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Application of Homotopy Perturbation Method to Nonlinear Drinfeld-Sokolov-Wilson Equation

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Abstract: Homotopy perturbation method has been applied to solve many functional equations so far. In this work, we propose this method (HPM), for solving Drinfeld-Sokolov-Wilson equation [14-15]. Numerical solutions obtained by the homotopy perturbation method are compared with the exact solutions. The results for some values for the variables are shown in the tables and the solutions are presented as plots as well, showing the ability of the method.

Key words: Homotopy perturbation method · Drinfeld-Sokolov-Wilson

INTRODUCTION

Large varieties of physical, chemical and biological phenomena are governed by nonlinear evolution equations. Except a limited number of these problems, most of them do not have precise analytical solutions so that they have to be solved using other methods. Homotopy perturbation method has been used by many mathematicians and engineers to solve various functional equations. This method continuously deforms a simple problem, easy to solve, into a difficult problems under study [1-2]. Almost all perturbation methods are based on the assumption of the existence of a small parameter in the equation. But most non-linear problems have no such a small parameter. This method has been proposed to eliminate the small parameter. In recent years the application of homotopy perturbation theory has appeared in many researches [3-13].

Solution of System of Partial Differential Equations by Homotopy Perturbation Method: We first consider the system of partial differential equations written in an operator form

$$\begin{aligned}
&\frac{\partial u_1}{\partial t}+\frac{\partial u_2}{\partial x_1}+\ldots+\frac{\partial u_n}{\partial x_{n-1}}+N_1=g_1,\\
&\frac{\partial u_2}{\partial t}+\frac{\partial u_1}{\partial x_1}+\ldots+\frac{\partial u_n}{\partial x_{n-1}}+N_2=g_2,\\
&\vdots\\
&\frac{\partial u_n}{\partial t}+\frac{\partial u_2}{\partial x_1}+\ldots+\frac{\partial u_1}{\partial x_{n-1}}+N_n=g_n.
\end{aligned} \tag{1}$$

with initial conditions

$$\begin{aligned}
&u_1(x_1,x_2,\ldots x_{n-1},0)=f_1(x_1,x_2,\ldots x_{n-1}),\\
&u_2(x_1,x_2,\ldots x_{n-1},0)=f_2(x_1,x_2,\ldots x_{n-1}),\\
&\vdots\\
&u_n(x_1,x_2,\ldots x_{n-1},0)=f_n(x_1,x_2,\ldots x_{n-1}).
\end{aligned} \tag{2}$$

Where $N_1,N_2,......N_n$ are nonlinear operators and $g_1,g_2,......g_n$ are inhomogeneous terms.

To solve system (1) by homotopy perturbation method, we construct the following homotopies:

$$\begin{aligned}
&(1-p)(\frac{\partial U_1}{\partial t}-\frac{\partial u_{10}}{\partial t})+p(\frac{\partial U_1}{\partial t}+\frac{\partial U_2}{\partial x_1}+\ldots+\frac{\partial U_n}{\partial x_{n-1}}+N_1-g_1)=0,\\
&(1-p)(\frac{\partial U_2}{\partial t}-\frac{\partial u_{20}}{\partial t})+p(\frac{\partial U_2}{\partial t}+\frac{\partial U_1}{\partial x_1}+\ldots+\frac{\partial U_n}{\partial x_{n-1}}+N_2-g_2)=0,\\
&\vdots\\
&(1-p)(\frac{\partial U_n}{\partial t}-\frac{\partial u_{n0}}{\partial t})+p(\frac{\partial U_n}{\partial t}+\frac{\partial U_2}{\partial x_1}+\ldots+\frac{\partial U_1}{\partial x_{n-1}}+N_n-g_n)=0.
\end{aligned} \tag{3}$$

Let's present the solution of the system (3) as the following

$$\begin{aligned}
&U_1=U_{10}+pU_{11}+p^2U_{12}+\ldots,\\
&U_2=U_{20}+pU_{21}+p^2U_{22}+\ldots,\\
&U_3=U_{30}+pU_{31}+p^2U_{32}+\ldots,\\
&\vdots\\
&U_n=U_{n0}+pU_{n1}+p^2U_{n2}+\ldots.
\end{aligned} \tag{4}$$

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Equating the coefficients of the terms with the identical powers of p, leads to

$$p^0:\begin{cases}\dfrac{\partial U_{10}}{\partial t}-\dfrac{\partial u_{10}}{\partial t}=0,\\ \dfrac{\partial U_{20}}{\partial t}-\dfrac{\partial u_{20}}{\partial t}=0,\\ \vdots\\ \dfrac{\partial U_{n0}}{\partial t}-\dfrac{\partial u_{n0}}{\partial t}=0,\end{cases}$$

$$p^1:\begin{cases}\dfrac{\partial U_{11}}{\partial t}+\dfrac{\partial u_{10}}{\partial t}+\dfrac{\partial U_{20}}{\partial x_1}+\ldots+\dfrac{\partial U_{n0}}{\partial x_{n-1}}+M_{10}-g_1=0,\\ \dfrac{\partial U_{22}}{\partial t}+\dfrac{\partial u_{20}}{\partial t}+\dfrac{\partial U_{10}}{\partial x_1}+\ldots+\dfrac{\partial U_{n0}}{\partial x_{n-1}}+M_{20}-g_2=0,\\ \vdots\\ \dfrac{\partial U_{n1}}{\partial t}+\dfrac{\partial u_{10}}{\partial t}+\dfrac{\partial U_{20}}{\partial x_1}+\ldots+\dfrac{\partial U_{n0}}{\partial x_{n-1}}+M_{n0}-g_n=0,\end{cases}$$

$$p^2:\begin{cases}\dfrac{\partial U_{12}}{\partial t}+\dfrac{\partial U_{21}}{\partial x_1}+\ldots+\dfrac{\partial U_{n1}}{\partial x_{n-1}}+M_{11}=0,\\ \dfrac{\partial U_{22}}{\partial t}+\dfrac{\partial U_{11}}{\partial x_1}+\ldots+\dfrac{\partial U_{n1}}{\partial x_{n-1}}+M_{21}=0,\\ \vdots\\ \dfrac{\partial U_{n2}}{\partial t}+\dfrac{\partial U_{21}}{\partial x_1}+\ldots+\dfrac{\partial U_{11}}{\partial x_{n-1}}+M_{n1}=0,\end{cases}$$

$$\vdots$$

$$p^j:\begin{cases}\dfrac{\partial U_{1j}}{\partial t}+\dfrac{\partial U_{2j-1}}{\partial x_1}+\ldots+\dfrac{\partial U_{nj-1}}{\partial x_{n-1}}+M_{1j-1}=0,\\ \dfrac{\partial U_{2j}}{\partial t}+\dfrac{\partial U_{1j-1}}{\partial x_1}+\ldots+\dfrac{\partial U_{nj-1}}{\partial x_{n-1}}+M_{2j-1}=0,\\ \vdots\\ \dfrac{\partial U_{nj}}{\partial t}+\dfrac{\partial U_{2j-1}}{\partial x_1}+\ldots+\dfrac{\partial U_{1j-1}}{\partial x_{n-1}}+M_{nj-1}=0.\end{cases}$$

$$\vdots$$

Where M_{ij}, $i = 1,2,......n$, $j = 0,1,2,......n\text{-}1$, are terms that obtain with equating the coefficients of the nonlinear operators N_{ij}, $i = 1,2,......n$, $j = 0,1,2,......n\text{-}1$, with the identical powers of P

For simplicity we take

$$\begin{aligned}U_{10}&=u_{10}=f_1(x_1,x_1,\ldots,x_{n-1}),\\ U_{20}&=u_{20}=f_2(x_1,x_1,\ldots,x_{n-1}),\\ &\vdots\\ U_{n0}&=u_{n0}=f_n(x_1,x_1,\ldots,x_{n-1}).\end{aligned}\tag{5}$$

We have the following scheme

$$\begin{aligned}U_{11}(x,t)&=-\int_0^t\Big(\frac{\partial U_{20}}{\partial x_1}+\ldots+\frac{\partial U_{n0}}{\partial x_{n-1}}+M_{10}-g_1\Big)dt,\\ U_{21}(x,t)&=-\int_0^t\Big(\frac{\partial U_{10}}{\partial x_1}+\ldots+\frac{\partial U_{n0}}{\partial x_{n-1}}+M_{20}-g_2\Big)dt,\\ &\vdots\\ U_{n1}(x,t)&=-\int_0^t\Big(\frac{\partial U_{20}}{\partial x_1}+\ldots+\frac{\partial U_{10}}{\partial x_{n-1}}+M_{n0}-g_n\Big)dt.\end{aligned}$$

Having this assumption we get the following iterative equations

$$\begin{aligned}U_{1j}(x,t)&=-\int_0^t\Big(\frac{\partial U_{2j-1}}{\partial x_1}+\ldots+\frac{\partial U_{n0}}{\partial x_{n-1}}+M_{1j-1}\Big)dt,\ j=2,3,\ldots,\\ U_{2j}(x,t)&=-\int_0^t\Big(\frac{\partial U_{1j-1}}{\partial x_1}+\ldots+\frac{\partial U_{nj-1}}{\partial x_{n-1}}+M_{2j-1}\Big)dt,\ j=2,3,\ldots,\\ &\vdots\\ U_{nj}(x,t)&=-\int_0^t\Big(\frac{\partial U_{2j-1}}{\partial x_1}+\ldots+\frac{\partial U_{1j-1}}{\partial x_{n-1}}+M_{nj-1}\Big)dt,\ j=2,3,\ldots.\end{aligned}$$

The approximate solution of (1) can be obtained by setting $p=1$

$$\begin{aligned}u_1&=\lim_{p\to 1}U_1=U_{10}+U_{11}+U_{12}+\ldots,\\ u_2&=\lim_{p\to 1}U_2=U_{20}+U_{21}+U_{22}+\ldots,\\ u_3&=\lim_{p\to 1}U_3=U_{30}+U_{31}+U_{32}+\ldots,\\ &\vdots\\ u_n&=\lim_{p\to 1}U_n=U_{n0}+U_{n1}+U_{n2}+\ldots.\end{aligned}\tag{6}$$

Applications: Consider the following Drinfeld-Sokolov-Wilson equation

$$\begin{aligned}&\frac{\partial u}{\partial t}+3v\frac{\partial v}{\partial x}=0,\\ &\frac{\partial v}{\partial t}-2\frac{\partial^3 v}{\partial x^3}+\frac{\partial u}{\partial x}v+2u\frac{\partial v}{\partial x}=0.\end{aligned}\tag{7}$$

With the following initial condition

$$\begin{aligned}u(x,0)&=3\sec h^2(x),\\ v(x,0)&=2\sec h(x).\end{aligned}\tag{8}$$

For solving Eq (7) with initial conditions (8) according to the homotopy perturbation, we construct the following homotopy:

$$(1-p)(\frac{\partial U}{\partial t}-\frac{\partial u_0}{\partial t})+p(\frac{\partial U}{\partial t}+3V\frac{\partial^3 V}{\partial x^3})=0,$$

$$(1-p)(\frac{\partial V}{\partial t}-\frac{\partial v_0}{\partial t})+p(\frac{\partial V}{\partial t}-2\frac{\partial^3 V}{\partial x^3}+\frac{\partial U}{\partial x}V+2U\frac{\partial V}{\partial x})=0,$$

or

$$\begin{aligned}&\frac{\partial U}{\partial t}-\frac{\partial u_0}{\partial t}+p(\frac{\partial u_0}{\partial t}++3V\frac{\partial^3 V}{\partial x^3})=0,\\&\frac{\partial V}{\partial t}-\frac{\partial v_0}{\partial t}+p(\frac{\partial v_0}{\partial t}-2\frac{\partial^3 V}{\partial x^3}+\frac{\partial U}{\partial x}V+2U\frac{\partial V}{\partial x})=0.\end{aligned}\qquad(9)$$

Suppose the solution of Eq. (9) has the for

$$\begin{aligned}U&=U_0+pU_1+p^2U_2+\ldots\\V&=V_0+pV_1+p^2V_2+\ldots.\end{aligned}\qquad(10)$$

Substituting (10) into (9) and equating the coefficients of the terms with the identical powers of p

$$P^0:\begin{cases}\frac{\partial U_0}{\partial t}-\frac{\partial u_0}{\partial t}=0,\\\frac{\partial V_0}{\partial t}-\frac{\partial v_0}{\partial t}=0,\end{cases}$$

$$P^1:\begin{cases}\frac{\partial U_1}{\partial t}+\frac{\partial u_0}{\partial t}+\frac{\partial^3 U_0}{\partial x^3}3V_0\frac{\partial^3 V_0}{\partial x^3}=0,\\\frac{\partial V_1}{\partial t}+\frac{\partial v_0}{\partial t}-2\frac{\partial^3 V_0}{\partial x^3}+\frac{\partial U_0}{\partial x}V_0+2U_0\frac{\partial V_0}{\partial x}=0,\end{cases}$$

$$P^2:\begin{cases}\frac{\partial U_2}{\partial t}+3V_0\frac{\partial V_1}{\partial x}+3V_1\frac{\partial V_0}{\partial x}V_1=0,\\\frac{\partial V_2}{\partial t}+-2\frac{\partial^3 V_1}{\partial x^3}+\frac{\partial U_1}{\partial x}V_0+\frac{\partial U_0}{\partial x}V_1+2U_1\frac{\partial V_0}{\partial x}+2U_0\frac{\partial V_1}{\partial x}=0.\end{cases}$$

$$P^j:\begin{cases}\frac{\partial U_j}{\partial t}+3\sum_{0}^{j-1}V_k\frac{\partial V_{j-1-k}}{\partial x}=0,\\\frac{\partial V_j}{\partial t}-2\frac{\partial^3 V_{j-1}}{\partial x^3}+\sum_{k=0}^{j-1}\frac{\partial U_k}{\partial x}V_{j-1-k}+2\sum_{k=0}^{j-1}U_k\frac{\partial V_{j-1-k}}{\partial x}=0.\end{cases}$$

$\vdots$

We start with initial approximations $u(x,0)$ and $v(x,0)$ given by Eq. (8)

$$\begin{aligned}U_0&=u_0=3\operatorname{sec}h^2(x),\\V_0&=v_0=2\operatorname{sec}h(x).\end{aligned}\qquad(11)$$

And we have the following recurrent equations for $j = 1,2,3,...$

$$\begin{aligned}U_j&=-3\int_0^t\left(-2\frac{\partial^3 V_{j-1}}{\partial x^3}+\sum_{k=0}^{j-1}\frac{\partial U_k}{\partial x}V_{j-1-k}+2\sum_{k=0}^{j-1}U_k\frac{\partial V_{j-1-k}}{\partial x}\right)dt,\\V_j&=-3\int_0^t\left(\sum_{0}^{j-1}V_k\frac{\partial V_{j-1-k}}{\partial x}\right)dt.\end{aligned}\qquad(12)$$

With the iteration formula (12) we get

$$U_1=-12sec^2(x)tan(x)t,$$
$$V_1=24sec(x)tan(x)^3t+20sec(x)$$
$$tan(x)t-24sec(x)^3tan(x)t,$$
$$U_2=-360t^2sec(x)^2tan(x)^4-396t^2sec(x)^2tan(x)^2$$
$$-60t^2sec(x)^2+360t^2sec(x)^4tan(x)^2+72t^2sec(x)^4$$
$$V_2=72t^2sec(x)^5+244t^2sec(x)+2880t^2sec(x)tan(x)^6$$
$$-3240t^2sec(x)^3tan(x)^4+360t^2sec(x)^5tan(x)^2+2648t^2sec(x)tan(x)^2$$
$$+5280t^2sec(x)tan(x)^4-2832t^2sec(x)^3tan(x)^2-312t^2sec(x)^3,$$

An approximation to the solution of (7) can be obtained by setting $p = 1$

$$u=\lim_{p\to 1}U=U_0+U_1+U_2+\ldots,$$
$$v=\lim_{p\to 1}V=V_0+V_1+V_2+\ldots.$$

Suppose $u^*=\sum_{j=0}^{3}U_j$, and $v^*=\sum_{j=0}^{3}V_j$, the results are presented in Table 1 and Fig. 1.

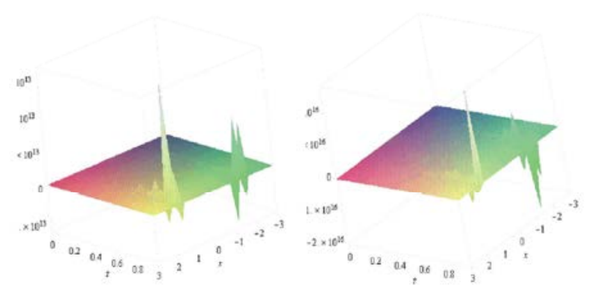

Fig. 1: The numerical results for are, respectively (a) and (c)

CONCLUSIONS

In this article, we have applied homotopy perturbation method for the solving the nonlinear Drinfeld-Sokolov-Wilson equation The results show that the homotopy perturbation method is a powerful mathematical tool for solving systems of nonlinear partial differential equations having wide applications in sciences and engineering. This method is introduced to overcome the difficulty arising in calculating Adomain polynomial in Adomian method. In our work, we use the maple package to carry the computations.

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