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Homotopy Perturbation Method for Nonlinear Thermoelasticity

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Abstract: In this paper, homotopy perturbation method (HPM) is applied to solve the Cauchy problem arising in one dimensional nonlinear thermoelasticity. It is observed that the proposed technique is fully compatible with such nonlinear problems. Numerical results explicitly reveal the complete reliabilltiy of the propoded algorithm (HPM).

Key words: Homotopy perturbation method · Cauchy problem · Nonlinear one dimensional thermoelasticity

INTRODUCTION

The homotopy perturbation method (HPM) was first proposed by the Chinese mathematician Ji-Huan He [1-3]. Unlike classical techniques, the homotopy perturbation method leads to an analytical approximate and exact solutions of the nonlinear equations easily and elegantly without transforming the equation or linearizing the problem and with high accuracy, minimal calculation and avoidance of physically unrealistic assumptions. As a numerical tool, the method provide us with numerical solution without discretization of the given equation and therefore, it is not effected by computation round-off errors and one is not faced with necessity of large computer memory and time. This technique has been employed to solve a large variety of linear and nonlinear problems [4-18]. The interested reader can see the Refs. [19-22] for latest development of HPM.

The object of this study is to employ HPM to solve a real-life problem that exhibits coupling between the mechanical and thermal fields. Let consider the following nonlinear system arising in thermo elasticity [23-25].

$$u_{tt} - a(u_x,\theta)\, u_{xx} + b(u_x,\theta)\theta_x = f(x,t) \tag{1}$$

$$c(u_x,\theta)\,\theta_1 + b(u_x,\theta)u_{xt} - d(\theta)\,\theta_{xx} = g(x,t) \tag{2}$$

subject to the initial conditions of

$$u(x,0) = u^0(x),\ u_1(x,0) = u^1(x),\ \theta(x,\theta) = \theta^0(x) \tag{3}$$

where $u(x,t)$ is the body displacement form equilibrium and $\theta(x,t)$ is the difference of the body's temperature from a reference $T_0 = 0$, subscripts denote partial derivatives and a, b, c and d are given smooth functions. For more details about the physical meaning of the model, see [23, 26]. Recently Ganji *et al* [27] used Adomian's decomposition method for solving the governing problem.

Solution Procedure: In order to illustrate the effectiveness of the method, an artificial model is used. Let us define a, b, c, d,u^0, u^1 and θ^0 by [25]:

$$a(u_x,\theta) = 2 - u_x b(u_x\ \theta) = 2 + u_x\theta = 1,\ d(u_x,\theta) = \theta \tag{4}$$

$$u^0(x) = \frac{1}{1+x^2},\ u^1(x) = 0,\ \theta^0(x) = \frac{1}{1+x^2} \tag{5}$$

and replace the right-hand side of above equations by :

$$f(x,t) = \frac{2}{1+x^2} - \frac{2(1+t^2)(3x^2-1)}{(1+x^2)^3}a(w,v) - \frac{2x(1+t)}{(1+x^2)^2}b(w,v) \tag{6}$$

$$g(x,t) = \frac{2}{1+x^2}c(w,v) - \frac{4xt}{(1+x^2)^2}b(w,v) - \frac{2(1+t)(3x^2-1)}{(1+x^2)^3}d(v) \tag{7}$$

$$w \equiv w(x,t) = -\frac{2x(1+t^2)}{(1+x^2)^2},\ v \equiv v(x,t) = \frac{1+t}{1+x^2} \tag{8}$$

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where a, b, c and d are defined by Eq. (4) and the exact solution of two equations are given by [25] :

$$u(x,t)=\frac{1+t^2}{\left(1+x^2\right)^2},\ \theta(x,t)=\frac{1+t}{1+x^2} \tag{9}$$

If we put (4) into (1) and (2), then we get :

$$u - (2-u_x\theta)\, u_{xx}+(2+u_x\theta)\, \theta_x - f(x,t) = 0 \tag{10}$$

$$\theta_1 + (2+u_x\theta)\, u_{xx}-\theta\theta_{xx}-g(x,t) = 0 \tag{11}$$

and we get

$$u_{tt} + p\,\{-2u_{xx} + u_xu_{xx}\theta + 2\theta_x+u_x\theta_x\theta_x-f(x,t)\} = 0 \tag{12}$$

$$\theta_1 + 2u_{xt} + u_xu_{xx}\theta - \theta\theta_{xx}+ g(x,t)\} = 0 \tag{13}$$

We construct the following homotopies

$$u_{tt} + p\,\{\ -2u_{xx} + u_xu_{xx}\theta + 2\theta_x + u_x\theta_x\theta - f(x,t)\ \} = 0 \tag{14}$$

$$\theta_t + p\,\{\ 2u_{xt} + u_xu_{xt}\theta - \theta\theta_{xx} - g(x,t)\ \} = 0 \tag{15}$$

Assume the solution of Eqs. (14,15) to be in the form:

$$u = u_0 + pu_1 + p^2u_2 + p^3u_3 + \ldots \tag{16}$$

$$\theta = \theta_0 + p\theta_1 + p^2\theta_2 + p^3\theta_3 + \ldots \tag{17}$$

Substituting (16-17) into (14,15) and equating the coefficients of like powers p, we get the following set of differential equations

$$p^0 : (u_0)_{tt} = 0$$

$$(\theta_0)_t = 0$$

$$p^2 : (u_2)_t - 2(u_1)_{xx} + (u_0)_x(u_0)_{xx}\theta_1 + (u_0)_x(u_1)_{xx}\theta_0 + (u_1)_x(u_0)_{xx}\theta_0 + 2(\theta_1)_x + (u_0)_x(\theta_1)_x\theta_0 + (u_0)_x(\theta_0)_x\theta_1 + (u_1)_x(\theta_0)_x\theta_0 - f(x,t) = 0$$

$$(\theta_2)_t + 2(u_1)_{xt} + (u_0)_x(u_0)_{xt}\theta_1 + (u_0)_x(u_1)_{xt}\theta_0 + (u_1)_x(u_0)_{xt}\theta_0 - \theta_0(\theta_1)_{xx} - \theta_1(\theta_0)_{xx} - g(x,t) = 0$$

$$\begin{aligned}&(\theta_3)_t + 2(u_2)_{xt} + (u_0)_x(u_0)_{xt}\theta_2 + (u_0)_x(u_1)_{xt}\theta_1 + (u_0)_x(u_2)_{xt}\theta_0 + (u_1)_x(u_0)_{xt}\theta_1 + (u_1)_x(u_1)_{xt}\theta_1\\&+(u_2)_x(u_0)_{xt}\theta_0 - \theta_0(\theta_2)_{xx} - \theta_2(\theta_0)_{xx} - \theta_1(\theta_1)_{xx} - g(x,t) = 0\end{aligned}$$

and so on, the rest of the polynomials can be constructed in a similar manner. With the initial conditions Eq. (3) gives

$$u_0(x,t)=\frac{1}{1+x^2} \tag{18}$$

$$u_1(x,t)=\frac{1}{105\left(1+x^2\right)^6}\begin{pmatrix}10t^7\left(x-3x^3\right)+14t^6\left(x+x^2-3x^3+x^4\right)\\+42t^5\left(x+x^2-3x^3+x^4\right)\\+35t^4\left(1+x+2x^2-6x^3-2x^4-8x^6-3x^8\right)\\+70t^3\left(2x^2-7x^3+2x^4-6x^5-4x^7-x^9\right)\\+105t^2\left(1+5x^2+10x^4+10x^6+5x^8+x^{10}\right)\end{pmatrix} \tag{19}$$

$$p^1 : (u_1)_{tt} - 2(u_0)_{xx} + (u_0)_x(u_0)_{xx}\theta_0 + 2(\theta_0)_x + (u_0)_x(\theta_0)_x\theta_0 - f(x,t) = 0\ (\theta_1)_t + 2(u_0)_{xt} + (u_0)_x(u_0)_{xt}\theta_0 - \theta_0(\theta_0)_{xx} - g(x,t) = 0$$

$$\theta_0(x,t)=\frac{1}{1+x^2} \tag{20}$$

$$\theta_1(x,t)=\frac{1}{15\left(1+x^2\right)^5}\begin{pmatrix}24t^5x^2+30t^4x^2+10t^3\left(1+2x^2-3x^4\right)\\+30t^2\left(1-2x-6x^3-3x^4-6x^5-2x^7\right)\\+15t\left(1+4x^2+6x^4+4x^6+x^8\right)\end{pmatrix} \tag{21}$$

Proceeding in the same way, we can obtain $u_2(x,t)$, $\theta_2(x,t)$ and higher order approximations. Here, the numerical results are evaluated using terms approximation of the recursive relations.

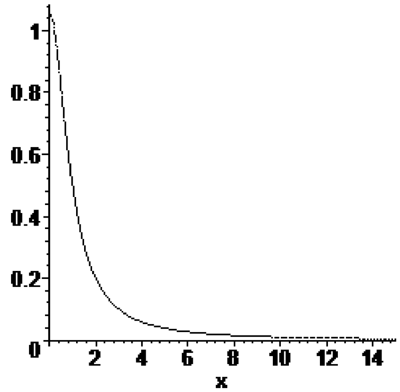

Fig. 1: u(x,t) when t=0.25 Line:HPM, Point:exact

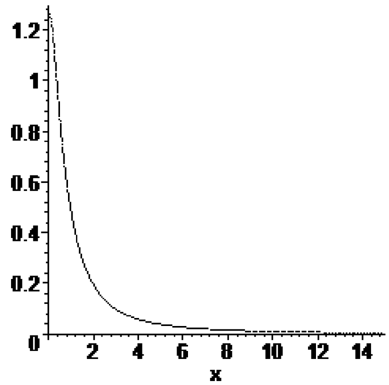

Fig. 2: u(x,t) when t=0.25 Line:HPM, Point:exact

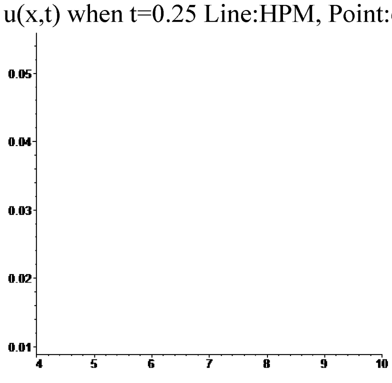

Fig. 3: Absolute error when t=0.25

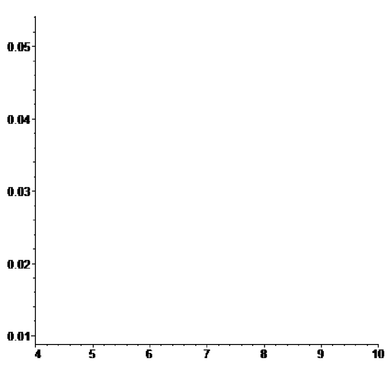

Fig. 4: Absolute error when t=0.5

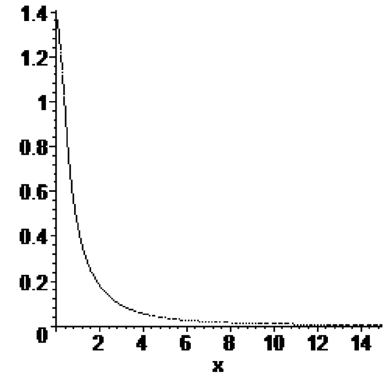

Fig. 5: $\theta(x,t)$ when t=0.25 Line:HPM, Point:exact

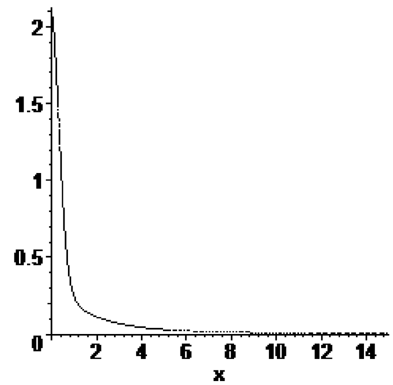

Fig. 6: $\theta(x,t)$ when t=0.5 Line:HPM, Point:exact

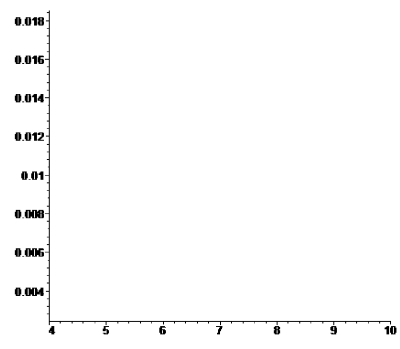

Fig. 7: Absolute error when t=0.25

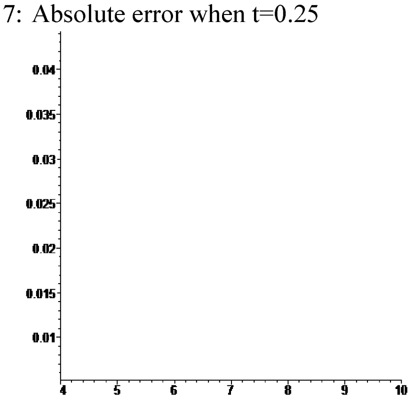

Fig. 8: Absolute error when t=0.5

CONCLUSIONS

In this study, we have successfully applied HPM to obtain an approximation of the analytic solution of the Cauchy problem arising in one dimensional nonlinear thermoelasticity. In this method, the solution is found in the form of a convergent series with easily computed components. The results obtained by homotopy perturbation method are compared with those of the exact solution, which shows very good agreement, even using only few terms of the recursive relations. In general, this method provides highly accurate numerical solutions and can be applied to wide class of nonlinear problems. Homotopy perturbation method does not require small parameters which are needed by perturbation method. Also the method avoids linearization and physically unrealistic assumptions.

REFERENCES

1. He, J.H., 1999. Homotopy perturbation technique, Computer Methods in Applied Mechanics and Engineering, 178: 257-262 .
2. He, J.H., 2003. Homotopy perturbation method: a new nonlinear analytical technique. Applied Mathematics and Computation, 135: 73-79.
3. He, J.H., 2006. Homotopy perturbation method for solving boundary value problems. Physics Letters A., 350: 87.
4. Dehghan, M. and F. Shakeri, 2008. Solution of an integro-differential equation arising in oscillating magnetic fields using He’s homotopy perturbation method, Progress in Electromagnetic Research, PIER, 78: 361-376.
5. Dehghan, M., F. Shakeri, 2008. Use of He’s homotopy perturbation method for solving a partial differential equation arising in modeling of flow in porous media, J. Porous Media, 11: 765-778.
6. Saadatmandi, A., M. Dehghan and A. Eftekhari, 2009. Application of He’s homotopy perturbation method for non-linear system of second-order boundary value problems, Nonlinear Analysis: Real World Applications, 10: 1912-1922.
7. Yıldırım, A., 2008. Solution of BVPs for Fourth-Order Integro-Differential Equations by using Homotopy Perturbation Method, Computers and Mathematics with Applications, 56: 3175-3180.
8. Yıldırım, A., The Homotopy Perturbation Method for Approximate Solution of the Modified KdV Equation, Zeitschrift für Naturforschung A,A J. Physical Sci., 63a: 621 .
9. Yıldırım, A., 2008. Application of the Homotopy perturbation method for the Fokker-Planck equation, Communications in Numerical Methods in Engineering, 2008 (in press)
10. Achouri, T. and K. Omrani, 2009. Application of the homotopy perturbation method to the modifed regularized long wave equation, Numerical Methods for Partial Differential Equations, DOI 10.1002/num.20441 (in press)
11. Ghanmi, I., K. Noomen, K. Omrani, 2009. Exact solutions for some systems of PDE's by He's homotopy perturbation method, Communication in Numerical Methods in Engineering, (in press).
12. Dehghan, M. and F. Shakeri, 2007. Solution of a partial differential equation subject to temperature Overspecification by He’s homotopy perturbation method, Physica. Scripta, 75: 778.
13. Shakeri, F., M. Dehghan, 2008. Solution of the delay differential equations via homotopy perturbation method, Mathematical and Computer Modelling, 48: 486.
14. Yıldırım, A., 2009. Homotopy perturbation method for the mixed Volterra-Fredholm integral equations, Chaos, Solitons and Fractals, 42: 2760-2764.
15. Koçak, H. and A. Yıldırım, 2009. Numerical solution of 3D Green’s function for the dynamic system of anisotropic elasticity, Physics Letters A.,373:3145-50.
16. Mohyud-Din, S.T., M.A. Noor and K.I. Noor, 2009. Travelling wave solutions of seventh-order generalized KdV equations using He’s polynomials, Int. J. Nonlin. Sci. Num. Sim., 10(2): 223-229.
17. Noor M.A. and S.T. Mohyud-Din, 2007. An efficient algorithm for solving fifth order boundary value problems, Math. Comput. Model., 45: 954-964.
18. Noor, M.A. and S.T. Mohyud-Din, 2008. Homotopy perturbation method for solving sixth-order boundary value problems, Comput. Math. Appl., 55(12): 2953-2972.
19. He, J.H., 2008. An elementary introduction to recently developed asymptotic methods and nanomechanics in textile engineering, International J. Modern Physics B., 22: 3487.
20. He, J.H., Recent development of the homotopy perturbation method, Topological Methods in Nonlinear Analysis, 31: 205.
21. He, J.H., 2006. Some asymptotic methods for strongly nonlinear equations, International J. Modern Physics B., 20: 1141.
22. He, J.H., 2006. New interpretation of homotopy perturbation method, International J. Modern Physics B., 20: 2561

23. Jiang. S., 1990. Numerical solution for the cauchy problem in nonlinear 1-d-thermoelasticity. Computing, 44: 147-158.
24. Slemrod. M., 1981. Global existence, uniqueness and asymptotic stability of classical solutions in one dimensional nonlinear thermoelasticity. Arch. Rational Mech. Anal., 76: 97-133.
25. Sweilam, N.H. and M.M. Khader, 2005. Variational iteration method for one dimensional nonlinear thermo-elasticity. Chaos, Solitons and Fractals, In press.
26. Moura, C.A.D. 1983. A linear uncoupling numerical scheme for the nonlinear coupled thermodynamics equations. Berlin-Springer, 204-211. In: V. Pereyra, A. Reinoze (Editors), Lecture notes in mathematics, 1005.
27. Sadighi, A., D.D. Ganji, A study on one dimensional nonlinear thermoelasticity by Adomian decomposition method, World J. Modelling and Simulation, 4: 19-25.