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Application of Optimal Control Theory to Adjust the Production Rate of Deteriorating Inventory System (Case Study: Dineh Iran Co.)

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Abstract: In this paper, optimal control of production inventory system with deteriorating items is considered. It is assumed that deterioration rate follows the Weibull distribution. It is, also, assumed that the demand of the manufacturing product is time dependent.For building our model we will assume that the firm has set an inventory goal level and production goal rate. We will study a periodic policy review and Lagrangian technique is used. The objective is to determine the optimal production rate. We adjust the optimal production rate to minimum total (production and inventory) costs. The assumption of a time variant demand is more real in the world and makes the result of this research more practical for Industries. This model has been applied in Dineh Iran Co. The Group of Pharmaceutical, Hygienic & Food Industries.

Key words: Production planning · Deterioration items · Weibull distribution · Optimal control theory · Lagrangian technique · Inventory control system

INTRODUCTION

The control and maintenance inventories for deteriorating items with shortages have received much attention of several researchers in the recent years because most of the physical goods deteriorate over time. In reality, some of the items are either damaged or decayed or affected by some other factors and is not in a perfect condition to satisfy the demand. Deterioration is defined as change, damage, decay, spoilage obsolescence and loss of utility or loss of original value in a commodity that results in the decreasing usefulness from the original one. Food items, drugs, pharmaceuticals, radioactive substances are examples of such items. So decay or deterioration of physical goods in stock is a very realistic feature and researchers felt the necessity to use this factor into consideration in developing inventory models. In classical inventory models the demand rate is assumed to be a constant. In reality demand for physical goods may be time dependent, stock dependent and price dependent.

In this paper, we consider a production inventory system that produces a single product at a certain production rate (actual rate) and seeks an alternative production rate (desired rate).

Literature Review: The system is dynamic by nature and an optimal control approach seems particularly well suited to achieve its optimization. One fascinating aspect of optimal control theory is its wide range of applications. It has found successful applications in various areas of management science and operations research; see Sethi and Thompson [1]. We are especially interested in applications of optimal control theory. Ghare and Schrader [2] are the first persons that consider an inventory model with a constant rate of deterioration and exponentially deterioration rate. Covert and Philip [3] developed a model with Weibull distribution deterioration. A large number of theoretical papers make the assumption that the deterioration rate follows the Weibull distribution, see for example [4-11].

The problem that is interesting in this paper is the production planning problem. We consider a firm that produces a single product, selling some units and stocking the remaining units in a warehouse. The problem is presented as an optimal control problem with one control variable (production rate) and one state variable (inventory level) and a closed form solution is obtained using the maximum principle. Typically, the firm has to balance these costs and find the quantity which should produce in order to keep the total cost minimum. There are

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a few papers that used an optimal control approach to study dynamic production models. Padmanabhan and Vrat [12] considered an EOQ model for perishable items with stock-dependent demand. Chung and Dye [13] presented the necessary and sufficient conditions of the existence and uniqueness of the optimal solutions of the profit per unit time functions without backlogging and with complete backlogging. In Padmanabhan and Vrat_s [1] model, the backlogging function is assumed to be dependent on the amount of demand backlogged. Therefore, the more the amount of demand backlogged, the smaller the demand to accept backlogging would be. Their definition of backlogging rate, however, seems to be inappropriate under some circumstances. In real life, for fashionable commodities and high-tech products with short product life cycle, the length of the waiting time for the next replenishment is the main factor for deciding whether the backlogging will be accepted or not. The willingness of a customer to wait for backlogging during a shortage period is decline with the length of the waiting time. To reflect this phenomenon,

Chang and Dye [14] developed an inventory model in which the proportion of customers who would like to accept backlogging is the reciprocal of a linear function of the waiting time. Khemlnitsky and Gerchak [15] used an optimal control approach to solve a production system where demand depends on the inventory level. KiesmÄuller [16] was interested in the optimal control of recovery systems, where attention is given to recycling and remanufacturing of used products in order to reduce waste. Salama [17] considered the optimal control of an unreliable manufacturing system with restarting costs. Zhang *et al.* [18] were concerned with the scheduling of a marketing production system with a demand dependent on the marketing status. To build our model, we will assume that the demand rate is a general function of time. We will also assume that the firm has set an inventory goal level and a production goal rate. The inventory goal level is a safety stock that the company wants to keep on hand. The production goal rate is the most efficient rate desired by the firm. The objective is to determine the optimal production rate that will keep the inventory level and the production rate as close as possible to the inventory goal level and production goal rate, respectively. We will study a periodic policy review and Lagrangian technique is used. Then we adjust the optimal production rate to minimum total (production and inventory) costs.

Assumptions and Notations: The following notations and assumptions are used for developing the model.

I(t) = Inventory level at time t
P(t) = Production rate at time t
D(t) = Demand rate at time t
I_0 = Initial inventory level
$\hat{I}(t)$ = Inventory goal level at time t
$\hat{P}(t)$ = Production goal rate at time t
T = Planning horizon
h = Holding cost
k = Production cost
$\theta(t)$ = Deterioration rate

Time to deterioration is a random variable following the two-parameter Weibull distribution. The probability density function for two-parameter Weibull distribution is given by

$$f(t) = \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}}, t > 0$$

where

α= scale parameter, α>0
β= shape parameter, β>0

The probability distribution function is

$$F(t) = 1 - e^{-\alpha t^{\beta}}, t > 0$$

The instantaneous rate of deterioration of the on hand inventory is given by

$$\theta(t) = \alpha\beta t^{\beta-1}, t > 0$$

We will assume that the demand rate is general function of time. Also the firm has set an inventory goal level and production goal rate. We will determine the optimal production rate that will keep the inventory level and we develop the periodic-review policy and Lagrangian technique is used. Divide the planning horizon [0,T] in to N subintervals of equal length and denote respectively by I(k), P(k) and D(k).

The rate of deterioration-time relationship for the two-Parameter Weibull distribution is shown in Fig. 1.

The Mathematical Model and its Analysis

Model Discretization: We start by discretizing the model [19]. The approximate discrete form of equation (1) is

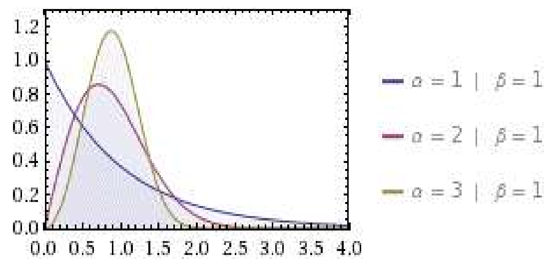

Fig. 1: Probability density function of Weibull distribution

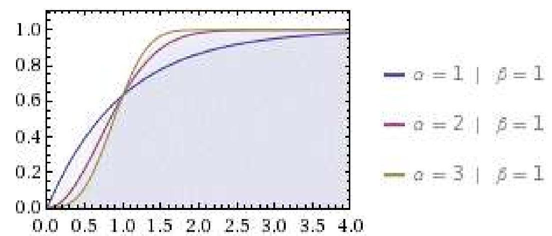

Fig 2: Cumulative distribution function

$$\frac{I(k+1)-I(k)}{T} = P(k) - D(k) - \alpha\beta\ k^{\beta-1} I(k) \quad (1)$$

Rearranging the terms in (1) gives

$$I(k+1) = [1 - T_s\alpha\beta k^{\beta-1}]I(k) + T_s[P(k - D(k) \quad (2)$$

Since I and P satisfy (2), we have

$$\hat{I} = [1 - T_s\alpha\beta k^{\beta-1}]\hat{I} + T_s[\hat{P}(k) - D(k)] \quad (3)$$

Now subtracting expression (3) from (2) yields

$$\Delta I(k+1) = [1 - T_s\alpha\beta k^{\beta-1}]\Delta I(k) + T_s\Delta P(k)$$

To simplify this expression, we let

$$a(k) = 1 - T_s\alpha\beta k^{\beta-1}$$

so

$$\Delta I(k+1) = a(k)\Delta I(k) + T_s\Delta P(k) \quad (4)$$

The objective function is

$$J = \frac{1}{2}\sum_{0}^{N}[h\Delta I(k)^2 + K\Delta P(k)^2\ ] \quad (5)$$

There for the problem is to determine the production rate P(k)>=0 that minimize (5) subject to constraint (4).

Analytical Solution: The problem is nonlinear and can be solved by Lagrangian technique.

$$L = \frac{1}{2}\sum_{0}^{N}\{h\Delta I(k)^2 + K\Delta P(k)^2\} + \lambda(k+1) \quad (6)$$
$$[-\Delta I(k+1) + a(k) + T_s\Delta P(k)]$$

The necessary optimality conditions

$$\nabla_{\Delta P(k)}L = 0, \nabla_{\Delta I(k)}L = 0, \nabla_{\lambda(k+1)}L = 0$$

Are respectively equivalent to

$$\Delta P(k) = -\frac{T_s}{K}\lambda(k+1) \quad (7)$$

$$\lambda(k) = h\ \Delta I(k) + a(k)\ \lambda(k+1) \quad (8)$$

and the constraint (4). To solve these equations, we use the sweep method of Bryson and Ho [2]. For k=0,…, N, introduce the positive quantities s(k) such that

$$\lambda(k) = s(k)\Delta I(k) \quad (9)$$

substituting (9) into (7) yields

$$\Delta P(k) = -\frac{T_s}{K}s(k+1)\Delta I(k+1) \quad (10)$$

Substituting (4) into (10) yields

$$\Delta P(k) = -\frac{T_s}{K}s(k+1)\left[a(k)\Delta I(k) + T_s\Delta P(k)\right] \quad (11)$$

We solve this equation for ΔP(k) to get

$$\Delta P(k) = -\frac{T_s a(k)s(k+1)}{K + T_s^2 s(k+1)}\Delta I(k) \quad (12)$$

Now, substitute (9) into (8)

$$s(k)\Delta I(k) = h\Delta I(k) + a(k)s(k+1)\Delta I(k+1) \quad (13)$$

Also, substitute (4) into (13)

$$s(k)\Delta I(k) = [h\Delta I(k) + a(k)^2 s(k+1)] \quad (14)$$
$$\Delta I(k) + T_s a(k)s(k+1)\Delta P(k)$$

and (12) into (14)

$$s(k)\Delta I(k) = \left[h + \frac{Ka(k)^2 s(k+1)}{K + T_s^2 s(k+1)}\right]\Delta I(k) \quad (15)$$

Hence, we obtain the discrete Ricatti equation

$$s(k) = h + \frac{Ks(k+1)}{K + T_s^2 s(k+1)} a(k)^2 \tag{16}$$

which needs to be solved backwards, starting from
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$$s(N)=h \tag{17}$$

since ΔP(N) = 0. Now, we turn to determining I(k). First, substitute (12) into (4)

$$\Delta I(k+1) = a(k)\left[1 - \frac{T_s s(k+1)}{K + T_s^2 s(k+1)}\right]\Delta I(k) \tag{18}$$

Then starting from $I(0)=I_0$. We compute recursively(forward)

$$I(k+1) = \hat{I} + a(k)\left[1 - \frac{T_s s(k+1)}{K + T_s^2 s(k+1)}\right]\left[I(k) - \hat{I}\right] \tag{19}$$

To determine P(k), again from (4) we have

$$\Delta P(k) = \frac{1}{T_s}\left[\Delta I(k+1) - a(k)\Delta I(k)\right] \tag{20}$$

And for k=0, …, N -1

$$\Delta P(k) = \hat{P}(k) + \frac{1}{T_s}\left[\Delta I(k+1) - a(k)\Delta I(k)\right] \tag{21}$$

Since only a nonnegative production rate is allowed, the optimal production rate is chosen equal to

$$\max\left\{\hat{P}(k) + \frac{1}{T_s}\left[\Delta I(k+1) - a(k)\Delta I(k)\right], 0\right\} K=0,......,N-1 \tag{22}$$

Numerical Example and Case Study: Here we have considered the 10 years previous data in the Dinah Iran Pvt. Ltd. which is the group of pharmaceutical, hygienic and food Industries was founded in 1980. Since then, Dineh Iran has continued its thriving achievement with new horizons, day by day. The company today is not only a top leader in manufacturing of herbal medicines but is also one the largest private owned body, producing and distributing herbal medicines in Iran. It produces herbal medicines in the form of tablets, ointments, syrups, granules, ext. The company today has 4,000,000 square meters of agricultural farm, 30,000 square meters in production units. It employs more than 500 employees and stepping in the fields of chemical medicine production too. Dineh Iran has blended science & technology and moreover, we are highly focused towards customer satisfaction with absolute safety. All products of Dineh Iran are highly effective, economical and safe. Today Dineh Iran is expanding its horizons in the Middle East, African countries and to most of the south East Asian countries.

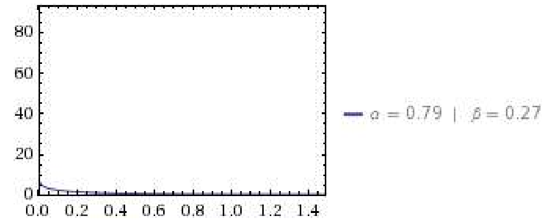

Fig 3: Probability density function

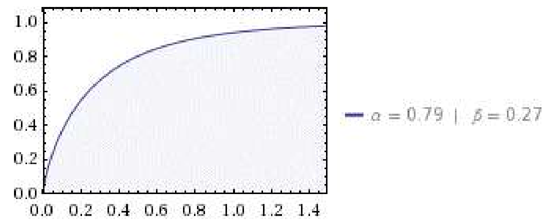

Fig 4: Cumulative distribution function

We have calculated the optimal production rate and minimize costs (production and inventory) for an especial drug. The result shown the following values

$\alpha = 0.79$
$\beta = 0.27$

The deterioration rate function is
$e = 0.2133\ t^{-0.21}$
$T_s = 24$
K=4266
H=84
I(0)= 145058
$\hat{I} = 133025$

The first step is to compute the production goal rate from (3) as

$$\hat{P}(k) = \max\{D(k) + \alpha\beta k^{\beta-1}\hat{I}, 0\}$$

Next we successively compute the vector s from (16), the optimal inventory level I from (19) and the optimal production rate P from (22). Figure 5 (left) shows the optimal inventory level and as can be seen, I converge toward $\hat{I}$. It also shows (right) the optimal production rate P which, as can be seen, converges toward $\hat{P}$. The new method shows the optimal amount "J=1.6396E+12" while the previous non-optimal amount was "J=3.08041E+18"

p(k)	$\hat{P}(t)$	D(k)	I(k+1)	k
454586	454505	426131	145058	1
890398	890557	777060	85620	2
560215	560020	304652	227580	3
941887	942047	488059	16011	4
1287063	1286975	577619	229099	5
1710068	1710100	688628	79886	6
1806592	1806584	416247	152386	7
2218256	2218257	402306	128703	8
2666242	2666242	367929	133514	9
3414284	3414284	576861	133013	10
4105467	4105467	672185	133024	11
4613054	4613054	527165	133025	12
4943533	4943533	148288	133025	13
6311487	6311487	750137	133025	14
6822584	6822584	438382	133025	15
7700321	7700321	436517	133025	16
8837395	8837395	637242	133025	17
9535484	9535484	342233	133025	18
10915367	10915367	672269	133025	19
12087273	12087273	737580	133025	20
12705237	12705237	192200	133025	21
14063533	14063533	330404	133025	22
15596054	15596054	586085	133025	23
17276238	17276238	932680	133025	24

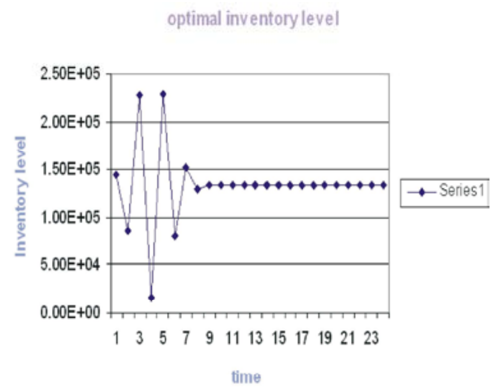

Fig. 5: Optimal inventory level

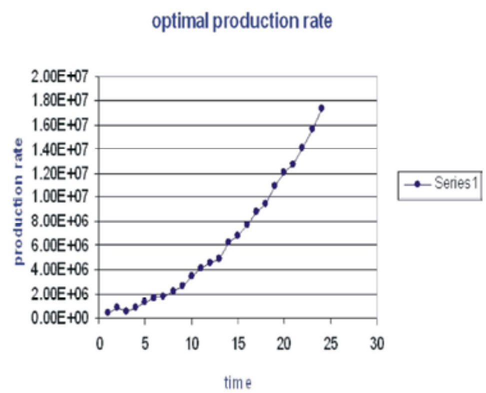

Fig 6: Optimal production rate

CONCLUSIONS

In this paper, optimal control of production inventory system with deteriorating items is considered. It is assumed that deterioration rate follows the Weibull distribution. It is, also, assumed that the demand of the manufacturing product is time dependent.For building our model we will assume that the firm has set an inventory goal level and production goal rate.

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