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Exp-Function Method for Generalized Regularized Long-wave Equation

[1]*Syed Tauseef Mohyud-Din,* [2,3]*Ahmet Yıldırım and* [2]*Zehra Pınar*

[1]Department of Mathematics, HITEC University Taxila Cantt, Pakistan
[2]Ege University, Department of Mathematics, 35100 Bornova, Izmir, Turkey
[3]University of South Florida, Department of Mathematics and Statistics, Tampa, FL, 33620-5700, USA

Abstract: In this paper, exp-function method is used to construct generalized solitonary solutions of the generalized regularized long-wave equation. It is shown that the Exp-function method, with the help of symbolic computation, provides a straightforward and powerful mathematical tool to solve nonlinear evolution equations with forcing term in mathematical physics.

Key words: Exp-function method · Solitary solution · Periodic solution · Generalized regularized long-wave equation

INTRODUCTION

The nonlinear partial differential equations [**1-291**] plays a pivotal role in the mathematical modeling of diversified physical phenomena. Finding exact solutions [1-21] of nonlinear evolution equations (NLEEs) has become one of the most exciting and extremely active areas of research investigation. The investigation of exact travelling wave solutions to nonlinear evolution equations plays an important role in the study of non-linear physical phenomena. The wave phenomena are observed in fluid dynamics, plasma, elastic media, optical fibres, etc. Many effective methods have been presented such as variational iteration method [1], homotopy perturbation method [2], Adomian decomposition method [3] and others [4]. The aim of the present paper is to extend the exp-function method to find new solitary solutions and periodic solutions for generalized regularized long-wave equation. Recently Jafari *et al*. [5] used the sine-cosine and the tanh methods to obtain solutions of the generalized regularized long-wave equation.

Exp-function Method: The exp-function method was first proposed by He and Wu [6, 7] and systematically studied to a class of nonlinear partial differential equations [8-11, 19-21]. We consider the general nonlinear partial differential equation of the type.

$$P(u,u_t,u_x,u_{tt},u_{xx},u_{xxx},...)=0 \qquad (1)$$

Using a transformation

$$\eta = kx + wt \qquad (2)$$

Where:
k and w are constants, we can rewrite Eq. (1) in the following nonlinear ODE.

$$Q(u,u',u'',u''',u^{(iv)},u^{(v)},...)=0 \qquad (3)$$

According to the exp-function method, which was developed by He and Wu [6, 7], we assume that the wave solutions can be expressed in the following form.

$$u(\eta)=\frac{\sum_n^d=-_c a_n \exp(n\eta)}{\sum_m^q=-_p b_m \exp(m\eta)} \qquad (4)$$

Where:
p,q,d and c are positive integers which are known to be further determined, a_n and b_m are unknown constants. We can rewrite equation (4) in the following equivalent form.

$$u(\eta)=\frac{a_c\ \exp(c\eta)+...+a_{-d}\exp(-d\eta)}{b_p\ \exp(p\eta)+...+b_{-q}\exp(-q\eta)} \qquad (5)$$

This equivalent formulation plays an important and fundamental part for finding the analytic solution of problems._To determine the value of c and p, we balance the linear term of highest order of equation (4) with the highest order nonlinear term. Similarly, to determine the value of d and q, we balance the linear term of lowest order of equation (3) with lowest order non linear term.

Corresponding Author: Dr. Syed Tauseef Mohyud-Din, Department of Mathematics, HITEC University Taxila Cantt, Pakistan.

The Generalized Long-wave Equation: We consider the generalized and regularized long-wave equation (RLW) [12]:

$$u_t + u_x + a(u^p)x - \beta u_{xxt} = 0 \tag{6}$$

Where:
p is a positive integer, α and β are positive constant. This equation was first put forward as a model for small amplitude long waves on the surface of water in channel by Peregine [13, 14] and later by Benjamin *et al*. [15]. Introducing a transformation as $\eta = kx + wt$ and $p = 2$, we can covert equation (6) into ordinary differential equations

$$wu' = ku' + 2akuu' - \beta k^2 wu''' \tag{7}$$

Where:
the prime denotes the derivative with respect to η. The solution of the equation (7) can be expressed in the form of equation (6).

$$u(\eta) = \frac{a_c \exp(c\eta) + \ldots + a_{-d}\exp(-d\eta)}{b_p \exp(p\eta) + \ldots + b_{-q}\exp(-q\eta)}$$

To determine the value of c and p, we balance the linear term of highest order of equation (7) with the highest order nonlinear term.

$$u''' = \frac{c_1\exp[(7p + c)\eta]) + \ldots}{c_2\exp[(8p)\eta]) + \ldots} \tag{8}$$

and

$$uu' = \frac{c_3\exp[(p + 2c)\eta]) + \ldots}{c_4\exp[(3p)\eta]) + \ldots} = \frac{c_3\exp[(6p + 2c)\eta]) + \ldots}{c_4\exp[(8p)\eta]) + \ldots} \tag{9}$$

Where:
c_t are determined coefficients only for simplicity; balancing the highest order of exp-function in (8) and (9), we have.

$$7p = c = 6p + 2c \tag{10}$$

Which in turn gives

$$p = c \tag{11}$$

To determine the value of d and q, we balance the linear term of lowest order of equation (7) with the lowest order non-linear term.

$$u''' = \frac{+\ldots d_1\exp[(-d - 7q)\eta])}{+\ldots d_2\exp[(-8q)\eta])} \tag{12}$$

and

$$uu' = \frac{+\ldots d_3\exp[(-2d - q)\eta])}{+\ldots d_4\exp[(-3q)\eta])} = \frac{+\ldots d_3\exp[(-2d - 6q)\eta])}{+\ldots d_4\exp[(-8q)\eta])} \tag{13}$$

Where:
d_t are determined coefficients only for simplicity. Now, balancing the lowest order of exp-function in (12) and (13), we have

$$-2d - 6q = -d - 7q \tag{14}$$

Which in turn gives

$$q = d \tag{15}$$

We can freely choose the values of c and d, but we will illustrate that the final solution does not strongly depend upon the choice of values of c and d. For simplicity, we set $p = c = 1$ and $q = d = 1$, then the trial solution, equation (6) reduces to.

$$u(\eta) = \frac{a_1 \exp(\eta) + a_0 + a_{-1}\exp(-\eta)}{b_1 \exp(\eta) + b_0 + b_{-1}\exp(-\eta)} \tag{16}$$

Substituting equation (16) into equation (7) we have

$$\frac{1}{A}[a_3 \exp(3\eta) + c_2\exp(2\eta) + c_1\exp(\eta) + c_0 + c_{-1}\exp(-\eta) + c_{-2} \exp(-2\eta) + c_{-3} \exp(-3\eta)] = 0 \tag{17}$$

Equating the coefficients of (ηn) to be zero, we obtain

$$\{c_{-3} = 0, c_{-2} = 0, c_{-1} = 0, c_{-0} = 0, c_{-1} = 0, c_{-2} = 0, c_{-3} = 0\} \tag{18}$$

Solution of (18) will yield

$$a_{-1} = -a_1 b_0^2 + a_0 b_0,\ b_{-1} = 0,\ w = \frac{w}{\beta k^2 - 1} \tag{19}$$

We, therefore, obtained the following generalized solitary solution $u(x,t)$ of equation (6).

$$u(x,t) = \frac{(-a1b_0^2 + a_0 b_0)e(-kx - \frac{k}{\beta k^2 - 1}t) + a_0 + a_1 e(kx + \frac{k}{\beta k^2 - 1}t)}{b_0 + b_1 e(kx + \frac{k}{\beta k^2 - 1}t)} \tag{20}$$

Where: a_1, b_0, b_1, a_0, k and β are real numbers.

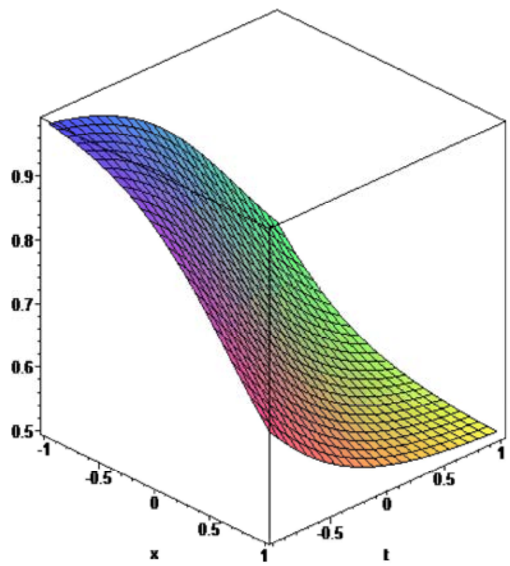

Fig. 1: Soliton solution for the problem.(Eq.(20))

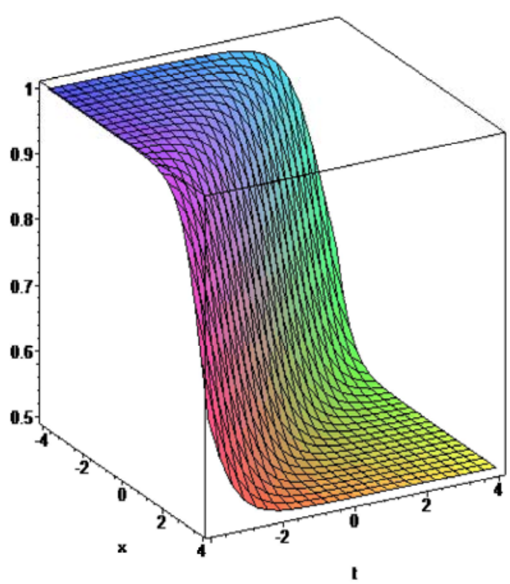

Fig. 2: Soliton solution for the problem.(Eq.(22))

$$u(x,t)=\frac{(-a_1b_0^2+a_0b_0+a_1)\cosh(kx+\frac{k}{\beta k^2-1})+(-a_1b_0^2+a_0b_0+a_1)\sinh(kx+\frac{k}{\beta k^2-1})+a_0h}{b_1\cosh(kx+\frac{k}{\beta k^2-1})+b_1\sinh(kx+\frac{k}{\beta k^2-1})} \quad (21)$$

Soliton solution of equation (6), when $b_1 = 2$, $k = 2$, $\beta = 1/2$, $b_0 = 1$, $a_0 = 1$, is

$$u(x,t)=\frac{\cosh(2x+2t)+\sinh(2x+2t)+1}{2\cosh(2x+2t)+2\sinh(2x+2t)+1} \quad (22)$$

CONCLUSION

The exp-function method has been used to obtain generalized solitonary solutions of the generalized regularized long-wave equation. This method can also be extended to other NLEEs. The Exp-function method is a promising and powerful new method for NLEEs arising in mathematical physics. Its applications are worth further studying.

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