyakhovich and Marnelius (2001) a condition placed on $\bar{x}^i(x)$ as,

$$\{\bar{x}^i(x), \bar{x}^j(x\} = \{x^i, x^j\}_D \mid_{x\to\bar{x}(x)} \tag{9}$$

This condition is to restrict the choice of gauge theory and is remove in this spirit; one can instead search for a bracket on M with property,

$$\{A(\bar{x}(x)), B(\bar{x}(x))\}_M = \{A(x), B(x)\}_D \mid_{x\to\bar{x}(x)} \tag{10}$$

When {,} and $\{,\}_D$ are the Poisson and the Dirac brackets, respectively and $\{,\}_M$ is a new bracket on M. $A(\bar{x}(x))$ and $B(\bar{x}(x))$ are arbitrary gauge invariant observable. On the manifold, the Jacobi identity is satisfied by the new bracket $\{,\}_M$ as,

$$\begin{aligned}&\{\{A(\bar{x}), B(\bar{x})\}_M, C(\bar{x})\}_M + (-1)^{\varepsilon^A(\varepsilon^B+\varepsilon^C)}\\&\{\{B(\bar{x}), C(\bar{x})\}, A(\bar{x})\}_M + (-1)^{\varepsilon^C(\varepsilon^A+\varepsilon^B)}\\&\{\{C(\bar{x}), A(\bar{x})\}_M, B(\bar{x})\}\}_M = 0\end{aligned} \tag{11}$$

Where $\{\{A(\bar{x}), B(\bar{x})\}_M, C(\bar{x})\}_M = \{\{A(x), B(x)\}_D, C(x)\}_D \mid_{x\to\bar{x}}$. The Batalin–Marnelius gauge invariant action [16, 17], show's the equation of motion of a particle on the manifold M, as $\omega_{Mij}(x)\dot{x}^j = \vec{\partial}_i H(\bar{x}(x)), where\ \omega_{Mij}$ is a degenerate function, so x^j is not unique and we have,

$$\dot{x}^j = \{\dot{x}^j, H(\bar{x},(x))\} \tag{12}$$

Canonical momentum may be written as $p^i = \frac{\partial}{\partial x^i} L(x, \dot{x})$.

As it well known, two non-commutating relations for any given state, in a Hilbert space are as,

$$[\hat{x}^i, \hat{x}^j] = i\hbar\omega^{ij}{}_M(\hat{x}) \tag{13}$$

$$[\hat{x}^\mu, \hat{p}^\nu] = i\hbar\omega^{\mu\nu}{}_M(\bar{x}(x)) \tag{14}$$

From eq. (14) one obtains,

$$[\hat{x}^\mu, \hat{p}^\nu] = \hat{x}^\mu\hat{p}^\nu - (-1)^{\varepsilon^\mu\varepsilon^\nu p^i}\hat{p}^\nu\hat{x}^\mu \tag{15}$$

Note that $\omega^{ij}{}_M = \omega^{ij}$. An appropriate choice of $\bar{x}^i(x)$ allow to non-degenerate canonical coordinates x^1, x^2. Consider two particles localized at x^1, x^2, respectively. Relation between x^1, x^2 is obtained by eq.(13). The total uncertainty on a manifold could be obtained by solution of the Jacobi identity as,

$$[x^i, [x^j, p^k]] + cyclic(i, j, k) \tag{16}$$

In the 8-dimensional manifold with higher non-trivial topology eq. (16) can be solved as,

$$\Delta x^i \Delta p^j = \frac{\hbar}{2} w^{ij}{}_M(1 + \bar{x}(x)) \tag{17}$$

The space-time manifold has a foam structure in the large-scale compared to the Planck scale. If we identify $\sqrt{\alpha'}$ as the Planck length, the minimal length on a manifold is $\sqrt{\alpha'}$ and we can write,

$\bar{x}(x) \approx \sqrt{\alpha'}$. From eq. (17) we obtain,

$$\Delta x^i \Delta p^j = \frac{\hbar}{2} w^{ij}{}_M(1 + \sqrt{\alpha'}) \tag{18}$$

CONCLUSION

The foamy space-time has the manifold structure in the Planck scale regime. Using a gauge invariant action, the modified space-time uncertainty in a foam structure of the space-time is constructed. Using the fact that the equation of motion of a particle on a constraint surface obeys from a gauge invariant action, a generalized version of space-time uncertainty principle is obtained. It is shown that, usual uncertainty principle receives a correction.

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