Middle-East Journal of Scientific Research 10 (5): 647-653, 2011
ISSN 1990-9233

An ILC Method for Discretized Two-Dimensional Systems Based on GR Model

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Abstract: This paper intends to introduce a new method of discretized Iterative learning control of two-dimensional systems. Recently, 2-D systems play very important role in industry. As a result, it is of great importance to pay more attention to this kind of dynamics. Generally, in the 1-D systems the various quantities are function of time. On the other hand in many phenomena in the nature, some quantities are function of two independent variables which mainly none of these two variables is time. Consequently, for controlling of 2-D systems many attempts have been made and some kind of former 1-D methods have been extended to 2-D systems. In this article, by using GR model of 2-D systems a Novel ILC method for 2-D systems is presented.

Key words: Two-Dimensional systems · 2-D stability · Discretized 2-D system

INTRODUCTION

In the theory of 1-D systems, generally various quantities are functions of time. On the other hand many processes in the nature have quantities which are function of two independent variables and generally none of them is time. As an example, the intensity of an image is function of horizontal and vertical axes of image. In this case, for modeling of this kind of systems 2-D signals are used. In these systems the processing operation is done in two independent axes. Two Dimensional (2-D) systems are mostly investigated in the literature as a multi-dimensional system. 2-D systems are often applied to theoretical aspects like filter design, image processing and recently, Iterative Learning Control methods (see for example Roesser, 1975; Hinamoto, 1993; Whalley, 1990; Al-Towaim, 2004; Hladowski *et al*. 2008). The main applications of 2-D systems are such as distributed systems, heat transfer, image processing, biologic systems, earthquake signals processing, sonar etc. To describe 2-D systems in addition to state space equation, transfer functions and difference equations are also used [1]. Similar to 1-D systems, if the system is time variant, state space equation or difference equation is used. For various applications, one of the description methods is used. For example in the case of stability analysis of 2-D systems, generally state space equation and transfer function are used [1,2]. Also for issues such as state and parameter estimation, state space equation and difference equation are used [3-10]. For optimal control of 2-D systems, mostly state space equation form has been used [11-15]. Some new results on the stability of 2-D systems have been presented - specifically with regard to the Lyapunov stability condition which has been developed for GR (Lu, 1994). Motivated by the applications in digital picture processing, seismic data processing, X-ray image processing, etc. the continuous-time 2-D system is often converted into an equivalent discrete-time 2-D system via some approximation methods, such as the first difference method [16] and the high-order discretization method [17] with the assumptions that the sampling time and the sampling distance are sufficiently small. Then, by using the approximate discrete-time 2-D model together with a discrete-time performance index suited to discrete-time 2-D systems, many digital linear quadratic regulators (LQRs) are developed for optimal digital control of discrete-time 2-D system [16,18-22].

Iterative learning control (ILC) is a technique to control the systems doing a defined task repetitively and periodically in a limited and constant time interval. Since the iterative learning control concept was proposed [23] (widely credited to Arimoto), a very large number of approaches have been considered. Rogers *et al*. [24] firstly noted the 2-D dynamic characteristics of the ILC

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system and explored the convergence of the system based on the stability criterion for the 2-D system. Geng *et al*. [25] first proposed ILC system as a 2-D system for design and analysis. Their resulted ILC scheme, however, is a conventional ILC. Based on a 2-D Roesser model, Kurek *et al*. [26] and Fang *et al*. [27] developed feedback feed-forward ILC schemes for deterministic repetitive processes. Shi *et al*. [28, 29] extended robust control concept to 2-D Roesser system resulting in an integrated design of robust feedback control and feedforward ILC for uncertain batch processes. Using ILC for 2-D systems was introduced in [30] for the first time based on FM model [31]. In this paper, the ILC method is used for discretized 2-D systems based on GR model. The organization of this paper is as follows: Section 2 presents 2-D system various descriptions. In section 3, the discretized 2-D systems are introduced. The problem of producing a proper ILC method for discretized 2-D systems in GR model and the stability of the method is explained in section 4. In section 5, simulation results are given. Finally conclusion is presented.

2-D Systems Descriptions: As it was mentioned previously, for describing the 2-D systems there are some methods. In this section these descriptions are introduced briefly.

Transfer Function: For finding the transfer function of discrete 2-D systems, similar to 1-D systems the z-transform is used. In this case, it is called 2-D z-transform [1]. The general form of transfer function is as follows

$$P(z,w) = \frac{N(z,w)}{D(z,w)} \tag{1}$$

Which z and w are the shift operator of horizontal and vertical axes respectively. Also $N(z,w)$ and $D(z,w)$ are polynomials based on z and w. For instance, consider the following transfer function

$$P(z,w) = \frac{1 + zw - z - w}{2 - z - w}$$

The roots of numerator and denominator of transfer function are considered as zeros and poles of 2-D system respectively.

2-Difference Equation: Difference equation for a 2-D system is as follows

$$\begin{aligned} y(m,n) &= ay(m-1,n) + by(m,n-1) + cy(m-1,n-1) + \ldots \\ &\quad + du(m-1,n) + eu(m,n-1) + \ldots \end{aligned} \tag{2}$$

Where u and y are the input and output of system respectively. Also this system is assumed linear. With taking the z-transform of above equation the transfer function is obtained. In the case that the mentioned 2-D system is stochastic this form is known as ARMA model.

State Space Equation: One of the most significant points of the subject of 2-D systems is the multiplicity of presented models for state space equation. This fact is due to this point that in this equations local state vector is used instead of state vector because 2-D systems have state vectors with infinite dimension. These models include GR (Givone-Roesser) model [18], FM (Fornasini-Marchesini) model [31] etc. In the following the GR description is presented.

GR Model: This model was introduced in 1972 by Givone and Roesser [18]. This model was used in consideration of recursive 2-D systems. After that GR model is used in other problems such as stability, image processing, control and prediction.

This model has the following formulation

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(i,j) \tag{3}$$

$$y(i,j) = [C_1 \quad C_2] \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}$$

Where $x^h \in R^n$ and $x^v \in R^m$ are local horizontal state vector and local vertical state vector respectively. Also u and y are respectively the input and output vectors of system. I and j are the indices in horizontal and vertical direction. Matrices A,B and C generally are functions of i and j. As can be seen the local state vector in every point is dependent to the previous state vectors located in a cell before the recent point. This matter is known as First Quadrant Causality. More information about this issue is found in [1].

The Discretized 2-d System Description: Consider the continuous-time two dimensional system in GR model [18] described by:

$$\begin{bmatrix} \dfrac{\partial x_c^h(x,t)}{\partial x} \\ \dfrac{\partial x_c^v(x,t)}{\partial t} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_c^h(x,t) \\ x_c^v(x,t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u_c(x,t) \quad (4)$$

$$\dot{x}_c(x,t) = Ax_c(x,t) + Bu_c(x,t)$$

(6-2)

With boundary values $x_c^h(0,t)$ and $x_c^v(x,0)$ that

$$\dot{x}_c(x,t) = \begin{bmatrix} \dfrac{\partial x_c^h(x,t)}{\partial x} \\ \dfrac{\partial x_c^v(x,t)}{\partial t} \end{bmatrix} \qquad x_c(x,t) = \begin{bmatrix} x_c^h(x,t) \\ x_c^v(x,t) \end{bmatrix} \quad (5)$$

$$x_c^h(x,t) \in R^{n_1}, \qquad x_c^v(x,t) \in R^{n_2}, \qquad u_c(x,t) \in R^m$$

Furthermore, A_{11}, A_{12}, A_{21}, A_{22}, B_1 and B_2 are all real matrices with an appropriate dimensions, v and h are vertical and horizontal components. Furthermore for optimal control of this continuous-time 2D system, it is discretized as following:

Suppose that $u_c(x,t)$ is a function with the following definition:

$$u_c(x,t) = u_d(iX, jT) \ for\ iX \le x < (i+1)X$$
$$jT \le t < (j+1)T \quad (6)$$

X and *T* are explaining the sampling period of *x* and *t* axles, now by this definition the (4) discrete-time system will present so:

$$\begin{bmatrix} x_d^h((i+1)X, jT) \\ x_d^v(iX,(j+1)T) \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} x_d^h(iX,jT) \\ x_d^v(iX,jT) \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} u_d(iX,jT) \quad (7)$$

$$x_d'(iX,jT) = Gx_d(iX,jT) + Hu_d(iX,jT) \quad (8)$$

That

$$x_d'(iX,jT) = \begin{bmatrix} x_d^h((i+1)X, jT) \\ x_d^v(iX,(j+1)T) \end{bmatrix}, \quad x_d(iX,jT) = \begin{bmatrix} x_d^h(iX,jT) \\ x_d^v(iX,jT) \end{bmatrix} \quad (9)$$

and

$$G = \begin{bmatrix} e^{A_{11}X} & (e^{A_{11}X} - I_{n_1})A_{11}^{-1}A_{12} \\ (e^{A_{22}X} - I_{n_2})A_{22}^{-1}A_{21} & e^{A_{22}X} \end{bmatrix}$$

$$H = \begin{bmatrix} (e^{A_{11}X} - I_{n_1})A_{11}^{-1}B_1 \\ (e^{A_{22}X} - I_{n_2})A_{22}^{-1}B_2 \end{bmatrix} \quad (10)$$

For instance, in a continuous GR model (4) suppose that A_{11}=-3, A_{12}=3.2, A_{21}=1, A_{22}=-1, B_1=0.3, B_2=0 and also by considering *X*=0.1 and *T*=0.1, the discretized 2-D dynamic is as following:

$$\begin{bmatrix} x_d^h((i+1)X, jT) \\ x_d^v(iX,(j+1)T) \end{bmatrix} = \begin{bmatrix} 0.7408 & 0.2765 \\ 0.0952 & 0.9048 \end{bmatrix} \begin{bmatrix} x_d^h(iX,jT) \\ x_d^v(iX,jT) \end{bmatrix} + \begin{bmatrix} 0.0259 \\ 0 \end{bmatrix} u_d(iX,jT)$$

ILC Method for Discretized 2-D System: For controlling an iterative discretized 2-D system, the following 2-D ILC algorithm can be used.

For this reason it is possible to define $u_d(iX,jT)$ for k+1th iteration as following:

$$u_{d(k+1)}(iX,jT) = u_{d(k)}(iX,jT) + \Upsilon \begin{bmatrix} e_{d(k)}^h((i+1)X, jT) \\ e_{d(k)}^v(iX,(j+1)T) \end{bmatrix} \quad (11)$$

In which

$$\begin{bmatrix} e_{d(k)}^h((i+1)X, jT) \\ e_{d(k)}^v(iX,(j+1)T) \end{bmatrix} = \begin{bmatrix} y_{des}((i+1)X, jT) - y_{d(k)}^h((i+1)X, jT) \\ y_{des}(iX,(j+1)T) - y_{d(k)}^v(iX,(j+1)T) \end{bmatrix} \quad (12)$$

That y_{des} explains the desired system output.

Now, for all iterations the control input of discretized 2-D system can be achieved by using the relation (11).

Without reducing the generality of the problem and for the reason of simplicity we assume the dynamic (7) as follows:

$$\begin{bmatrix} x_d^h(i+1,j) \\ x_d^v(i,j+1) \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} x_d^h(i,j) \\ x_d^v(i,j) \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} u_d(i,j) \quad (13)$$

Furthermore, the boundary condition for (13) is

$$\begin{cases} x_0^h = x^h(0,j) \\ x_0^v = x^v(i,0) \end{cases} \quad (14)$$

Definition 1: If the state vector norms $\left\| x^h(i,j) \right\|$ and $\left\| x^v(i,j) \right\|$, converge to zero when $i+j \to \infty$ then the 2-D system (13) is asymptotically stable.

Here, we want to restate the Lyapunov stability for the GR that has been presented in many papers [33-35].

Lemma 1[33]: The 2-D system (13) with $u^h(i,j)=0$ and $u^v(i,j)=0$ is asymptotically stable if there exist two positive definite matrices P_1 and P_2 such that

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} - \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} < 0 \tag{15}$$

With respect to (15), let's define two functions as follows.

$$v(i,j) = \frac{1}{2}\left(\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \right) \tag{16}$$

$$v_{11}(i,j) = \frac{1}{2}\left(\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} \right) \tag{17}$$

The two functions (3) and (4) are called 2-D Lyapunov functions with different delays. In addition, the difference of the 2-D Lyapunov functions can be defined as

$$\Delta v(i,j) = v_{11}(i,j) - v\ (i,j) \tag{18}$$

Lemma 2: The zero input 2-D system (1) is asymptotically stable if

$$\Delta v(i,j) < 0 \tag{19}$$

Proof: Note, the zero input system (13) is

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \tag{20}$$

By replacing the state vector (20) into (17) we can get

$$v_{11}(i,j) = \frac{1}{2}\left(\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \right) \tag{21}$$

Therefore, the difference equation $\Delta v(i,j)$ is

$$\begin{aligned}\Delta v(i,j) &= \frac{1}{2}\left(\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \right) - \\ &\quad \frac{1}{2}\left(\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \right) \\ &= \frac{1}{2}\left(\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}^T Q \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \right)\end{aligned} \tag{22}$$

Where

$$Q = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}^T \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} - \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$$

It is clear that the expression $\Delta v(i,j) = 0$ results in the stability condition (15). In other words, both expressions (15) and (19) are equivalent. This completes the proof.

In General, in order to consider the stability of dynamic (13), the following relations are mentioned. By using z-transform for dynamic (13) we have

$$\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} = \left(\begin{bmatrix} z_1 & 0 \\ 0 & z_2 \end{bmatrix} - \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \right)^{-1} \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} u(i,j) \tag{23}$$

As a result, the output y is as follows

$$\begin{bmatrix} y^h(i,j) \\ y^v(i,j) \end{bmatrix} = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \left(\begin{bmatrix} z_1 & 0 \\ 0 & z_2 \end{bmatrix} - \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \right)^{-1} \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} u(i,j) \tag{24}$$

So for an iterative 2-D system the relation (24) can be rewritten as follows

$$\begin{bmatrix} y_k^h(i,j) \\ y_k^v(i,j) \end{bmatrix} = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \left(\begin{bmatrix} z_1 & 0 \\ 0 & z_2 \end{bmatrix} - \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \right)^{-1} \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} u_k(i,j) \tag{25}$$

In which, the subscript "k" in previous relation denotes a trial of the system which should be controlled. So, by using the relation (25) the dynamic (13) can be expressed in the form

$$Y_k = RU_k \tag{26}$$

Where R is a lower-triangular Toeplitz matrix whose elements are the Markov parameters of the system to be controlled. We call R the system impulse response matrix. For linear, timevarying systems and some classes of affine nonlinear systems, a similar representation can be developed, with the key feature being that the matrix R is lower-triangular. Using this formulation, the problem then becomes to design a learning gain matrix γ so the resulting "closed-loop system" in the iteration domain, given by $E_{k+1} = (I - R\gamma)E_k$ where $E_k = Y_{des} - Y_k$ is the error, for some desired trajectory Y_k, is either asymptotically and/or monotonically convergent (stable) along the iteration axis in an appropriate norm topology. Such stability conditions have been analyzed in Moore and Chen (2002) and design issues have been considered in Chen and Moore (2002).

Now, defining the error vector on the iteration trial (12) as

$$\begin{cases} E_k = Y_{des} - Y_k \\ E_{k+1} = Y_{des} - Y_{k+1} \end{cases} \tag{27}$$

Furthermore, the update equation (11) can be rewritten as

$$U_{k+1} = U_k + \gamma E_k \tag{28}$$

By using (27) and (28) we have

$$\begin{aligned} E_{k+1} - E_k &= -E_k + E_{k+1} = \\ -RU_{k+1} + RU_k &= -R\gamma U_k \end{aligned} \tag{29}$$

Which yields the error evolution law:

$$E_{k+1} = (I - R\gamma)E_k \tag{30}$$

In ILC, there are two stability concepts: asymptotic stability and monotonic convergence. In asymptotic stability, two concepts should be differentiated according to the ILC gain matrix structure.

For the learning gain matrix, the following definition is used.

Definition 2: [32] The learning gain matrix γ consists of a combination of Arimoto-like gains, causal gains and non-causal gains defined as

- Arimoto-like gains: The learning gains are placed in diagonal terms of γ, i.e. γ_{ij}, $i = j$.
- Causal gains: The learning gains are placed in the lower triangular part of γ, i.e. γ_{ij}, $i > j$.
- Non-causal gains: The learning gains are placed in the upper triangular part of γ, i.e. γ_{ij}, $i < j$.

When Arimoto-like gains and purely causal gains are used, the asymptotic stability condition is defined as

$$\left|1 - \gamma_{ij} r_1\right| < 1 \qquad i = 1,\ldots,n. \tag{31}$$

Where r_i is the first non-zero Markov parameter. When noncausal gains are used, the asymptotic stability condition is defined as: $\rho(I\text{-}R\gamma)<1$, where ρ is the spectral radius of $(I\text{-}R\gamma)$. However, from (30), using the relationship

$$\|E_{k+1}\| = \|(I - R\gamma)E_k\| \le \|(I - R\gamma)\|\|E_k\| \tag{32}$$

the monotonic convergence condition can also be simply defined in an appropriate norm topology such as

Definition 3: If $\|(I - R\gamma)\|_1 < 1$, then $\|E_k\|$ is monotonically convergent to zero in l_1-norm topology.

Definition 4: If $\|(I - R\gamma)\|_\infty < 1$, then $\|E_k\|$ is monotonically convergent to zero in l_∞-norm topology.

SIMULATION RESULTS

Let a 2-D system in GR model with the following parameters

$$G = \begin{bmatrix} 0.7015 & -0.7846 \\ -1.6573 & -0.7190 \end{bmatrix}$$
$$H = \begin{bmatrix} 1.2632 \\ 0.3524 \end{bmatrix}$$

Suppose the learning parameters are $\gamma_1 = 0.5$ and $\gamma_2 = 0.01$. Also, assume the matrix C is a proper unique matrix. Furthermore, the boundary conditions are supposed to be zero. The value of control law in the first iteration also supposed to be zero. The desired trajectory which 2-D system should be achieved is assumed to be zero. The number of iterations is 50.

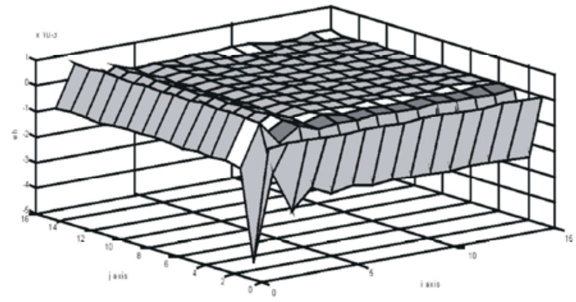

Fig. 1: System error $e^{h}(i,j)$

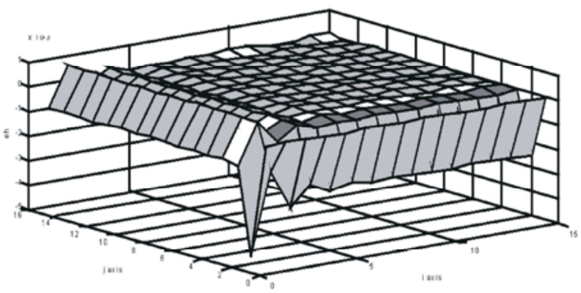

Fig. 2: System error $e^{v}(i,j)$

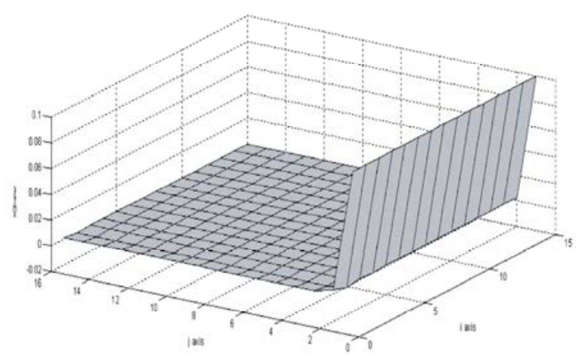

Fig. 3: System state e^{h} (i,j)

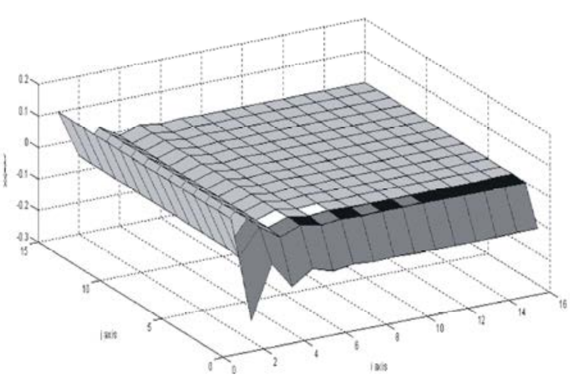

Fig. 4: System state $e^{v}(i,j)$

$$\begin{bmatrix} y_{des}^{h} \\ y_{des}^{v} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

With these assumptions the following results are available.

CONCLUSION

In this article, a new method of ILC for discretized 2-D systems by using GR model is presented. In this regard, the 2-D models given in literature are considered firstly and the stability of the presented method has been considerd. Finally, the simulation results have been used to figure out the validity of the method.

REFERENCES

1. Tzafestas, S.G., 1986. “ Multidimensional systems, Technique and Applications” Marcel Decker Pub.,
2. Shafiee, M., 1987. “ 2-D stabilization of the wave model “ PHD thesis, Baton Rouge Louisiana state univ. Dec.,
3. Heath, W.P. and P.E. Wellstead, 1995. “ Self tunning prediction and control for two dimensional process-part 1: Fixed parameter algorithms “ Int. J. Control., 62: 239-269.
4. ………” Self tunning prediction and control for two dimensional processes-part 2 : parameter estimation, set point tracking and offset handling “ Int. J. Control., 1995. 62: 239-269.
5. Wellstead, P.E. and J.R. Caldas-Pinto, 1985. “Self tunning filters and predictors for two dimensional systems part: 1 Algorithms “ Int. J. Control., 42: 457-478.
6. Biemond, J. and F.G. Van Der Putten and J.W. Woods, 1988. ” Identification and restoration images with symmetric noncausal blurs” IEEE Trans.Circuit and Systems, 35: 385-393.
7. Kaufman, H., *et al.,* 1983. “ Estimation and identification of 2-D images “ IEEE Trans. Aut. Contr., 28: 745-756.
8. Azimi-Sajadi, M.R. and J. Bannour, 1991. ” Two dimensional recursive parameter identification for adaptive Kalman filtering ” IEEE Trans. Circuit and Systems, 38: 1077-1081.
9. Teklap rtal, A.M., 1989. “Edga-adaptive Kalman filtering for image restoration with ringing suppression “ IEEE Trans. ASSP. 37: 892-899.

10. ………………, 1985. " Fast recursive estimation of parameters of a spacevarying autoregressive image model" IEEE Trans. ASSP. 29: 469-472.
11. Kalmka, J., 1992. " Controllability and optimal control for 2-D linear systems" MMAR92, pp: 234-239. Poland.
12. Kaczorec, T., 1994. " Mininmum energy control of 2-D continuos discrete linear systems " Bull. Of the Polish Academy of Science and Technol., pp: 456-462.
13. Lee, E.B. and Y. You, 1988. "Optimal control of two dimensional linear systems" Proc. 26-th CDC Los Angeles, CA, pp: 1572-1576.
14. Li, C. and M.S. Fadali, 1991. " Optimal control of 2-D systems" IEEE Trans. Automat. Contr., AC-36(2): 223-228.
15. Kaczorec, T. and J. Klamka, 1988. " Minimum energy control for general model of 2-D linear systems " Int. Jour. Contr., 47(5): 1555-1562, 1988.
16. Kaczorek, T., 1985. Lecture Notes in Control and Information Sciences 68: Two-Dimensional Linear System. New York: Springer-Verlag,
17. Galkowski, K., 2000. "High order discretization of 2-D systems," IEEE Trans.Circuits and Systems I, 4: 713-722.
18. Roesser, R.P., 1975. "A discrete state-space model for linear image processing," IEEE Trans. Automat. Contr., 20: 1-10.
19. Lewis, F.L., 1992. "A review of 2-D implicit systems," Automatica, 28: 345-354.
20. Chen, C.W., J.S.H. Tsai and L.S. Shieh, 2000. "Two-dimensional discrete continuous model conversion," Circuits, Syst. Signal Processing, 18: 565-585.
21. Li, J.S., J.S. H. Tsai and L.S. Shieh, 1999. "Optimal control for two-dimensional linear systems with variable coefficients," Asian J. Control., 1: 245-257..
22. Jagannathan, M. and V.L. Syrmos, 1996. "A linear-quadratic optimal regulator for two-dimensional systems," in Proc. IEEE Conf. Decision and Control., pp: 4172-4177.
23. Arimoto, S., S. Kawamura and F. Miyazaki, 1984. "Bettering operation of robots by learning," J. Robotic Systems, 1(2): 123-140.
24. Rogers, E. and D.H. Owens, 1992. Stability Analysis for Linear Repetitive Processes. Springer-Verlag, Berlin Heidelberg.
25. Geng, Z., R. Carroll and J. Xie, 1990. "Two dimensional model and algorithm analysis for a class of iterative learning control systems". International J. Control., 52(4): 833-862.
26. Kurek, J.E. and M.B. Zaremba, 1993. "Iterative learning control synthesis based on 2-D system theory". IEEE Transactions on Automatic Control., 38(1): 121-125.
27. Fang, Y. and T.W.S. Chow, 2003. "2-D analysis for iterative learning controller for discrete-time systems with variable initial conditions". IEEE Transactions on Circuits and Systems -I: Fundamental Theory and Applications, 50(5): 722-727.
28. Shi, J., F. Gao and T.-J. Wu, 2005. "Robust design of integrated feedback and iterative learning control of a batch process based on a 2D Roesser system". J. Process Control., 15: 907-924.
29. Shi, J., F. Gao and T.J. Wu, 2005. "Integrated design and structure analysis of robust iterative learning control system based on a two-dimensional model". Industrial and Engineering Chemistry Res., 44(21): 8095-8105.
30. Roopaei, M., A. Argha, P. Karimaghaee, 2009. "On the Control of 2-D Systems by Intelligent Schemes". World Applied Sciences J., 6(3): 385-392
31. Frornasini, W. and Marchesini, 1976. "State space realization theory of 2-D filters " IEEE Trans. Aut. Contr., pp: 21.
32. Hyo-Sung Ahna, Kevin L. Mooreb and YangQuan Chen" 2007. Stability analysis of discrete-time iterative learning control systems with interval uncertainty" Automatica, 43: 892-902.
33. Anderson, B.D.O., P.A. Agathoklis, E.I. Jury and M. Mansour, 1986. "Stability and the matrix Lyapunov equation for discrete 2-dimensional systems," IEEE Trans. Circuits Sys., 33(3): 261-267.
34. Kar, H., 2008. "A new sufficient condition for the global asymptotic stability of 2-D state-space digital filters with saturation arithmetic," Signal Processing, Elsevier, 88: 86-98.
35. Singh, V., 2008. "On global asymptotic stability of 2-D discrete systems with state saturation," Physics Letters A, Elsevier, 372: 5287-5289.
36. Adloo, H., P. Karimaghaee and A.S. Sarvestani, "An Extension of Sliding Mode Control Design for the 2-D systems in Roesser Model